Math 2200

4.1 Graphical Solutions of Quadratic Equations

A **quadratic equation** is a second-degree equation with standard form $ax^2 + bx + c = 0$, where $a \neq 0$.

The **roots** of a quadratic equation are the solutions to that equation.

The **zeros** of a function are the values of x for which $f(x) = 0$.

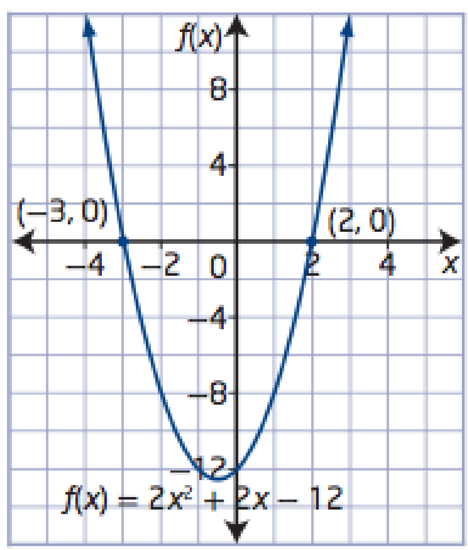
It is important for students to distinguish between the terms roots, zeros and x-intercepts, and to use the correct term in a given situation. The x-intercepts of the graph or the zeros of the quadratic function correspond to the roots of the quadratic equation.

You would:

- find the roots of the equation $x^2 - 7x + 12 = 0$
- find the zeros of $f(x) = x^2 - 7x + 12$
- determine the x-intercepts of $y = x^2 - 7x + 12$

In each case they are solving $x^2 - 7x + 12 = 0$ and arriving at the solution $x = 3$ or $x = 4$.

Example 1:
What are the zeros of the quadratic function $f(x) = 2x^2 + 2x - 12$?



$(-3, 0), (2, 0)$

or

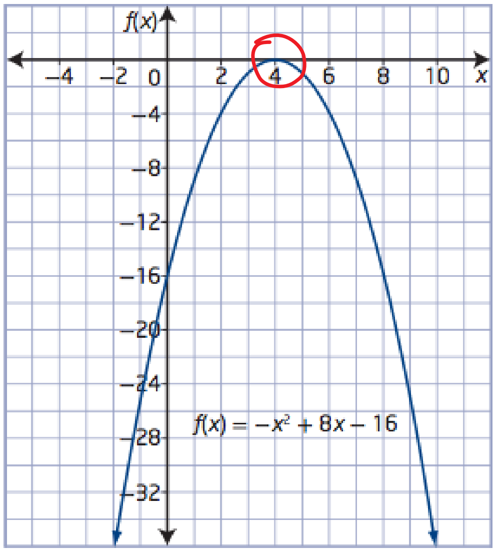
$x = -3, x = 2$

Example 2:

What are the roots of the equation $-x^2 + 8x - 16 = 0$?

We can either create our own graph by creating a table of values, or we can use our calculators to find the solution.

x	$f(x)$
0	-16
1	-9
2	-4
3	-1
4	0
5	-1
6	-4
7	-9
8	-16



We can see that the vertex is (4,0).

So, the root of the equation is x = 4. Notice there is only a single root because the vertex is the intersection point.

Another way of conceptualizing this topic is to think of a quadratic equation equal to 0 as two separate functions. One a quadratic and the other a horizontal line $y = 0$.

For example:

$$2x^2 + 3x - 7 = 0$$

Can be thought of as two functions, namely:

$y = 2x^2 + 3x - 7$; $y = 0$

The solution to these two equations is where they intersect, which is always the line $y = 0$ or also called the x-axis.

Graphical Solutions Using Technology

We can also use technology such as graphing calculators or computer software to solve quadratic equations.

Example 3:
Solve $2x^2 + x = -2$ by graphing.

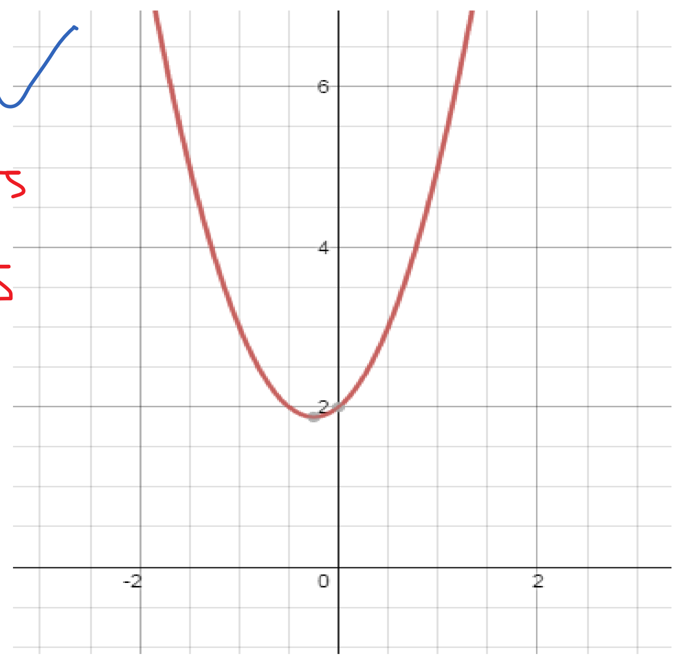
First, rearrange the equation so that it is in the form $ax^2 + bx + c = 0$.

$$2x^2 + x + 2 = 0$$

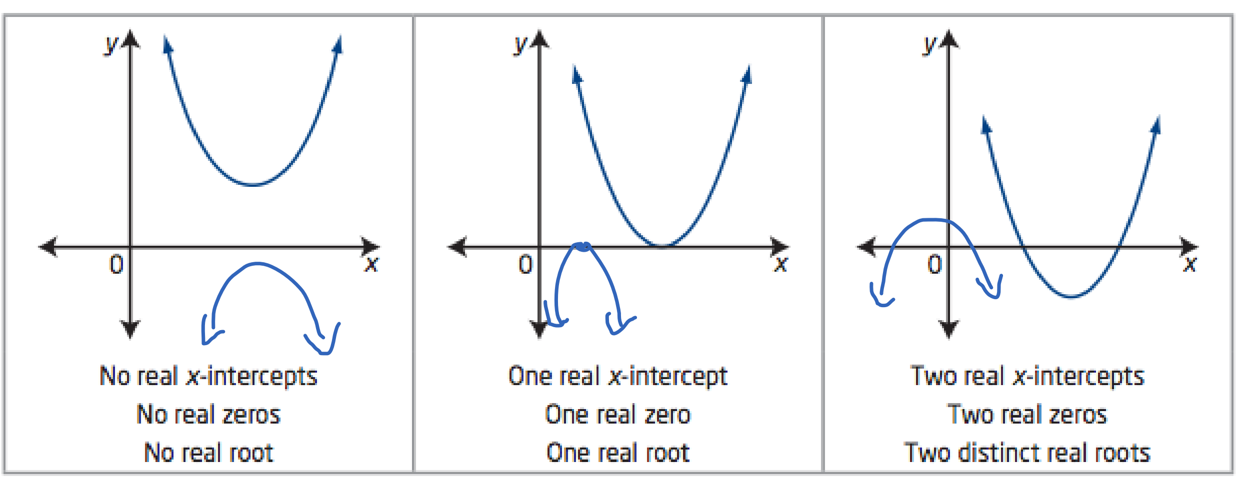
Replacing 0 with y and using a Desmos Graphing we get:

What does this mean?

- no solution(s) ✓
- no real roots
- imaginary roots



Recall from Chapter 3:



Example 4:
Two numbers have a sum of of 7 and a product of 12.

(A) What single variable equation in the form of $ax^2 + bx + c = 0$ can be used to represent the product of the two numbers?

x & y

① $x + y = 7$

② $x \cdot y = 12$

Solve ① for y.

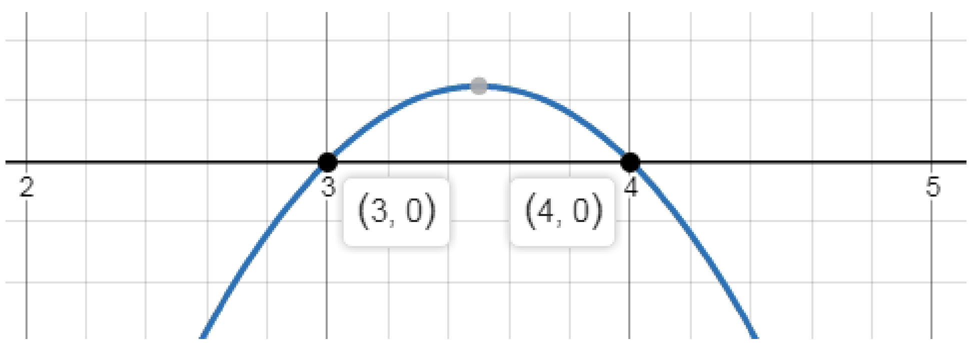
$\rightarrow y = -x + 7$

Sub ① into ②

$x(-x+7) = 12$

$-x^2 + 7x - 12 = 0$

(B) Determine the numbers by graphing the corresponding quadratic function.



roots: (3, 0), (4, 0)

or

$x = 3$, $x = 4$

Textbook Questions: page 215 - 217, # 1, 2, 3, 5, 6, 7, 8, 10, 13