Math 2200

4.2 Factoring Quadratic Equations

Review of Factoring

Type I Trinomial: $x^2 + bx + c$
We simply need two numbers that multiply to give us c and add to give us b. We usually refer to this as the **Product/Sum** method.

Example 1:

$$x^2 + x - 6 = 0$$

6
1, 6
2, 3

$(x-2)(x+3)=0$
$x-2=0, x+3=0$
$x=2, x=-3$

What relationship did you notice between the factored form of a quadratic equation and its roots? opposite sign.

Type II Trinomial: $ax^2 + bx + c$, $a \neq 1$, $a \neq 0$
We typically use **decomposition** to factor this type of quadratic. The process involves multiplying $a \cdot c$ and choosing two factors of this product that can be added to make b.

Example 2:

$$2x^2 - 9x - 5 = 0$$

10
1, 10
2, 5

$(2x^2+x)(-10x-5)=0$
$x(2x+1)-5(2x+1)=0$
$(2x+1)(x-5)=0$
$2x+1=0, x-5=0$
$\frac{2x}{2}=\frac{-1}{2}, x-5=0$
$x=-1/2, x=5$

Check: Are the binomials the same?

Difference of Squares:
These are quadratics of the form $a^2x^2 - b^2$. If you can recognize this pattern, factoring becomes quite easy.

Example 3:

$$9x^2 - 49 = 0$$

$(3x+7)(3x-7)=0$

$3x+7=0$, $3x-7=0$

$3x=-7$, $3x=7$

$x=-7/3$, $x=7/3$

Example 4:
Determine two values of n that will allow the polynomial $25x^2 + nx + 49$ to be a perfect square trinomial.

$25x^2+nx+49$

$\sqrt{25} = 5$

$\sqrt{49} = 7$

$2(5\times7)=70$

or -70

Example 5:
List all possible binomial factors of the following expression $x^2 + bx + 24$ that can be factored.

$\underline{c=24}$

1, 24

2, 12

3, 8

4, 6

$1+24=25$

$-1+-24=-25$

$2+12=14$

$-2+-12=-14$

$3+8=11$

$-3+-8=-11$

$4+6=10$

$-4+-6=-10$

Example 6:

Answer the following:

(A) Find the zeros of $f(x) = 2x^2 + 5x - 7$

$2x^2 + 5x - 7 = 0$

14

$1, 14$

$2, 7$

$(2x^2 - 2x)(+7x - 7)$

$2x(x-1) + 7(x-1) = 0$

$(x-1)(2x+7) = 0$

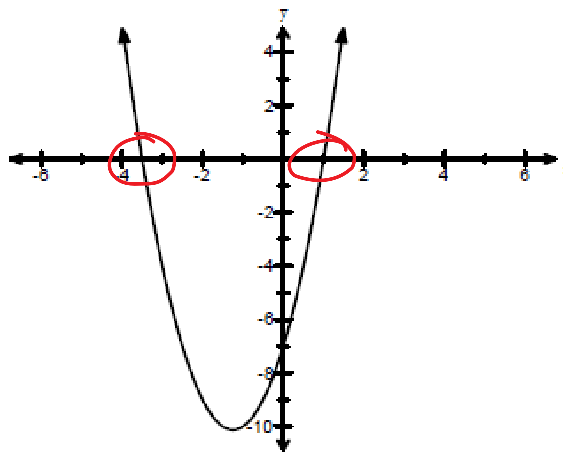
$x - 1 = 0, \quad 2x + 7 = 0$

$x = 1, \quad 2x = -7$

$x = -7/2$

$(1, 0), \left(-\frac{7}{2}, 0\right)$

(B) Identify the x-intercepts of the graph.



$x = -3.5, \quad x = 1$

(C) Find the roots of $2x^2 + 5x - 7 = 0$.

$x = -\frac{7}{2}, \quad x = 1$

(D) What do you notice about the answers to the above questions?

They are all the same.

Factoring Polynomials Having a Quadratic Pattern

Recognizing patterns in mathematics is very important. Sometimes a new problem can be solved using techniques typically used for other situations. Take the following polynomial. How can you factor it? Notice that it has the pattern of a typical quadratic equation.

$$3(x+2)^2 - 13(x+2) + 12$$

You could multiply it all out, and work backwards, but it is easier to substitute in another variable. Substitution of a variable for a common element in an algebraic expression is a common technique used to organize and simplify.

Example 7:

$$3(x+2)^2 - 13(x+2) + 12$$

Method 1:

Substitution

Let $r = x+2$

$3r^2 - 13r + 12$

36
1, 36
2, 18
3, 12
4, 9
6, 6

$(3r^2 - 9r)(-4r + 12)$

$3r(r-3) - 4(r-3)$

$(r-3)(3r-4)$

$[(x+2)-3][3(x+2)-4]$

$(x-1)(3x+6-4)$

$(x-1)(3x+2)$

$x = 1, \; x = -\frac{2}{3}$

Method 2:

Expansion

$$3(x+2)^2-13(x+2)+12$$

$$=3(x^2+4x+4)-13x-26+12$$

$$=3x^2+12x+12-13x-14$$

$$3x^2-x-2$$

$$\frac{6}{1,6}$$

$2,3$

$$(3x^2-3x)(+2x-2)$$

$$3x(x-1)+2(x-1)$$

$$(x-1)(3x+2)$$

$$x-1=0, \quad 3x+2=0$$

$$x=1, \quad \frac{3x}{3}=\frac{-2}{3}$$

$$x=-\frac{2}{3}$$

Which do you prefer?

Example 8:
Notice that both terms are perfect squares. So, this is a difference of squares. We just need the square roots of each term.

$$9(2t+1)^2 - 4(s-2)^2$$

Let $r = 2t+1$
$w = s-2$

$$9r^2 - 4w^2$$
$$= (3r+2w)(3r-2w)$$
$$= [3(2t+1)+2(s-2)][3(2t+1)-2(s-2)]$$
$$= (6t+3+2s-4)(6t+3-2s+4)$$
$$= (6t+2s-1)(6t-2s+7)$$

Problems Involving Fractions
The best way to eliminate fractions in quadratic equations is to multiply by a common denominator. This works because the right hand side of the equation is equal to zero and will remain zero once the workings are completed.

Example 9:

$$\frac{1}{2}x^2 - x - 4 = 0$$

$$2\cdot\left(\frac{1}{2}x^2 - x - 4\right) = 2\cdot(0)$$
$$x^2 - 2x - 8 = 0$$
$$(x+2)(x-4) = 0$$
$$x = -2,\ x = 4$$

Word Problems Involving Roots/Zeros

Just like max/min problems, we can create our own quadratic functions using given information and find the zeros of that function. The process is similar until the end. Instead of finding the vertex you find the roots by factoring.

Example 10:

The length of an outdoor lacrosse field is 10 m less than twice the width. The area of the field is 6600 m^2. Determine the dimensions of the outdoor lacross field.

$\boxed{6600 m^2}$

$l \cdot w = A$

① $l \cdot w = 6600$

② $l = 2w - 10$

Sub ② into ①

$(2w-10)w = 6600$

$\frac{2w^2 - 10w - 6600}{2} = \frac{0}{2}$

$w^2 - 5w - 3300 = 0$

$(w+55)(w-60) = 0$

$w+55=0, \quad w-60=0$

$\cancel{w=-55}, \quad \boxed{w = 60m}$

$l = 2(60) - 10$

$l = 120 - 10$

$l = 110m$

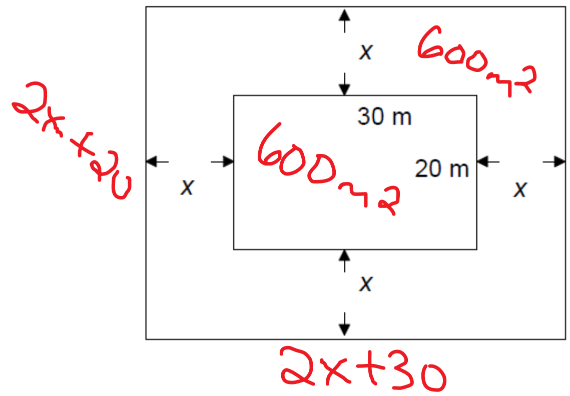
The dimensions of the lacross field are 60m X 110m.

Example 11:

A rectangular swimming pool has length 30 m and width 20 m. There is a deck of uniform width surrounding the pool. The area of the pool is the same as the area of the deck. Write a quadratic equation to model this situation and use it to determine the width of the deck.

total area: $1200m^2$

$l \cdot w = A$



$(2x+20)(2x+30) = 1200$

$4x^2 + 60x + 40x + 600 - 1200 = 0$

$\frac{4x^2 + 100x - 600}{4} = \frac{0}{4}$

$x^2 + 25x - 150 = 0$

$(x+30)(x-5) = 0$

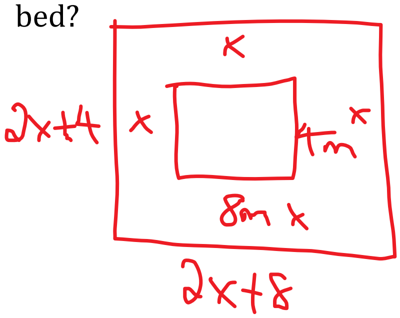
$x = \cancel{-30}, \; x = 5m$

The deck is 5m wide.

Your turn:

Example 12

A rectangular lawn measuring 8 m by 4 m is surrounded by a flower bed of uniform width. The combined area of the lawn and flower bed is 140m^2 . What is the width of the flower bed?



$$l \cdot w = A$$

$$(2x+4)(2x+8) = 140$$

$$4x^2 + 24x + 32 - 140 = 0$$

$$\frac{4x^2 + 24x - 108}{4} = \frac{0}{4}$$

$$x^2 + 6x - 27 = 0$$

$$(x-3)(x+9) = 0$$

$$\boxed{x = 3}, \ \cancel{x = -9}$$

The flower bed is 3m wide.

Other Types of Word Problems

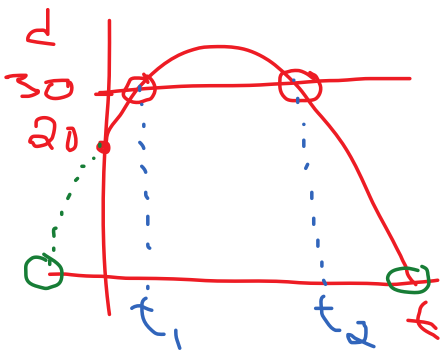
Example 13

A diver's path when diving off a platform is given by $d = -5t^2 + 15t + 20$, where d is the distance above the water, in feet, and t is the time from the beginning of the dive, in seconds.

(A) How high is the diving platform?

20ft.

(B) When is the diver 30 feet above the water?



$30 = -5t^2 + 15t + 20$

$5t^2 - 15t - 20 + 30 = 0$

$\frac{5t^2 - 15t + 10}{5} = \frac{0}{5}$

$t^2 - 3t + 2 = 0$

$(t-1)(t-2) \rightarrow t=1, t=2$

(C) When does the diver enter the water?

$0 = -5t^2 + 15t + 20 \rightarrow \cancel{t=-1}, t=4$

$\frac{5t^2 - 15t - 20}{5} = \frac{0}{5}$

$t - 3t - 4 = 0$

$(t+1)(t-4)$

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Textbook Questions: Page 229 – 233 #1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 20, 21, 23, 24

#14. two consecutive, even integers

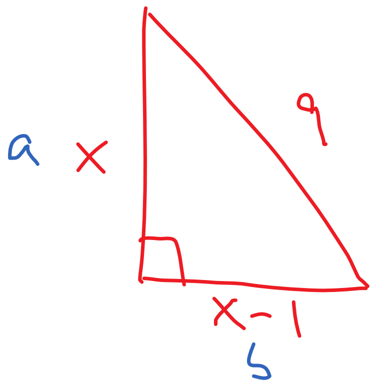
$x, x+2$

p232

20.

$a^2 + b^2 = c^2$

$x^2 + (x-1)^2 = 24^2$



$x + x - 2x + 1 = 841$

$\frac{2x^2 - 2x - 840}{2} = \frac{0}{2}$

$x^2 - x - 420 = 0$

$(x + 20)(x - 21) = 0$

$\cancel{x = -20}, x = 21$

legs are: 21 cm

21-1 = 20 cm

29 cm