Math 2200

4.3 Solving Quadratic Equations By Completing the Square

Recall from Chapter 3, completing the square involves adding a value to and subtracting a value from a quadratic polynomial so that it contains a perfect square trinomial. You can then rewrite this trinomial as the square of a binomial. In other words, rewrite it in vertex form.

First, let's look at an easier example to demonstrate the concept.

Example 1:

$$x^2 - 9 = 0$$

$(-3)^2 = 9$

$\therefore \sqrt{9} = -3$

$x^2 = 9$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

$x = 3, x = -3$

principle square root

secondary square root

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = -3, x = 3$

Example 2:

$$(x-1)^2 - 49 = 0$$

$(x-1)^2 = 49$

$\sqrt{(x-1)^2} = \sqrt{49}$

$x - 1 = \pm 7$

$x - 1 = 7$, $x - 1 = -7$

$x = 7 + 1$, $x = -7 + 1$

$x = 8$, $x = -6$

Now let's combine completing the square with solving for the roots of a quadratic equation.

Example 3:

$(x+3)(x+7)=0$

$x=-3,\ x=-7$

$$x^2 + 10x + 21 = 0$$

$(x^2+10x)+21=0$

$(x^2+10x+25)+21-25=0$

$(x+5)^2-4=0$

$(x+5)^2=4$

$\sqrt{(x+5)^2}=\sqrt{4}$

$x+5=\pm 2$

$x+5=2$

$x=2-5$

$x=-3$

$x+5=-2$

$x=-2-5$

$x=-7$

Example 4:

$$y = 3x^2 - 12x - 9$$

$(3x^2-12x)-9=0$

$3(x^2-4x)-9=0$

$3(x^2-4x+4)-9-12=0$

$3(x-2)^2-21=0$

$\frac{3(x-2)^2}{3}=\frac{21}{3}$

$\sqrt{(x-2)^2}=\sqrt{7}$

$x-2=\pm\sqrt{7}$

$x-2=\sqrt{7}$

$\boxed{x=\sqrt{7}+2}$

$x-2=-\sqrt{7}$

$\boxed{x=-\sqrt{7}+2}$

Example 5:

$$y = 6x^2 + 24x + 17$$

$$(6x^2 + 24x) + 17 = 0$$

$$6(x^2 + 4x + 4) + 17 - 24 = 0$$

$$6(x+2)^2 - 7 = 0$$

$$\frac{6(x+2)^2}{6} = \frac{7}{6}$$

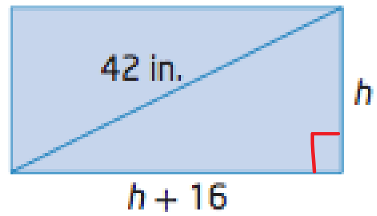
$$\sqrt{(x+2)^2} = \sqrt{\frac{7}{6}}$$

$$x + 2 = \pm\sqrt{\frac{7}{6}}$$

$$x = \sqrt{\frac{7}{6}} - 2$$

$$x = -\sqrt{\frac{7}{6}} - 2$$

Example 6:
A wide-screen television has a diagonal measure of 42 in. The width of the screen is 16 in. more than the height. Determine the dimensions of the screen, to the nearest tenth of an inch.



$$a^2+b^2=c^2$$

$$(h+16)^2+h^2=42^2$$

$$h^2+32h+256+h^2=1764$$

$$2h^2+32h+256-1764=0$$

$$(2h^2+32h)-1508=0$$

$$2(h^2+16h+64)-128-1508=0$$

$$2(h+8)^2-1636=0$$

$$\frac{2(h+8)^2}{2}=\frac{1636}{2}$$

$$\sqrt{(h+8)^2}=\sqrt{818}$$

$$h+8=\pm 28.6$$

$$h+8=28.6$$

$$h=28.6-8$$

$$\boxed{h=20.6}$$

$$h+8=-28.6$$

$$h=-28.6-8$$

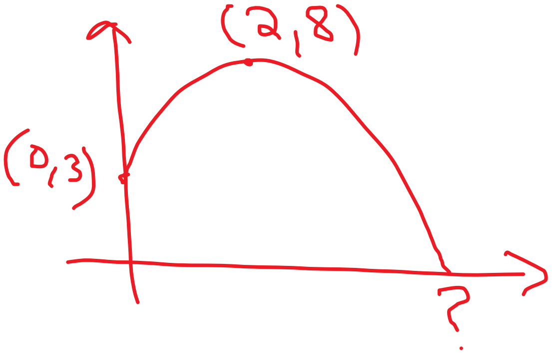
$$h=\cancel{-36.6}$$

$$w=20.6\text{ in}$$

$$l=20.6+16=36.6\text{ in}$$

Example 7

A baseball is thrown from an initial height of 3 m and reaches a maximum height of 8 m, 2 seconds after it is thrown. At what time does the ball hit the ground?



$y = a(x-p)^2 + q$

$3 = a(0-2)^2 + 8$

$3 - 8 = 4a$

$-5 = 4a$

$\frac{-5}{4} = \frac{4a}{4}$

$a = -\frac{5}{4}$ $\quad y = -\frac{5}{4}(x-2)^2 + 8$

$0 = -\frac{5}{4}(x-2)^2 + 8$

$\frac{5}{4}(x-2)^2 = 8$

$\cancel{\frac{4}{5}} \cdot \cancel{\frac{5}{4}}(x-2)^2 = \frac{4}{5} \cdot 8$

$\sqrt{(x-2)^2} = \sqrt{\frac{32}{5}}$

$x - 2 = \pm\sqrt{\frac{32}{5}}$

$\rightarrow x - 2 = \pm 2.5$

$x = 2.5 + 2 = \boxed{4.5}$

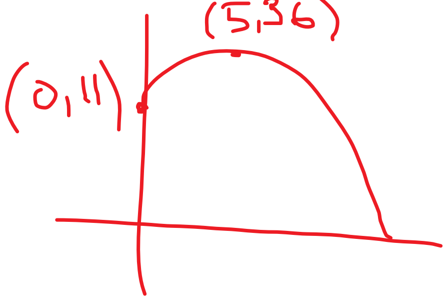
$x = -2.5 + 2 = \cancel{-0.5}$

The ball hits the ground at 4.5 s.

Example 8

A ball is thrown from a building at an initial height of 11 metres and reaches a maximum height of 36 metres, 5 seconds after it is thrown.

(A) Write a quadratic equation which models this situation.



$$y = a(x-p)^2 + q$$

$$11 = a(0-5)^2 + 36$$

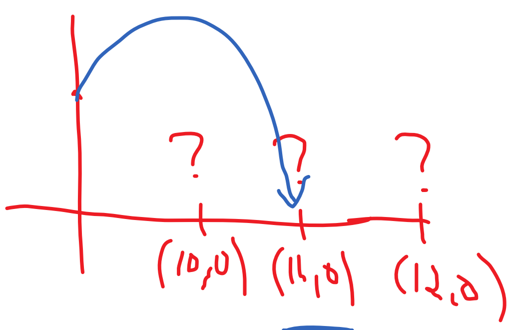
$$11 - 36 = 25a$$

$$\frac{-25}{25} = \frac{25a}{25}$$

$$a = -1$$

$$y = -(x-5)^2 + 36$$

(B) Three targets are placed at different locations on the ground. One is at (10,0), another at (11,0) and a final target is placed at (12, 0). Which target does the ball hit?



$$-(x-5)^2 + 36 = 0$$

$$\frac{-(x-5)^2}{-1} = \frac{-36}{-1}$$

$$\sqrt{(x-5)^2} = \sqrt{36}$$

$$x - 5 = \pm 6$$

$$x = 6 + 5 \quad , \quad x = -6 + 5$$

$$\boxed{x = 11} \quad , \quad \cancel{x = -1}$$

The ball lands on the target placed at (11,0).

Textbook Questions: Page 240 – 243 #2, 3 (a, b, c), 4 (a), 5, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 19