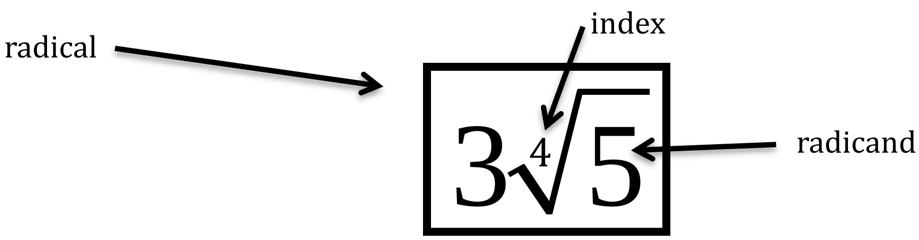
Math 2200

5.1 Working With Radicals

Recall from Level 1, radicals:



Radicals with the same index and radicand are called **Like Radicals.**

Like Radicals	**Unlike Radicals**
$5\sqrt{7}$ and $-\sqrt{7}$	$2\sqrt{5}$ and $5\sqrt{3}$
$\frac{2}{3}\sqrt[3]{5x^2}$ and $\sqrt[3]{5x^2}$	$\sqrt[4]{5a}$ and $\sqrt[5]{5a}$

When adding and subtracting radicals, only like radicals can be combined. However, you can multiply and divide radicals that have the same index but different radicands.

Principal and Secondary Square Roots

Every positive number has two square roots. For example, the square root of 49 is 7 since $7^2 = 49$. Likewise $(-7)^2 = 49$ so -7 is also a square root of 49. The value $\sqrt{49} = 7$ is called the **principal square root** and $\sqrt{49} = -7$ is the **secondary square root**.

Even Index vs Odd Index

If a radical has an even index, the radicand must be non-negative. If a radical has an odd index, the radicand can be any real number, including negative numbers. For example,

$\sqrt{9}$

$= \pm 3$

$\sqrt{-16}$

$= \sqrt{16}\sqrt{-1}$

$= \pm 4i$

$\sqrt[3]{-27}$

$= (-3)$

Converting Mixed Radicals to Entire Radicals
To do this, simply raise the coefficient outside the radical to the same power as the index of the radical and at the same time take the root. Then combine radicands and simplify.

Example 1:

(A) $7\sqrt{2}$

$=\sqrt{7^2}\cdot\sqrt{2}$

$=\sqrt{49}\sqrt{2}$

$=\sqrt{98}$

(B) $a^4\sqrt{a}$

$\sqrt{(a^4)^2}\sqrt{a}$

$=\sqrt{a^8}\sqrt{a^1}$

$=\sqrt{a^9}$

(C) $5b\sqrt[3]{3b^2}$

$=\sqrt[3]{(5b)^3}\sqrt[3]{3b^2}$

$=\sqrt[3]{125b^3}\sqrt[3]{3b^2}$

$=\sqrt[3]{375b^5}$

Radicals in Simplest Form
A radical is in simplest form if the following are true:

- The radicand does not contain a fraction or any factor which could be removed.
- The radical is not part of the denominator of a fraction.

$\sqrt{18}$ - is not in simplest form since 18 has 9 as a factor, which is a perfect square.

$\sqrt{18}=\sqrt{9}\sqrt{2}=3\sqrt{2}$

$3\sqrt{2}$ – is in simplest form since there are no perfect squares that divide into 2

Reducing Radicals and Expressing Entire Radicals as Mixed Radicals

There are a couple methods for reducing radicals. The method you choose often depends on the type of radical.

Example 2:

(A) $\sqrt{32}$

$\sqrt{32}$

$4 \cdot 8$

$(2 \cdot 2)(2 \cdot 2)2$

$\sqrt{\cancel{2^2} \cdot \cancel{2^2} \cdot 2}$

$= 2 \cdot 2\sqrt{2}$

$= 4\sqrt{2}$

$\sqrt{32}$

$= \sqrt{16}\sqrt{2}$

$= 4\sqrt{2}$

(B) $\sqrt[4]{c^9}$

$\sqrt[4]{(c \cdot c \cdot c \cdot c)\cdot(c \cdot c \cdot c \cdot c)c}$

$= \sqrt[\cancel{4}]{c^{4} \cdot c^{4}\, c}$

$= c \cdot c\sqrt[4]{c}$

$= c^2\sqrt[4]{c}$

Recall: $\sqrt[n]{x} = x^{\frac{1}{n}}$

$\sqrt[4]{c^9}$

$= (c^9)^{\frac{1}{4}}$

$= c^{\frac{9}{4}}$

$= c^{\frac{8}{4}+\frac{1}{4}}$

$= c^{\frac{8}{4}} \cdot c^{\frac{1}{4}}$

$\rightarrow c^2\sqrt[4]{c}$

(C) $\sqrt{48y^5}$

$= \sqrt{48}\sqrt{y^5}$

$= \sqrt{16}\sqrt{3}\,(y^5)^{\frac{1}{2}}$

$= 4\sqrt{3}\,y^{\frac{5}{2}}$

$= 4\sqrt{3}\,y^{\frac{4}{2}}y^{\frac{1}{2}}$

$= 4\sqrt{3}\,y^2\sqrt{y}$

$\rightarrow = 4y^2\sqrt{3y}$

Example 3:
Compare and order the following radicals from least to greatest.

5	1	2	4	3
$4(13)^{\frac{1}{2}}$	$8\sqrt{3}$	14	$\sqrt{202}$	$10\sqrt{2}$
$= 4\sqrt{13}$	$= \sqrt{8^2}\sqrt{3}$	$= \sqrt{14^2}$		$= \sqrt{10^2}\sqrt{2}$
$= \sqrt{4^2}\sqrt{13}$	$= \sqrt{64}\sqrt{3}$	$= \sqrt{196}$		$= \sqrt{100}\sqrt{2}$
$= \sqrt{16}\sqrt{13}$	$= \sqrt{192}$			$= \sqrt{200}$
$= \sqrt{208}$				

$$8\sqrt{3},\ 14,\ 10\sqrt{2},\ \sqrt{202},\ 4(13)^{\frac{1}{2}}$$

Adding and Subtracting Radicals
As mentioned above, only like radicals can be added and subtracted. Radicals don't always appear to be like and might have to be reduced before an operation can be performed.

Example 4:
Simplify radicals and combine like terms.

(A) $\sqrt{50} + 3\sqrt{2}$

$= \sqrt{25}\sqrt{2} + 3\sqrt{2}$

$= 5\sqrt{2} + 3\sqrt{2}$

$= 8\sqrt{2}$

Think: $5x + 3x$

$= 8x$

(B) $-\sqrt{27}+3\sqrt{5}-\sqrt{80}-2\sqrt{12}$

$= -\sqrt{9}\sqrt{3}+3\sqrt{5}-\sqrt{16}\sqrt{5}-2\sqrt{4}\sqrt{3}$

$= -3\sqrt{3}+3\sqrt{5}-4\sqrt{5}-4\sqrt{3}$

$= -7\sqrt{3}-\sqrt{5}$

(C) $\sqrt{4c}-4\sqrt{9c},\ c\geq 0$

$= \sqrt{4}\sqrt{c}-4\sqrt{9}\sqrt{c}$

$= 2\sqrt{c}-4\cdot 3\sqrt{c}$

$= 2\sqrt{c}-12\sqrt{c}$

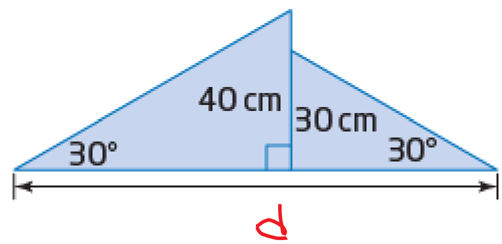
$= -10\sqrt{c}$

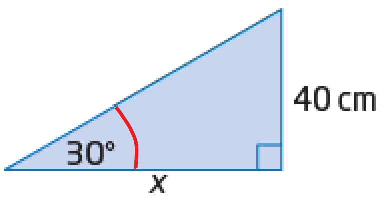
Example 5:
Consider the design shown for a skateboard ramp. What is the exact distance across the base? What is the approximate distance to the nearest tenth of a centimeter? Which would you use when building the ramp?

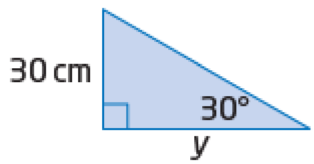
Recall: SOH CAH TOA

$\tan 30^\circ = \frac{40}{x}$

$\tan 30^\circ = \frac{30}{y}$







$d = x + y$

$x = \frac{40}{\tan 30^\circ}$

$\frac{1}{\sqrt{3}} = \frac{40}{x}$

$x = 40\sqrt{3}$

$\tan 30^\circ = \frac{30}{y}$

$\frac{1}{\sqrt{3}} = \frac{30}{y}$

$y = 30\sqrt{3}$

$= 40\sqrt{3} + 30\sqrt{3}$

$= 70\sqrt{3}$ cm.

(121.2 cm)

Example 6:

The voltage V required for a circuit is given by $V = \sqrt{PR}$ = where P is the power in watts and R is the resistance in ohms. How many more volts are needed to light a 100 -W bulb than a 75-W bulb if the resistance for both is 100 ohms? Solve the problem in exact and approximate form.

100 W

$V = \sqrt{100(100)}$

$V = 100$

75 W

$V = \sqrt{75(100)}$

$V = \sqrt{75}\sqrt{100}$

$V = \sqrt{25}\sqrt{3} \cdot 10$

$V = 5\sqrt{3} \cdot 10$

$V = 50\sqrt{3}$

$100 - 50\sqrt{3}$ V (13.4V) more volts are needed for the 100 W bulb.

Textbook Questions: page 278 – 281, #4, 5 (a) (c), 6 (a) (c), 9, 10, 11, 13, 15, 17, 19