**Math 2200**

# 5.2 Multiplying and Dividing Radical Expressions

**Multiplying Radicals**

When multiplying radicals, multiply the coefficients and multiply the radicands. You can only multiply radicals if they have the same index.

In general, $(m\sqrt[k]{a})(n\sqrt[k]{b}) = mn\sqrt[k]{ab}$ where $k$ is a natural number, and $m$, $n$, $a$, and $b$ are real numbers. If $k$ is even, then $a \geq 0$ and $b \geq 0$.

**Example 1:**

(A)

$$(-3\sqrt{2x})(4\sqrt{6}), x \geq 0$$

$$= (-3 \cdot 4)\sqrt{2x \cdot 6}$$
$$= -12\sqrt{12x}$$
$$= -12\sqrt{4}\sqrt{3x}$$
$$= -12 \cdot 2\sqrt{3x}$$
$$= -24\sqrt{3x}$$

(B)

$\sqrt{x}\sqrt{x} = \sqrt{x^2} = x$

$$7\sqrt{3}(5\sqrt{5} - 6\sqrt{3})$$

$$= (7\sqrt{3})(5\sqrt{5}) - (7\sqrt{3})(6\sqrt{3})$$
$$= 35\sqrt{15} - 42\sqrt{9}$$
$$= 35\sqrt{15} - 42 \cdot 3$$
$$= 35\sqrt{15} - 126$$

FOIL

(C)

$$(8\sqrt{2}-5)(9\sqrt{5}+6\sqrt{10})$$

$= 72\sqrt{10} + 48\sqrt{20} - 45\sqrt{5} - 30\sqrt{10}$

$= 72\sqrt{10} + 48\sqrt{4}\sqrt{5} - 45\sqrt{5} - 30\sqrt{10}$

$= 72\sqrt{10} + 96\sqrt{5} - 45\sqrt{5} - 30\sqrt{10}$

$= 42\sqrt{10} + 51\sqrt{5}$

(D)

$$9\sqrt[3]{2w}\left(\sqrt[3]{4w}+7\sqrt[3]{28}\right), w \geq 0$$

$= 9\sqrt[3]{8w^2} + 63\sqrt[3]{56w}$

$= 9\sqrt[3]{8}\sqrt[3]{w^2} + 63\sqrt[3]{8}\sqrt[3]{7w}$

$= 9 \cdot 2\sqrt[3]{w^2} + 63 \cdot 2\sqrt[3]{7w}$

$= 18\sqrt[3]{w^2} + 126\sqrt[3]{7w}$

**Dividing Radicals**

When dividing radicals, divide the coefficients and then divide the radicands. You can only divide radicals that have the same index.

In general, $\frac{m\sqrt[k]{a}}{n\sqrt[k]{b}} = \frac{m}{n} \cdot \sqrt[k]{\frac{a}{b}}$, $k$ is a natural number, and $m$, $n$, $a$, and $b$ are real numbers. $n \neq 0$ and $b \neq 0$. If $k$ is even, then $a \geq 0$ and $b > 0$.

**Example 2:**

$$\frac{4\sqrt[3]{6}}{2\sqrt[3]{3}}$$

$$= 2\sqrt[3]{2}$$

**Rationalizing the Denominator**

It's considered bad form to leave a radical in the denominator of a fraction. There are two methods we use to remedy this, depending on the type of expression that is in the denominator.

be ied

A monomial denominator can simply multiply by 1 in the form of that denominator over itself:

**Example 3:**

$$\frac{5}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{5\sqrt{3}}{2 \cdot 3}$$

$$= \frac{5\sqrt{3}}{6}$$

A binomial denominator requires a **conjugate** to remove any radicals. **Conjugates** are binomial factors whose product is a difference of squares. Since squaring a square root elimates a the radical and the middle terms cancel when expanding a difference of squares, we can use this property to remove an radical in a denominator.

**Example 4:**

$$\frac{5\sqrt{3}}{4-\sqrt{6}}$$

$$\frac{5\sqrt{3}}{(4-\sqrt{6})} \cdot \frac{(4+\sqrt{6})}{(4+\sqrt{6})}$$

$$= \frac{20\sqrt{3}+5\sqrt{18}}{16-6}$$

$$= \frac{20\sqrt{3}+5\sqrt{9}\sqrt{2}}{10}$$

$$= \frac{20\sqrt{3}+15\sqrt{2}}{10}$$

$$= \frac{4\sqrt{3}+3\sqrt{2}}{2}$$

$$(4-\sqrt{6})(4+\sqrt{6})$$

$$= 16+\cancel{4\sqrt{6}}-\cancel{4\sqrt{6}}-6$$

$$= 16-6$$

$$= 10$$

**Example 5:**

(A)

$$\frac{\sqrt{24x^2}}{\sqrt{3x}}, x > 0$$

$$= \sqrt{\frac{24x^2}{3x}}$$

$$= \sqrt{8x}$$

$$= \sqrt{4}\sqrt{2x}$$

$$= 2\sqrt{2x}$$

(B)

$\frac{4\sqrt{5n}}{3\sqrt{2}}, n \geq 0$

$$= \frac{4\sqrt{5n}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{4\sqrt{10n}}{3 \cdot 2}$$

$$= \frac{\cancel{4}^{2}\sqrt{10n}}{\cancel{6}_{3}} \rightarrow = \frac{2\sqrt{10n}}{3}$$

(C)

$\frac{4\sqrt{11}}{y\sqrt[3]{6}}, y \neq 0$

$$\frac{4\sqrt{11}}{y\sqrt[3]{6}} \cdot \frac{\sqrt[3]{6}\sqrt[3]{6}}{\sqrt[3]{6}\sqrt[3]{6}}$$

$$\sqrt[3]{6} \cdot \sqrt[3]{6} = \sqrt[3]{36}$$

$$= \frac{4\sqrt{11}\sqrt[3]{36}}{y \cdot 6}$$

$$= \frac{2\sqrt{11}\sqrt[3]{36}}{3y}$$

(D)

$$\frac{11}{\sqrt{5}+7} \cdot \frac{(\sqrt{5}-7)}{(\sqrt{5}-7)}$$

$$= \frac{11\sqrt{5}-77}{5-49}$$

$$= \frac{11\sqrt{5}-77}{-44}$$

$$= \frac{(\sqrt{5}-7)}{-4} \quad \frac{(-1)}{(-1)}$$

$$= \frac{-\sqrt{5}+7}{4}$$

$$= \frac{7-\sqrt{5}}{4}$$

(E)

$$\frac{2}{3\sqrt{5}-4}$$

$$\frac{2}{3\sqrt{5}-4} \quad \frac{(3\sqrt{5}+4)}{(3\sqrt{5}+4)}$$

$$= \frac{6\sqrt{5}+8}{9 \cdot 5-16}$$

$$= \frac{6\sqrt{5}+8}{29}$$

(F)

$$\frac{6}{\sqrt{4x}+1} \cdot \frac{(\sqrt{4x}-1)}{(\sqrt{4x}-1)}$$

$$= \frac{6\sqrt{4x}-6}{4x-1}$$

$$= \frac{6\sqrt{4}\sqrt{x}-6}{4x-1}$$

$$= \frac{(12\sqrt{x}-6)}{(4x-1)}$$

**Example 6:**

When asked to rationalize the denominator in the expression $\frac{4}{2+\sqrt{7}}$ your friend said he could just multiply the expression by $\frac{\sqrt{7}}{\sqrt{7}}$. Explain why this would not work.

$$\frac{4}{2+\sqrt{7}} \frac{(2-\sqrt{7})}{(2-\sqrt{7})}$$

$$= \frac{8-4\sqrt{7}}{4-7}$$

$$= \frac{8-4\sqrt{7}}{-3}$$

$$= \frac{-8+4\sqrt{7}}{3}$$

$$= \frac{4\sqrt{7}-8}{3}$$

**Textbook Questions:** page 289-293, #1 (a), (c), 2 (b), (d), 4, 5, 8, 9, 10, 11, 13, 15, 17, 20, 21, 22, 23, 24, 26