Math 2200

6.1A Rational Expressions

A **rational expression** is an algebraic fraction that can be written as the quotient of two polynomials, in the form $\frac{P(x)}{Q(x)}$, where $Q(x) \neq 0$. For example:

$$\frac{1}{x}, \quad \frac{y^2-1}{y^2+2x+1}, \quad \frac{m}{m+1}$$

Example 1:
Which of the following are rational expressions? Why?

$$\frac{4}{5}, \quad \frac{2y}{x}, \quad \frac{x^2-4}{x+1}, \quad \sqrt{5}, \quad 2\pi, \quad \frac{\sqrt{x}}{2y}$$

No Yes Yes No No Yes

Non-Permissible values
Just like radicals can have restrictions on what a variable can be because of even roots of negative numbers, rational expressions have **non-permissible values.** These are the values for a variable that makes an expression undefined. In a rational expression, this is a value that results in a denominator of zero.

To find the non-permissible values of a rational expression we can simply use logic to figure out which values make a denominator zero or we can solve the expression in the denominator for zero. This value is what the variable **cannot** be.

Example 2:

For each rational expression, find its non-permissible values:

(A) $\frac{x-7}{x-3}$, $x \neq 3$ or $x-3 \neq 0$

$x \neq 3$

(B) $\frac{3x}{x(2x-3)}$, $x \neq 0$ $2x-3 \neq 0$

$2x \neq 3$

$x \neq 3/2$

(C) $\frac{x-1}{3x^2-12}$ * Factor first

$= \frac{x-1}{3(x^2-4)}$

$= \frac{x-1}{3(x+2)(x-2)}$ $x+2 \neq 0$, $x-2 \neq 0$

$x \neq -2$, $x \neq 2$

(D) $\frac{2p-1}{p^2-p-12}$

$= \frac{2p-1}{(p+3)(p-4)}$ $p+3 \neq 0$, $p-4 = 0$

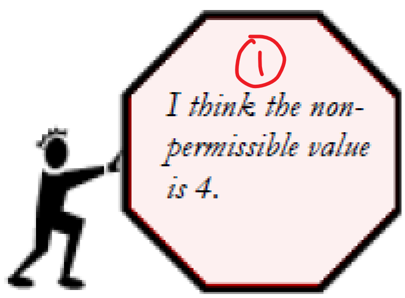
$p \neq -3$; $p \neq 4$

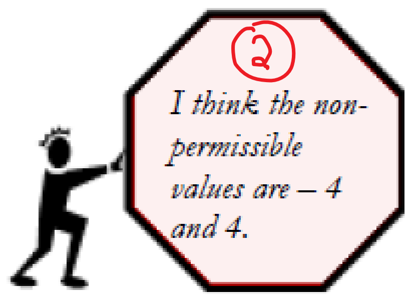
Example 3:
Write a rational expression for the following non-permissible values of 0, −2 and 3.

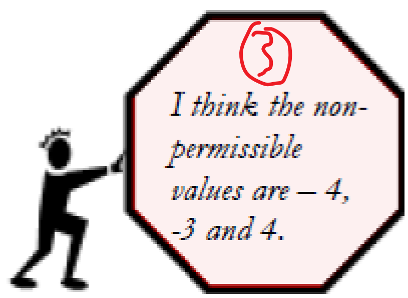
$$\frac{467852x^{27} - 30012x^{12} + x^{7}}{x(x+2)(x-3)}$$

Example 4:
From the Curriculum Guide:

What are the non-permissible values for $\frac{x+3}{x^2-16}$?







Who is correct? Justify your answer by solving the problem.

2

$$\frac{x+3}{(x+4)(x-4)}, \quad x \neq -4, 4$$

Equivalent Fractions

We can write a fraction that is equivalent to a given fraction by multiplying both the numerator and denominator by the same number.

Example 5:
Write a fraction that is equivalent to $\frac{2}{3}$.

$$\frac{2 \cdot 3}{3 \cdot 3} = \frac{6}{9}$$

When we multiply top and bottom of a fraction by the same value, we are essentially multiplying by 1. That is why the numerical value of the equivalent fractions remains the same, even though the two fractions may look different.

We can do something similar with rational expressions. Multiplying top and bottom of a rational expression by the same number will result in an equivalent rational expression. The same is true if we multiply top and bottom by some polynomials. However, if we multiply top and bottom by a polynomial that ends up changing the ~~restrictions~~ on our new rational expression, then the two expressions will NOT be equivalent. non-permissibles

Example 6:
Consider the rational expression $\frac{4}{x}$.

(A) Multiply top and bottom by 2. Did the restrictions change on the new rational expression? Is the new expression equivalent to the original one?

$$\frac{4}{x}, x \neq 0 \longrightarrow \frac{4 \cdot 2}{x \cdot 2} = \frac{8}{2x}, x \neq 0$$

1. Are top and bottom multiplied by the same value? ✓
Yes.

2. Do both expressions have the same non-permissibles? ✓
Yes. $\therefore$ Equivalent

(B) Multiply top and bottom by x. Did the restrictions change on the new rational expression? Is the new expression equivalent to the original one?

$\frac{4}{x}, x \neq 0 \longrightarrow \frac{4}{x} \cdot \frac{x}{x} = \frac{4x}{x^2}$

$x^2 \neq 0$

$\sqrt{x^2} \neq \sqrt{0}$

$x \neq 0$

1. Yes

2. Yes

$\therefore$ Equivalent

(C) Multiply top and bottom by $x - 1$. Did the restrictions change on the new rational expression? Is the new expression equivalent to the original one?

$\frac{4}{x}, x \neq 0 \longrightarrow \frac{4}{x} \cdot \frac{(x-1)}{(x-1)} = \frac{4(x-1)}{x(x-1)} = \frac{4x-4}{x^2-x}$

1. Yes

2. No

$\therefore$ Not Equivalent

$x \neq 0$ $\quad x - 1 \neq 0$

$x \neq 1$

Summary:

To determine whether two rational expressions are equivalent:

- The top and bottom of one expression must be multiplied by the same value to give the other expression.
- The two expressions must have the same ~~restrictions.~~ non-permissibles

Example 7:
Complete the following table:

Are the expressions equivalent?	Yes	No	Justify your choice
$\frac{x+3}{x-4}$ and $\frac{4x+12}{4x-16}$	✓		1. Top & bottom × 4 2. non-permissible $x \neq 4$
$\frac{5}{x-5}$ and $\frac{5x+25}{x^2-25}$		✓	2. Different non-permissibles
$\frac{x+2}{x-3}$ and $\frac{3x+6}{2x-6}$		✓	1. Top and bottom × different values

Example 8:

Angie thinks the expressions $\frac{x-3}{2x}$ and $\frac{(x-3)(x+1)}{2x(x+1)}$ are equivalent. Is she correct?

2. Different non-permissibles.

∴ Angie is incorrect.

Example 9:

Which expression is equivalent to $\frac{x-3}{x+2}$, $\neq -2$?

~~(A)~~ $\frac{x^2-3x}{x^2+2x}$

~~(B)~~ $\frac{6x-18}{x+2}$

~~(C)~~ $\frac{3x-3}{3x+2}$

(D) $\frac{4x-12}{4x+8}$

(A) $\frac{x(x-3)}{x(x+2)}$, $x \neq 0, x \neq -2$

(B) $\frac{6(x-3)}{x+2}$

(C) $\frac{3(x-1)}{3x+2}$

(D) $\frac{4(x-3)}{4(x+2)}$, $x \neq -2$

Textbook Questions: page 317, 318, #1, 2, 3, 4, 5