Math 2200

6.1B Simplifying Rational Expressions

Simplifying Rational Expressions

To simplify rational expressions, we need to find any common factors in the numerator and denominator. Recall from artithmetic, we can prime factorize any fraction to reduce.

Example 1:

$$\frac{9}{12}$$

$$= \frac{3 \cdot \cancel{3}}{2 \cdot 2 \cdot \cancel{3}}$$

$$= \frac{3}{2 \cdot 2}$$

$$= \frac{3}{4}$$

The process for reducing rational expressions is essentially the same. To simplify rational expressions, we need to find any common factors in the numerator and denominator.

Example 2:

Simplify:

$$\frac{m^3 t}{m^2 t^4}$$

Level I:

$$\frac{m^3 t}{m^2 t^4}$$

$$= m^{3-2} t^{1-4}$$

$$= m t^{-3}$$

$$= \frac{m}{t^3}$$

Level II

$$= \frac{\cancel{m} \cdot \cancel{m} \cdot m \cdot \cancel{t}}{\cancel{m} \cdot \cancel{m} \cdot \cancel{t} \cdot t \cdot t \cdot t}$$

$$= \frac{m}{t^3}$$

Example 3:
Simplify, and state the non-permissible values:

$$\frac{3x-6}{2x^2+x-10}$$

① Factor

$$\frac{3\cancel{(x-2)}}{\cancel{(x-2)}(2x+5)}$$

② Non-permissibles

$x-2 \neq 0$, $2x+5 \neq 0$

$x \neq 2$

$2x \neq -5$

$x \neq -5/2$

③ Simplify

$\frac{3}{2x+5}$, $x \neq -5/2, 2$

$2x^2+x-10$

$\frac{20}{1,20}$

$2,10$

$4,5$

$(2x^2-4x)(+5x-10)$

$2x(x-2)+5(x-2)$

$(x-2)(2x+5)$

Example 4:

Simplify, and state the non-permissible values:

$$\frac{16x^2 - 9y^2}{8x - 6y}$$

$$= \frac{(4x+3y)\cancel{(4x-3y)}}{2\cancel{(4x-3y)}}$$

$$4x - 3y \neq 0$$

$$= \frac{4x+3y}{2}, \quad x \neq \frac{3y}{4}$$

$$4x \neq 3y$$

$$x \neq \frac{3y}{4}$$

Common Mistakes:

When simplifying rational expressions, students often cancel terms rather than factors. For example, they may simplify:

$$\frac{x^2+x}{x^2-1}$$

$$=\frac{\cancel{x^2}+x}{\cancel{x^2}-1}$$

$$=\frac{x}{-1}$$

$$=-x$$

This is wrong. Cancelling a portion of the factor is incorrect. One way that helps students avoid this is to put brackets around all binomials. Students must then realize that a binomial can only be cancelled with the exact same binomial above or below it. Likewise a monomial can only be cancelled with the exact same monomial.

The correct solution is:

$$\frac{(x^2+x)}{(x^2-1)}$$

$$=\frac{x\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}$$

$x+1\neq 0$, $x-1\neq 0$

$x\neq -1$, $x\neq 1$

$$=\frac{x}{x-1},\ x\neq \pm 1$$

Another error occurs when students omit a numerator of 1 after the rational expression is simplified. For example:

$$\frac{\cancel{3}}{\cancel{6}x} = 2x$$

Even though the 3 divides into 6, there still has to be a numerator with 1 as the placeholder. The correct solution is:

$$\frac{\cancel{3}^{\,1}}{{}_{2}\cancel{6}x}$$

$$= \frac{1}{2x}, x \neq 0$$

Example 5:
Simplify and state the non-permissible values:

(A) $\frac{8x-12}{4x^2-6x}$

$$= \frac{{}^{2}\cancel{4}\cancel{(2x-3)}}{{}^{1}\cancel{2}x\cancel{(2x-3)}}$$

$$= \frac{2}{x}, x \neq 0, 3/2$$

$$2x - 3 \neq 0$$
$$2x \neq 3$$
$$x \neq 3/2$$

(B) $\frac{\cancel{8}}{\cancel{16}x}$ $\frac{1}{2}$

$$= \frac{1}{2x}, x \neq 0$$

Reversed Terms With a Difference
There is a shortcut when dealing with the following scenario:

$$\frac{(x-1)}{(1-x)}$$

$$= \frac{(x-1)}{(-x+1)}$$

$$= \frac{\cancel{(x-1)}}{-\cancel{(x-1)}}$$

$$= \frac{1}{-1}$$

* Example 6d.

$$= -1$$

Example 6:

$$\frac{8-2x^2}{2x-4}$$

$$= \frac{2(4-x^2)}{2(x-2)}$$

$$= \frac{-\cancel{2}\cancel{(2-x)}(2+x)}{\cancel{2}\cancel{(x-2)}}$$

$x-2 \neq 0$

$x \neq 2$

$$= -(2+x)$$

$$= -2-x, \; x \neq 2$$

Textbook Questions: page 318 - 321, #6, 7, 8, 11, 13, 15, 20 (a), 25