Math 2200

6.2 Multiplying and Dividing Rational Expressions

Multiplying Rational Expressions

What is the rule for multiplying fractions? Let's compare with arithmetic again:

$$\left(\frac{5}{8}\right)\left(\frac{4}{15}\right)$$

$$= \frac{5}{2 \cdot 2 \cdot 2} \cdot \frac{2 \cdot 2}{3 \cdot 5}$$

$$= \frac{1}{6}$$

Now with variables:

$$\left(\frac{4x^2}{3xy}\right)\left(\frac{y^2}{8x}\right)$$ think → $$\frac{4x^2y^2}{3xy8x}$$

$$= \frac{2 \cdot 2 \cdot x \cdot x}{3 \cdot x \cdot y} \cdot \frac{y \cdot y}{2 \cdot 2 \cdot 2x}$$

$$= \frac{y}{6}$$

Steps:

- Where possible, factor the numerators and denominators of both expressions.
- Cancel common factors.
- Determine the restrictions by calculating the non-permissible values.
- State the simplified answer along with restrictions.

Example 1:

(A) $\frac{4x^2-20x}{18x} \cdot \frac{30x}{x-5}$

$= \frac{4x(x-5)}{3\cancel{18x}} \cdot \frac{\cancel{30x}^{5}}{\cancel{(x-5)}}$

$= \frac{120x}{18}$

$= \frac{20x}{3}, x \neq 0, 5$

(B) $\frac{a^2-a-12}{a^2-9} \times \frac{a^2-4a+3}{a^2-4a}$

$= \frac{\cancel{(a-4)}(a+1)}{\cancel{(a+3)}\cancel{(a-3)}} \times \frac{(a-1)\cancel{(a-3)}}{a\cancel{(a-4)}}$

$= \frac{(a-1)}{a}, a \neq \pm 3, 0, 4$

(C) $\frac{x^2-9}{y^3-y} \times \frac{y^2-y}{x+3}$

$= \frac{(x+3)(x-3)}{y(y^2-1)} \times \frac{y(y-1)}{(x+3)}$

$= \frac{(x+3)(x-3)}{y(y+1)(y-1)} \times \frac{y(y-1)}{(x+3)}$

$= \frac{(x-3)}{(y+1)}, x \neq -3, y \neq 0, \pm 1$

Dividing Rational Expressions

Recall that when we divide fractions, we invert the denominator and multiply it to the numerator. This is how it works for rational expressions as well. The difference is we now have more than one denominator in the expression that we have to find non-permissible values for.

Steps:

- Keep the first fraction the same and multiply by the reciprocal of the second fraction.
- Where possible, factor the numerators and denominators of both expressions.
- Cancel common factors.
- Determine the restrictions by calculating the non-permissible values.
- State the simplified answer along with restrictions.

Example 2:

Dave stated that the permissible values for the quotient and the product of the expressions $\frac{x^2+4x+3}{x^2-16}$ and $\frac{x^2-2x-8}{x^2-7x+12}$ are the same. Do you agree or disagree with his statement. Justify you answer.

Product

$$\frac{x^2+4x+3}{x^2-16} \cdot \frac{x^2-2x-8}{x^2-7x+12}$$

$$= \frac{(x+1)(x+3)}{(x+4)(x-4)} \cdot \frac{(x-4)(x+2)}{(x-3)(x-4)}$$

$x \neq \pm 4, 3$

Quotient

$$\frac{x^2+4x+3}{x^2-16} \div \frac{x^2-2x-8}{x^2-7x+12}$$

$$\frac{(x+1)(x+3)}{(x+4)(x-4)} \cdot \frac{(x-3)(x-4)}{(x-4)(x+2)}$$

$x \neq \pm 4, 3, -2$

Example 3:

(A)

$$\frac{x^2-4}{x^2-4x} \div \frac{x^2+x-6}{x^2+x-20}$$

$$= \frac{x^2-4}{x^2-4x} \cdot \frac{x^2+x-20}{x^2+x-6}$$

denominator →

Non permissibles

$$\frac{\frac{x^2-4}{x^2-4x}}{\frac{x^2+x-6}{x^2+x-20}}$$

$$= \frac{(x+2)(x-2)}{x(x-4)} \cdot \frac{(x+5)(x-4)}{(x+3)(x-2)}$$

$$= \frac{(x+2)(x+5)}{x(x+3)}, x \neq 0, 4, -5, -3, 2$$

(B)

$$\frac{c^2-6c-7}{c^2-49} \div \frac{c^2+8c+7}{c^2+7c}$$

$$= \frac{c^2-6c-7}{c^2-49} \times \frac{c^2+7c}{c^2+8c+7}$$

$$= \frac{(c+1)(c-7)}{(c+7)(c-7)} \times \frac{c(c+7)}{(c+1)(c+7)}$$

$$= \frac{c}{c+7}, \quad c \neq \pm 7, 0, -1$$

(C)

$$\frac{2m^2-7m-15}{2m^2-10m} \div \frac{4m^2-9}{6} \times (3-2m)$$

$$= \frac{2m^2-7m-15}{(2m^2-10m)} \cdot \frac{6}{(4m^2-9)} \cdot \frac{(3-2m)}{1}$$

$2m^2-7m-15$ $\quad 30$, $\quad 3, 10$

$2m^2-10m+3m-15$

$2m(m-5)+3(m-5)$

$(m-5)(2m+3)$

$$= -\frac{(m-5)(2m+3)}{2m(m-5)} \cdot \frac{6 \cdot 3}{(2m+3)(2m-3)} \cdot \frac{(3-2m)}{1}$$

$$= \frac{-3}{m}, \quad m \neq 0, 5, \pm 3/2$$

$2m+3 \neq 0$

$2m \neq -3$

$m \neq -3/2$

(D)

$$\frac{3x+12}{3x^2-5x-12} \div \frac{12}{3x+4} \times \frac{2x-6}{x+4}$$

$3x^2 - 5x - 12 \quad 36$

$3x^2 - 9x + 4x - 12 \quad 9, 4$

$3x(x-3) + 4(x-3)$

$(x-3)(3x+4)$

$= \frac{3(x+4)}{(x-3)(3x+4)} \cdot \frac{(3x+4)}{12} \cdot \frac{2(x-3)}{(x+4)}$

$= \frac{6}{12}$

$= \frac{1}{2}, x \neq 3, -4/3, -4$

Textbook Questions: page 327 – 330, #1, 2, 4, 8, 14, 15, 16