Math 2200

6.3 Adding and Subtracting Rational Expressions

To begin this section we will take a look at adding and subtracting fractions in a different way:

Example 1:

Simplify the following.

(A) $\frac{6}{7}+\frac{2}{7}$

$= \frac{6+2}{7}$

$= \frac{8}{7}$

(B) $\frac{4}{5}-\frac{1}{5}$

$= \frac{4-1}{5}$

$= \frac{3}{5}$

(C) $\frac{1}{3}+\frac{7}{12}$ LCD: $2\cdot 2\cdot 3$

$= \frac{1}{3} + \frac{7}{2\cdot 2\cdot 3}$

$= \frac{2\cdot 2\cdot 1}{2\cdot 2\cdot 3} + \frac{7}{2\cdot 2\cdot 3}$

$= \frac{4}{2\cdot 2\cdot 3} + \frac{7}{2\cdot 2\cdot 3}$

$= \frac{4+7}{2\cdot 2\cdot 3}$

$= \frac{11}{2\cdot 2\cdot 3}$

$= \frac{11}{12}$

(D) $\frac{1}{3}+\frac{3}{4}$ LCD: $2\cdot 2\cdot 3$

$\frac{1}{3} + \frac{3}{2\cdot 2}$

$\frac{2\cdot 2\cdot 1}{2\cdot 2\cdot 3} + \frac{3\cdot 3}{3\cdot 2\cdot 2}$

$= \frac{4}{2\cdot 2\cdot 3} + \frac{9}{2\cdot 2\cdot 3}$

$= \frac{4+9}{2\cdot 2\cdot 3}$

$= \frac{13}{2\cdot 2\cdot 3}$

$= \frac{13}{12}$

Adding/Subtracting Rational Expressions

Steps:

- Reduce the rational expressions first if possible.
- Get a common denominator.
- Rewrite each fraction with equivalent fractions so that each one has the common denominator.
- Add/subtract the numerators.
- Keep the common denominator the same.
- Reduce/simplify the the answer if possible.
- State restrictions.

Example 2:
Complete the following table:

Rational Number	LCM	Rational Expression	LCM	Similarities
$\frac{4}{5}+\frac{3}{5}$	5	$\frac{6}{2x-1}+\frac{-2}{2x-1}$	$2x-1$	same denominator
$\frac{1}{5}-\frac{7}{15}$	$3\cdot 5$	$\frac{4x}{x-3}-\frac{5}{6x-18}$	$6(x-3)$	One denominator is a multiple of the other.
$\frac{7}{12}+\frac{3}{8}$	$2\cdot 2\cdot 2\cdot 3$	$\frac{2}{x^2-36}+\frac{4}{3x+18}$	$3(x+6)(x-6)$	Both denominators have a common factor.

Example 3:

$$\frac{3}{x+5}-\frac{1}{4x+20}$$

$$=\frac{3}{(x+5)}-\frac{1}{4(x+5)}$$

LCD: $4(x+5)$

$$=\frac{4\cdot 3}{4\cdot(x+5)}-\frac{1}{4(x+5)}$$

$$=\frac{12}{4(x+5)}-\frac{1}{4(x+5)}$$

$$=\frac{12-1}{4(x+5)}$$

$$=\frac{11}{4(x+5)},\ x\neq -5$$

Example 4:

$$\frac{x^2}{x+1} - \frac{1}{x-1} \quad LCD: (x+1)(x-1)$$

$$= \frac{(x-1)}{(x-1)} \frac{x^2}{(x+1)} - \frac{(x+1)}{(x+1)} \frac{1}{(x-1)}$$

$$= \frac{(x^3 - x^2)}{(x+1)(x-1)} - \frac{(x+1)}{(x+1)(x-1)}$$

$$= \frac{(x^3 - x^2) - (x+1)}{(x+1)(x-1)}$$

$$= \frac{x^3 - x^2 - x - 1}{(x+1)(x-1)}, x \neq \pm 1$$

Example 5:

$$\frac{3}{2x} + \frac{4}{(x-1)} \quad LCD: 2x(x-1)$$

$$= \frac{(x-1)\cdot 3}{(x-1)\cdot 2x} + \frac{2x\cdot 4}{2x\cdot(x-1)}$$

$$= \frac{(3x-3)}{2x(x-1)} + \frac{8x}{2x(x-1)}$$

$$= \frac{3x-3+8x}{2x(x-1)}$$

$$= \frac{11x-3}{2x(x-1)}, \; x \neq 0, 1$$

Example 6:

$$\frac{7}{x^2-9}+\frac{1}{4x+12}$$

$$= \frac{7}{(x+3)(x-3)} + \frac{1}{4(x+3)} \quad \text{LCD: } 4(x+3)(x-3)$$

$$= \frac{4}{4}\,\frac{7}{(x+3)(x-3)} + \frac{(x-3)}{(x-3)}\,\frac{1}{4(x+3)}$$

$$= \frac{28}{4(x+3)(x-3)} + \frac{(x-3)}{4(x+3)(x-3)}$$

$$= \frac{28+x-3}{4(x+3)(x-3)}$$

$$= \frac{x+25}{4(x+3)(x-3)}, \; x \neq \pm 3$$

Example 7:

$$\frac{x}{x^2-3x-4}-\frac{4}{x+1}$$

$$= \frac{x}{(x+1)(x-4)} - \frac{4}{(x+1)} \qquad \text{LCD: } (x+1)(x-4)$$

$$= \frac{x}{(x+1)(x-4)} - \frac{(x-4)}{(x-4)} \frac{4}{(x+1)}$$

$$= \frac{x}{(x+1)(x-4)} - \frac{(4x-16)}{(x+1)(x-4)}$$

$$= \frac{x-4x+16}{(x+1)(x-4)}$$

$$= \frac{-3x+16}{(x+1)(x-4)}, \; x \neq -1, 4$$

Common Mistakes

A common student error involves adding or subtracting the numerators without first writing the fractions with a common denominator. For example:

$$\frac{x}{5} + \frac{2}{3} = \frac{x+2}{8}$$

The correct solution is: LCD: $3 \cdot 5$

$$\frac{3 \cdot x}{3 \cdot 5} + \frac{5 \cdot 2}{5 \cdot 3} = \frac{3x}{3 \cdot 5} + \frac{10}{3 \cdot 5} = \frac{3x+10}{3 \cdot 5} = \frac{3x+10}{15}$$

Remind students to be careful when subtracting rational expressions. They sometimes forget to distribute the negative sign when there is more than one term in the numerator. For example:

$$\frac{3x-2}{(x+2)(x-2)} - \frac{(2x-4)}{(x+2)(x-2)} = \frac{3x-2-2x-4}{(x+2)(x-2)}$$

As with simplification, using brackets around binomials can help avoid this mistake. The correct solution is:

$$= \frac{3x-2-(2x-4)}{(x+2)(x-2)}$$

$$= \frac{3x-2-2x+4}{(x+2)(x-2)}$$

$$= \frac{\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$$

$$= \frac{1}{(x-2)}, \quad x \neq \pm 2$$

Example 8:

(A) $= \frac{(x+7)}{(2x+14)} - \frac{5x}{(-3x-21)}$

$= \frac{x+7}{2(x+7)} - \frac{5x}{-3(x+7)} \frac{(-1)}{(-1)}$

$= \frac{x+7}{2(x+7)} - \frac{(-5x)}{3(x+7)}$ LCD: $2 \cdot 3(x+7)$

$= \frac{3}{3}\frac{(x+7)}{2(x+7)} - \frac{2}{2}\frac{(-5x)}{3(x+7)}$

$= \frac{3x+21}{2 \cdot 3(x+7)} - \frac{(-10x)}{2 \cdot 3(x+7)}$

$= \frac{3x+21-(-10x)}{2 \cdot 3(x+7)}$

$= \frac{3x+21+10x}{2 \cdot 3(x+7)}$

$= \frac{13x+21}{6(x+7)}, x \neq -7$

(B) $\frac{(2x-6)}{x^2-x-6} - \frac{(3x+12)}{x^2+x-12}$

LCD: $(x+2)(x-3)(x+4)$

$$= \frac{2(x-3)}{(x+2)(x-3)} - \frac{3(x+4)}{(x-3)(x+4)}$$

$$= \frac{(x+4)}{(x+4)} \cdot \frac{2(x-3)}{(x+2)(x-3)} - \frac{(x+2)}{(x+2)} \cdot \frac{3(x+4)}{(x-3)(x+4)}$$

$$= \frac{2(x^2+x-12)}{(x-3)(x+2)(x+4)} - \frac{3(x^2+6x+8)}{(x-3)(x+2)(x+4)}$$

$$= \frac{2x^2+2x-24-3x^2-18x-24}{(x-3)(x+2)(x+4)}$$

$$= \frac{-x^2-16x-48}{(x-3)(x+2)(x+4)}$$

$$= \frac{-1(x^2+16x+48)}{(x-3)(x+2)(x+4)}$$

$$= \frac{-1\cancel{(x+4)}(x+12)}{(x-3)(x+2)\cancel{(x+4)}}$$

$$= \frac{-(x+12)}{(x-3)(x+2)}$$

$$= \frac{-x-12}{(x-3)(x+2)}, \quad x \neq -4, -2, 3$$

or

$$= \frac{2(x-3)}{(x+2)(x-3)} - \frac{3\cancel{(x+4)}}{(x-3)\cancel{(x+4)}}$$

$$= \frac{2(x-3)}{(x+2)(x-3)} - \frac{3}{(x-3)}$$

$$= \frac{2(x-3)}{(x+2)(x-3)} - \frac{(x+2)}{(x+2)} \cdot \frac{3}{(x-3)}$$

$$= \frac{2x-6}{(x+2)(x-3)} - \frac{(3x+6)}{(x+2)(x-3)}$$

$$= \frac{2x-6-3x-6}{(x+2)(x-3)}$$

$$= \frac{-x-12}{(x+2)(x-3)}, \quad x \neq -4, -2, 3$$

Complex Fractions

A complex fraction is an example of an expression involving two or more operations on a rational expression. In order to avoid errors, you should place brackets appropriately and use the order of operations correctly.

Steps:

- simplify both the numerator and denominator
- invert and multiply
- simplify

Example 9:

(A) Simplify and state all non-permissible values:

$$\frac{1+\frac{1}{x}}{x-\frac{1}{x}}$$

Method I: Rewrite Linearly

$$\left(1+\frac{1}{x}\right)\div\left(x-\frac{1}{x}\right) \quad \text{LCD: } x$$

$$=\left(\frac{x}{x}\cdot\frac{1}{1}+\frac{1}{x}\right)\div\left(\frac{x}{x}\cdot\frac{x}{1}-\frac{1}{x}\right)$$

$$=\left(\frac{x}{x}+\frac{1}{x}\right)\div\left(\frac{x^2}{x}-\frac{1}{x}\right)$$

$$=\left(\frac{x+1}{x}\right)\div\left(\frac{x^2-1}{x}\right)$$

$$=\left(\frac{\cancel{x+1}}{\cancel{x}}\right)\cdot\left(\frac{\cancel{x}}{(x-1)\cancel{(x+1)}}\right)$$

$$=\frac{1}{x-1},\ x\neq 0,\pm 1$$

Method 2: LCD

LCD: x

$$\frac{\left(1+\frac{1}{x}\right)x}{\left(x-\frac{1}{x}\right)x}$$

$$=\frac{\left(x+\frac{\cancel{x}}{\cancel{x}}\right)}{\left(x^2-\frac{\cancel{x}}{\cancel{x}}\right)}$$

$$=\frac{(x+1)}{(x^2-1)}$$

$$=\frac{\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$$

$$=\frac{1}{x-1},\ x\neq 0,\pm 1$$

(B) Simplify and state all non-permissible values:

$$\frac{\frac{1}{x+2}+\frac{1}{x-2}}{\frac{x}{x^2-4}+\frac{1}{x+2}}$$

LCD: $(x+2)(x-2)$

$$= \left[\frac{\frac{1}{(x+2)}+\frac{1}{(x-2)}}{\frac{x}{(x-2)(x+2)}+\frac{1}{(x+2)}}\right]\begin{matrix}(x+2)(x-2)\\ (x+2)(x-2)\end{matrix}$$

$$= \frac{\frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}}+\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}}{\frac{x\cancel{(x+2)}\cancel{(x-2)}}{\cancel{(x-2)}\cancel{(x+2)}}+\frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}}}$$

$$= \frac{x-2+x+2}{x+x-2}$$

$$= \frac{2x}{2x-2}$$

$$= \frac{\cancel{2}x}{\cancel{2}(x-1)}$$

$$= \frac{x}{x-1}, \ |x \neq \pm 2, 1|$$

(C)

$$\frac{\left(1+\frac{1}{x}\right)x^2}{\left(1-\frac{1}{x^2}\right)x^2}$$

LCD: $x \cdot x = x^2$

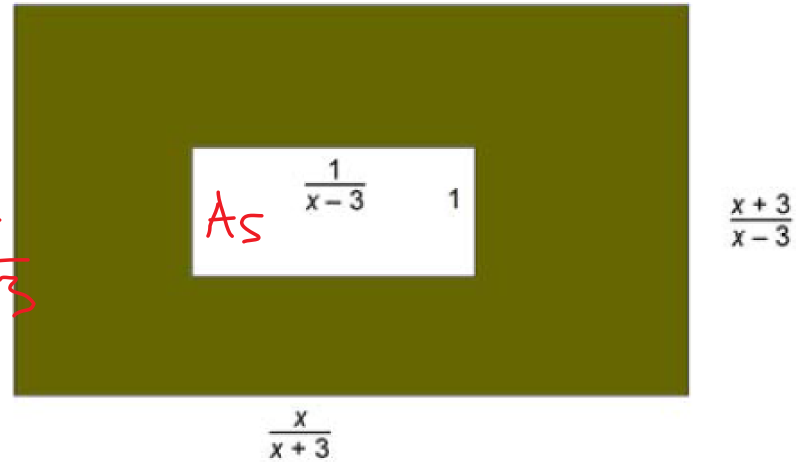
$$= \frac{\left(x^2 + \frac{x^{\cancel{2}}}{\cancel{x}}\right)}{\left(x^2 - \frac{\cancel{x^2}}{\cancel{x^2}}\right)}$$

$$= \frac{x^2 + x}{x^2 - 1}$$

$$= \frac{x\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$$

$$= \frac{x}{x-1},\ x \neq 0, \pm 1$$

Example 10:
Find the area of the shaded region:



$A_B = l \cdot w$

$= \frac{x}{\cancel{(x+3)}} \cdot \frac{\cancel{(x+3)}}{(x-3)} = \frac{x}{x-3}$

$A_S = l \cdot w$

$= \frac{1}{x-3} \cdot 1$

$A_{shaded} = A_B - A_S = \frac{x}{x-3} - \frac{1}{x-3} = \frac{x-1}{x-3}, x \neq \pm 3$

Textbook Questions: page 336 – 340 #1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 18, 19