Math 2200

6.4A Rational Equations

Now that we've learned to work with rational expressions, we can now piece them together and work with rational equations. A **rational equation** is an equation containing one or more rational expression. For example:

$$\frac{5}{x} = \frac{4}{x+2}$$

We can use the techniques of multiplying/dividing and adding/subtracting that we learned earlier in this chapter to solve an equation for a variable. The difference now is that we don't use the LCD to get a common denominator for two or more rational expressions. Instead we use the LCD to eliminate the denominator in each rational expression.

To solve a rational equation:

- factor each denominator
- identify the non-permissible values
- multiply both sides of the equation by the lowest common denominator
- solve by isolating the variable on one side of the equation
- check your answers

Example 1:
Solve the following equation for x:

(A)

$$\frac{x}{10} = \frac{2}{5}$$

CD: 5·10

$$5\cdot\cancel{10}\cdot\frac{x}{\cancel{10}} = \cancel{5}\cdot 10\cdot\frac{2}{\cancel{5}}$$

$$\frac{5x}{5} = \frac{20}{5}$$

$$x = 4$$

* You can "cross-multiply". However this only works for single expressions on either side.

$$\frac{x}{10} = \frac{2}{5}$$

$$\frac{5x}{5} = \frac{20}{5}$$

$$x = 4$$

(B)

$$\frac{x}{4} - \frac{7}{x} = 3 \qquad \text{CD: } 4x$$

$$\cancel{4}x \cdot \frac{x}{\cancel{4}} - 4 \cdot \cancel{x} \cdot \frac{7}{\cancel{x}} = 4 \cdot x \cdot 3$$

$$x^2 - 28 = 12x$$

$$x^2 - 12x - 28 = 0$$

$$(x+2)(x-14) = 0$$

$$x = -2, \; x = 14, \; x \neq 0$$

(C)

$$\frac{3}{x} + \frac{7}{2x} = \frac{1}{5} \qquad \text{CD: } 2 \cdot 5 \cdot x$$

$$2 \cdot 5 \cdot \cancel{x} \cdot \frac{3}{\cancel{x}} + \cancel{2} \cdot 5 \cancel{x} \cdot \frac{7}{\cancel{2x}} = 2 \cdot \cancel{5} \cdot x \cdot \frac{1}{\cancel{5}}$$

$$30 + 35 = 2x$$

$$\frac{65}{2} = \frac{2x}{2}$$

$$x = 65/2, \; x \neq 0$$

(D)

$$\frac{2x^2+1}{x+3} = \frac{x}{4} + \frac{5}{x+3}$$ CD: $4(x+3)$

$$4\cdot\cancel{(x+3)}\cdot\frac{(2x^2+1)}{\cancel{(x+3)}} = \cancel{4}(x+3)\cdot\frac{x}{\cancel{4}} + 4\cdot\cancel{(x+3)}\cdot\frac{5}{\cancel{(x+3)}}$$

$$8x^2+4 = x^2+3x+20$$

$$8x^2-x^2-3x+4-20=0$$

$$7x^2-3x-16=0$$

$$x = \frac{-(-3)\pm\sqrt{(-3)^2-4(7)(-16)}}{2(7)}$$

$$x = \frac{3\pm\sqrt{457}}{14}$$

$$x = \frac{3+\sqrt{457}}{14},\quad x = \frac{3-\sqrt{457}}{14},\quad x \neq -3$$

(E)

$$\frac{2}{z^2-4}+\frac{10}{6z+12}=\frac{1}{z-2}$$

CD: $6(z+2)(z-2)$

$$\frac{2}{(z+2)(z-2)}+\frac{10}{6(z+2)}=\frac{1}{(z-2)}$$

$$\cancel{6(z+2)(z-2)}\cdot\frac{2}{\cancel{(z+2)(z-2)}}+\cancel{6(z+2)}(z-2)\cdot\frac{10}{\cancel{6(z+2)}}=6(z+2)\cancel{(z-2)}\cdot\frac{1}{\cancel{(z-2)}}$$

$$12+10z-20=6z+12$$

$$10z-6z=12+8$$

$$\frac{4z}{4}=\frac{20}{4}$$

$$z=5,\ z\neq\pm2$$

(F)

$$\frac{9}{y-3}-\frac{4}{y-6}=\frac{18}{y^2-9y+18}$$

$$\frac{9}{(y-3)}-\frac{4}{(y-6)}=\frac{18}{(y-3)(y-6)} \qquad CD: (y-3)(y-6)$$

$$\cancel{(y-3)}(y-6)\cdot\frac{9}{\cancel{(y-3)}}-(y-3)\cancel{(y-6)}\cdot\frac{4}{\cancel{(y-6)}}=\cancel{(y-3)}\cancel{(y-6)}\frac{18}{\cancel{(y-3)}\cancel{(y-6)}}$$

$$9y-54-4y+12=18$$

$$5y=18+54-12$$

$$\frac{5y}{5}=\frac{60}{5}$$

$$y=12,\ y\neq 3,6$$

Extraneous Roots
We dealt with extraneous solutions in Chapter 5. As before, we simply plug our solutions back into the original equation and check to see if the left side equals the right side.

Example 2:
Solve for x. State all non-permissible values and any extraneous roots:

(A)

$$\frac{2x+3}{x+5}+\frac{1}{2}=\frac{-14}{2x+10}$$

$$\frac{(2x+3)}{(x+5)}+\frac{1}{2}=\frac{-14}{2(x+5)} \qquad CD: 2(x+5)$$

$$2\cancel{(x+5)}\frac{(2x+3)}{\cancel{(x+5)}}+\cancel{2}(x+5)\cdot\frac{1}{\cancel{2}}=\frac{\cancel{2}\cancel{(x+5)}(-14)}{\cancel{2}\cancel{(x+5)}}$$

$$4x+6+x+5=-14$$

$$5x=-14-11$$

$$\frac{5x}{5}=\frac{-25}{5}$$

$$x=-5, \quad x\neq-5$$

No solution.

(B)

$$\frac{4k-1}{k+2}-\frac{k+1}{k-2}=\frac{k^2-4k+24}{k^2-4} \quad CD: (k+2)(k-2)$$

$$\frac{(4k-1)}{(k+2)}-\frac{(k+1)}{(k-2)}=\frac{k^2-4k+24}{(k+2)(k-2)}$$

$$\cancel{(k+2)}(k-2)\cdot\frac{(4k-1)}{\cancel{(k+2)}}-(k+2)\cancel{(k-2)}\cdot\frac{(k+1)}{\cancel{(k-2)}}=\cancel{(k+2)}\cancel{(k-2)}\frac{(k^2-4k+24)}{\cancel{(k+2)}\cancel{(k-2)}}$$

$$4k^2-9k+2-(k^2+3k+2)=k^2-4k+24$$

$$4k^2-9k+2-k^2-3k-2-k^2+4k-24=0$$

$$\frac{2k^2-8k-24}{2}=\frac{0}{2}$$

$$k^2-4k-12=0$$

$$(k-6)(k+2)=0$$

$$k=6,\ \cancel{k=-2},\ k\neq\pm2$$

$$\frac{4(6)-1}{6+2}-\frac{6+1}{6-2}=\frac{6^2-4(6)+24}{6^2-4}$$

$$\frac{23}{8}-\frac{7\cdot2}{4\cdot2}=\frac{36}{32}$$

$$\frac{23}{8}-\frac{14}{8}=\frac{9}{8}$$

$$\frac{9}{8}=\frac{9}{8}\ \checkmark$$

(C)

$$\frac{3x}{x+2} - \frac{5}{x-3} = \frac{-25}{x^2 - x - 6}$$

$$\frac{3x}{(x+2)} - \frac{5}{(x-3)} = \frac{-25}{(x+2)(x-3)} \qquad \text{c.d.: } (x+2)(x-3)$$

$$\cancel{(x+2)}(x-3)\cdot\frac{3x}{\cancel{(x+2)}} - (x+2)\cancel{(x-3)}\cdot\frac{5}{\cancel{(x-3)}} = \cancel{(x+2)}\cancel{(x-3)}\frac{(-25)}{\cancel{(x+2)}\cancel{(x-3)}}$$

$$3x^2 - 9x - 5x - 10 = -25$$

$$3x^2 - 14x + 15 = 0 \qquad \frac{45}{5,9}$$

$$3x^2 - 9x - 5x + 15$$

$$3x(x-3) - 5(x-3)$$

$$(x-3)(3x-5) = 0$$

$$\cancel{x=3}, \quad x = \frac{5}{3}, \quad x \neq 3$$

Check:

$$\frac{\cancel{3}\left(\frac{5}{\cancel{3}}\right)}{\frac{5}{3} + 2\cdot\frac{3}{3}} - \frac{5}{\frac{5}{3} - 3\cdot\frac{3}{3}} = \frac{-25}{\left(\frac{5}{3}\right)^2 - \frac{5\cdot 3}{3\cdot 3} - 6\cdot\frac{9}{9}}$$

$$\frac{5}{\frac{11}{3}} - \frac{5}{\frac{-4}{3}} = \frac{-25}{\frac{25}{9} - \frac{15}{9} - \frac{54}{9}}$$

$$\frac{4\cdot 15}{4\cdot 11} + \frac{15\cdot 11}{4\cdot 11} = \frac{-25}{\frac{-44}{9}}$$

$$\frac{60}{44} + \frac{165}{44} = \frac{225}{44}$$

$$\frac{225}{44} = \frac{225}{44} \checkmark$$

Textbook Questions: page 348 #1, 2, 3, 5, 6, 27