Math 2200

6.4B Applications of Rational Equations

Here we will write rational equations to represent a variety of situations, and use them to solve for a variable. Once we solve for the variable, we not only need to check for extraneous roots, but we also need to consider inadmissible roots.

Inadmissible Roots

Values for the variable that do not make sense in the context of an application problem. For example, if we are solving for x and x represents time, then a negative x-value would be an inadmissible root since we cannot have negative time values. Also if a solution happens to be a non-permissible value, it is inadmissible. If this is the only solution, we say the problem has no solution.

Example 1:

When three more than an integer is divided by twice the integer, the result is the same as the original integer. Find all integers that satisfy these conditions.

Let integer be x.

$$\frac{x+3}{2x} = x \quad \text{LCD: } 2x$$

$$\cancel{2x} \cdot \frac{(x+3)}{\cancel{2x}} = 2x \cdot x$$

$$x+3 = 2x^2$$

$$0 = 2x^2 - x - 3 \qquad \frac{6}{\begin{array}{c}1,6\\2,3\end{array}}$$

$$0 = 2x^2 + 2x - 3x - 3$$

$$0 = 2x(x+1) - 3(x+1)$$

$$0 = (2x-3)(x+1)$$

$$\cancel{x = 3/2}, \quad x = -1$$

Check:

$$\frac{(-1)+3}{2(-1)} = -1$$

$$\frac{2}{-2} = -1$$

$$-1 = -1 \checkmark$$

Example 2:
One positive integer is 5 more than the other. When the reciprocal of the larger number is subtracted from the reciprocal of the smaller number, the result is $\frac{5}{14}$. Find the two integers.

$x, \; x+5$

$$\frac{1}{x} - \frac{1}{x+5} = \frac{5}{14} \qquad \text{LCD: } 14x(x+5)$$

$$14\cancel{x}(x+5)\cdot\frac{1}{\cancel{x}} - 14x\cancel{(x+5)}\cdot\frac{1}{\cancel{x+5}} = \cancel{14}x(x+5)\cdot\frac{5}{\cancel{14}}$$

$$\cancel{14x} + 70 - \cancel{14x} = 5x^2 + 25x$$

$$\frac{0}{5} = \frac{5x^2 + 25x - 70}{5}$$

$$0 = x^2 + 5x - 14$$

$$0 = (x-2)(x+7)$$

$$x = \underline{2}, \; \cancel{x = -7}$$

$$x + 5 = \underline{7}$$

Check:

$$\frac{1}{2} - \frac{1}{2+5} = \frac{5}{14}$$

$$\frac{7\cdot 1}{7\cdot 2} - \frac{2\cdot 1}{2\cdot 7} = \frac{5}{14}$$

$$\frac{7}{14} - \frac{2}{14} = \frac{5}{14}$$

$$\frac{5}{14} = \frac{5}{14} \checkmark$$

Work Rate Problem

We have two equivalent formulae to choose from when solving a work rate problem. If two people are working together on a job, their rates add and they can perform the job in shorter time.

If we let x = time it takes person 1 to complete the job, then the work rate is $\frac{1}{x}$. In other words, this person can complete the job in x hours.

If we let y = time it takes person 2 to complete the job, and t = time is takes with both working together, we get the following formulas:

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{t}$$

Example 3:

Sherry mows a lawn in 4 hours. Mary mows the same lawn in 5 hours. How long would it take both of them working together to mow the lawn?

Table of Values:

	Name	time	rate
r_1	Sherry	4	$1/4$
r_2	Mary	5	$1/5$
r_3	both	x	$1/x$

$r_1 + r_2 = r_3$

$\frac{1}{4}+\frac{1}{5}=\frac{1}{x}$ LCD $=4\cdot5\cdot x$

$\cancel{4}\cdot 5x\cdot\frac{1}{\cancel{4}}+4\cdot\cancel{5}\cdot x\cdot\frac{1}{\cancel{5}}=4\cdot5\cdot\cancel{x}\cdot\frac{1}{\cancel{x}}$

$4x+5x=20$

$9x=20$

$x=20/9=2.2h$

Sherry & Mary can both mow the lawn together in 2.2h.

Example 4:
It takes Mike 9 hours longer to construct a fence than it takes Jason. If they work together, they can construct the fence in 20 hours. How long would it take Mike to construct the fence alone?

Name	time	rate
Mike	$x+9$	$\frac{1}{x+9}$
Jason	x	$\frac{1}{x}$
both	20	$\frac{1}{20}$

$$\frac{1}{x+9} + \frac{1}{x} = \frac{1}{20} \quad \text{LCD: } 20x(x+9)$$

$$20x(x+9)\left[\frac{1}{x+9} + \frac{1}{x} = \frac{1}{20}\right]$$

$$20x + 20(x+9) = x(x+9)$$

$$20x + 20x + 180 = x^2 + 9x$$

$$0 = x^2 - 31x - 180$$

$$0 = (x+5)(x-36)$$

$$\cancel{x=-5}, \; x = 36$$

$$x+9 = 36+9 = 45$$

It takes Mike 45 hours to construct the fence alone.

Example 5:
When they work together, Stuart and Lucy can deliver flyers to all the homes in their neighbourhood in 42 minutes. When Lucy works alone, she can deliver the flyers in 13 minutes less than Stuart when he works alone. When Stuart works alone, how long does he take to deliver the flyers?

Up the River/Down the River

This type up problem usually involves one or more boats travelling on a river, two cars travelling along a highway or trains on a track. An organization table can often help here, as will the formula for speed, $v = \frac{d}{t}$.

Example 7:

Grand River has a current of 2 km/h. A motorboat can travel 15 km down the river in the same amount of time it takes to travel 9 km up the river. What is the speed of the boat in still water?

$$t \cdot v = \frac{d}{\cancel{t}} \cdot \cancel{t}$$

$$\frac{t \cdot \cancel{v}}{\cancel{v}} = \frac{d}{v}$$

$$t = \frac{d}{v}$$

$$t = t$$

$$\frac{d}{v} = \frac{d}{v}$$

$$\frac{15}{x+2} = \frac{9}{x-2}$$

$$9(x+2) = 15(x-2)$$

$$9x + 18 = 15x - 30$$

$$18 + 30 = 15x - 9x$$

$$\frac{48}{6} = \frac{\cancel{6}x}{\cancel{6}}$$

$$x = 8 \text{ km/h}$$

Example 8:

On the first part of a road trip, Jonas rode his bike 16 km. On the second part of the trip he road his bike 42 km. His average speed during the second part of the trip was 6 km/h faster than his average speed on the first part of the trip. Find her average speed for the second part of the trip if the total time for the trip was 5 hours.

$$t_1 + t_2 = 5$$

$$\frac{d_1}{v_1} + \frac{d_2}{v_2} = 5$$

$$\frac{16}{x} + \frac{42}{x+6} = 5 \quad \text{LCD: } x(x+6)$$

$$\cancel{x}(x+6)\cdot\frac{16}{\cancel{x}} + x\cancel{(x+6)}\cdot\frac{42}{\cancel{x+6}} = x(x+6)\cdot 5$$

$$16x + 96 + 42x = 5x^2 + 30x$$

$$0 = 5x^2 - 28x - 96$$

$$x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(5)(-96)}}{2(5)}$$

$$x = \frac{28 \pm \sqrt{2704}}{10}$$

$$x = \frac{28 \pm 52}{10}$$

$$x = \frac{28+52}{10}, \quad x = \frac{28-52}{10}$$

$$x = 8, \quad x = \cancel{-2.4}$$

1st part of trip: 8 km/h, 2nd part of trip: 14 km/h

Example 9:
Erica frequently drives 189 km to visit friends in St. John's. She has noticed that she saves 36 minutes if she travels 24 km/h faster than her average speed. What is her average speed?

$t_1 - t_1 = \frac{3}{5}$ or $t_2 + \frac{3}{5} = t_1$

$\frac{d_1}{v_1} - \frac{d_2}{v_2} = \frac{3}{5}$

$\frac{189}{x} - \frac{189}{x+24} = \frac{3}{5}$ LCD: $5x(x+24)$

$5x(x+24)\left[\frac{189}{x} - \frac{189}{x+24} = \frac{3}{5}\right]$

$5(x+24)\cdot 189 - 5x(189) = 3(x+24)$

$945x + 22680 - 945x = 3x^2 + 72x$

$\frac{0}{3} = \frac{3x^2 + 72x - 22680}{3}$

$0 = x^2 + 24x - 7560 = 0$

$x = \frac{-24 \pm \sqrt{24^2 - 4(1)(-7560)}}{2(1)}$

$x = \frac{-24 \pm \sqrt{30816}}{2}$

$x = \frac{-24 \pm 17.55}{2}$

$\rightarrow x = 75.8,\ x = \cancel{-99.5}$

Erika's average speed is 75.8 km/h.

Textbook Questions: page 348 - 351 #8, 9, 11, 12, 13, 14, 15, 16, 17, 18