Math 2200

7.3 Absolute Value Equations

An **absolute value equation** is an equation that includes the absolute value of an expression involving a variable.

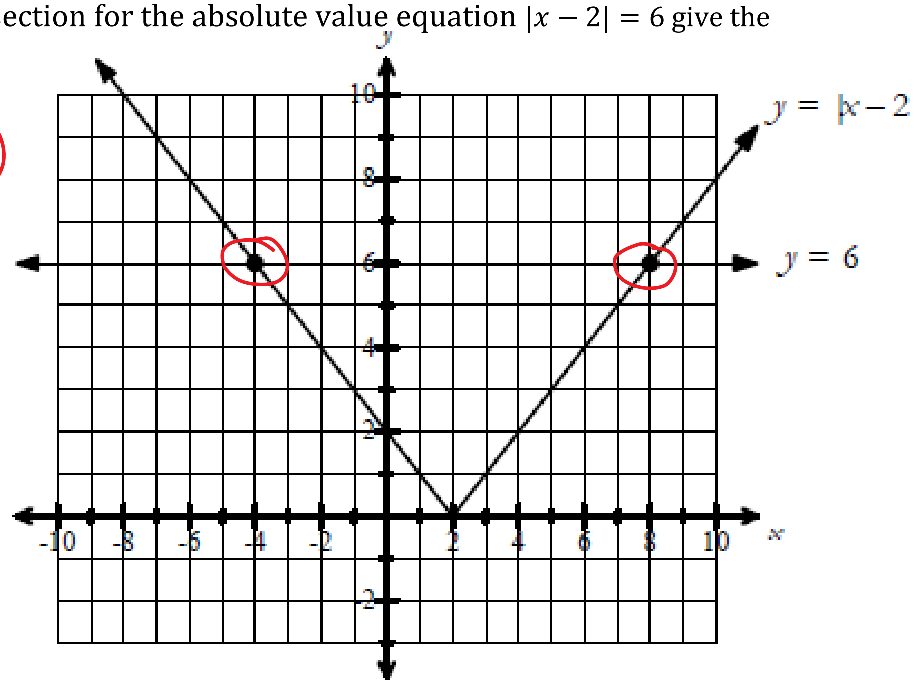
Graphical Solutions to Absolute Value Equations

What does it means to solve an equation such as $|x - 2| = 6$? You should look for points whose distance from 2 is 6. Using a number line, they should realize that both -4 and 8 are at a distance of 6 from 2. This reasoning will allow you to better understand the solutions when using a graph.

Example 1:

Determine the points of intersection for the absolute value equation $|x - 2| = 6$ give the graph.

$(-4, 6)$ & $(8, 6)$



Algebraic Solutions to Absolute Value Equations

When solving absolute value equations algebraically we use the definition of absolute value. There are two cases to consider:

Case 1: The expression inside the absolute value symbol is positive or zero.

Case 2: The expression inside the absolute value symbol is negative.

Example 2:
Solve: $|x-3| = 7$

Case 1:

$x-3=7$

$x=7+3$

$x=10$

Case 2:

$-(x-3)=7$

$-x+3=7$

$-x=7-3$

$-x=4$

$x=-4$

Check:

$|10-3|=7$

$|7|=7$

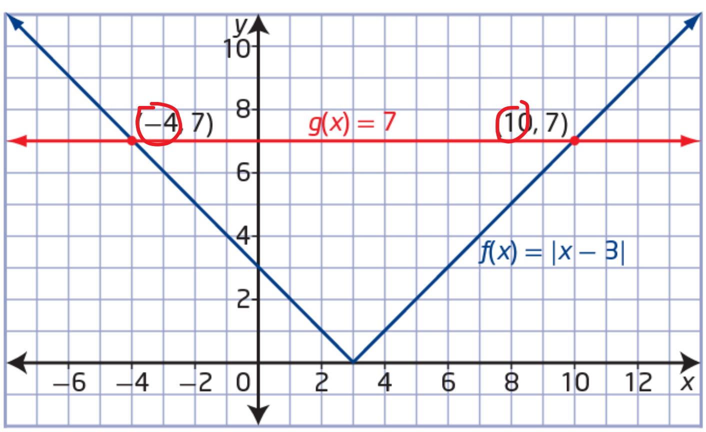
$7=7$ ✓

$|-4-3|=7$

$|-7|=7$

$7=7$ ✓

Note: Let's take a look at the graph again. Since $|x-3| = 7$ can be expressed as two functions, $f(x) = |x-3|$ and $g(x) = 7$, we can find the intersection points by graphing both functions.



Verifying solutions can be done at: https://www.desmos.com/calculator

Common Errors
Before we continue with some examples, lets look at a few common errors.

Common errors include:

- Treating the absolute value sign like parentheses.

- Multiplying a constant by the expression within the absolute value sign. For example: $-2|x-3| = |-2x+6|$

- Incorrectly placing the negative in front of the variable rather than the entire Expression. For example, when solving $|x-3| = 8$, students may write $-x-3=8$ instead of $-(x-3)=8$.

- Not identifying extraneous roots.

- Errors when using the quadratic formula.

Example 3:
Solve $|2x-5| = 5-3x$.

Case 1:

$2x-5 = 5-3x$

$2x+3x = 5+5$

$\frac{5x}{5} = \frac{10}{5}$

$\cancel{x = 2}$

Case 2:

$-(2x-5) = 5-3x$

$-2x+5 = 5-3x$

$-2x+3x = 5-5$

$x = 0$

Check:

$|2(2)-5| = 5-3(2)$

$|4-5| = 5-6$

$|-1| = -1$

$1 \neq -1$

$|2(0)-5| = 5-3(0)$

$|-5| = 5$

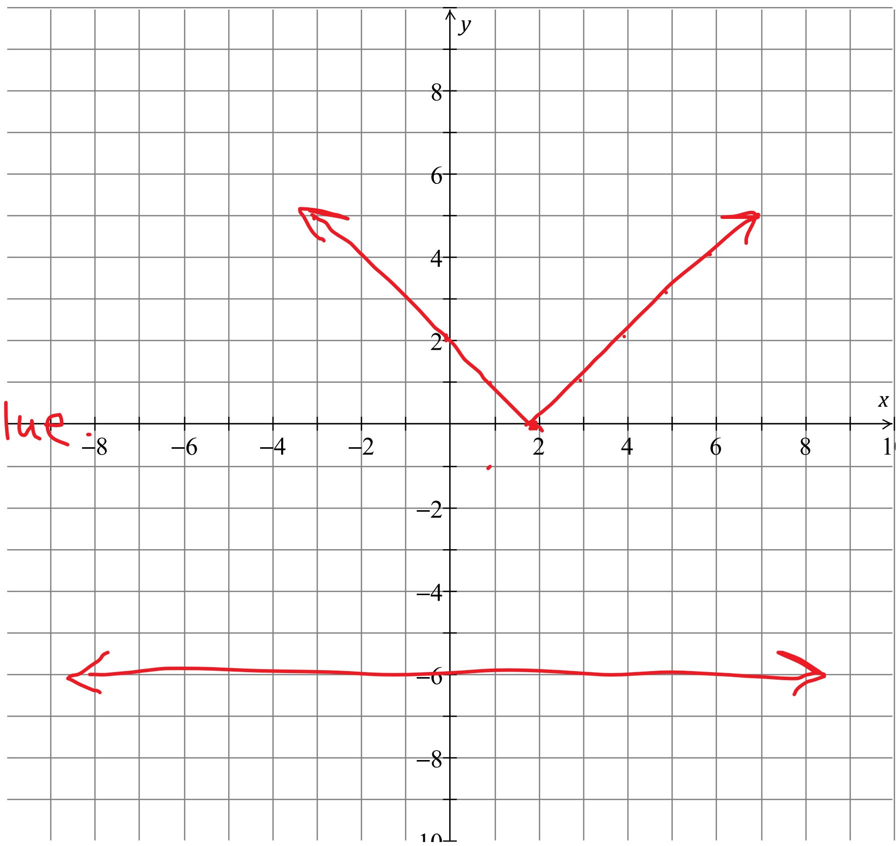
$5 = 5$ ✓

Example 4:
Illustrate with a graph why $|x - 2| = -6$ has no solution.

$y = |x-2|$

$y = -6$

An absolute value function can <u>never</u> be equal to a negative value.



Example 5:
Solve $|3x - 4| + 12 = 9$

$|3x-4| = 9-12$

$|3x-4| \neq -3$

No Solution.

Example 6:
Solve $|x^2 - 2x| = 1$

Case 1:

$$x^2 - 2x = 1$$

$$x^2 - 2x - 1 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = 1 \pm \sqrt{2}$$

$$x = 1+\sqrt{2},\ x = 1-\sqrt{2}$$

$$|(1+\sqrt{2})^2 - 2(1+\sqrt{2})| = 1$$

$$|1 \cancel{+2\sqrt{2}} \cancel{+2} \cancel{-2} \cancel{-2\sqrt{2}}| = 1$$

$$|1| = 1$$

$$1 = 1 \checkmark$$

$$|(1-\sqrt{2})^2 - 2(1-\sqrt{2})| = 1$$

$$|1 \cancel{-2\sqrt{2}} \cancel{+2} \cancel{-2} \cancel{+2\sqrt{2}}| = 1$$

$$|1| = 1$$

$$1 = 1 \checkmark$$

Case 2:

$$-(x^2 - 2x) = 1$$

$$-x^2 + 2x - 1 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)(x-1) = 0$$

$$x = 1$$

check:

$$|1^2 - 2(1)| = 1$$

$$|1-2| = 1$$

$$|-1| = 1$$

$$1 = 1 \checkmark$$

Example 7:
Solve $|x^2 - 4| = 3x$

Case 1:

$x^2 - 4 = 3x$

$x^2 - 3x - 4 = 0$

$(x+1)(x-4) = 0$

$\cancel{x = -1}$, $x = 4$

Check: $|4^2 - 4| = 3(4)$

$|12| = 12$

$12 = 12$ ✓

$|(-1)^2 - 4| = 3(-1)$

$3 \neq -3$

Case 2:

$-(x^2 - 4) = 3x$

$-x^2 + 4 = 3x$

$-x^2 - 3x + 4 = 0$

$x^2 + 3x - 4 = 0$

$(x-1)(x+4) = 0$

$x = 1$, $\cancel{x = -4}$

$|1^2 - 4| = 3(1)$

$3 = 3$ ✓

$|(-4)^2 - 4| = 3(-4)$

$12 \neq -12$

Example 8:
Solve $|x^2 - 3x| = x$

Case 1:

$x^2 - 3x = x$

$x^2 - 3x - x = 0$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$, $x = 4$

Check:

$|0^2 - 3(0)| = 0$

$|0| = 0$

$0 = 0$ ✓

$|4^2 - 4(4)| = 4$

$|16 - 12| = 4$

$|4| = 4$

$4 = 4$ ✓

Case 2:

$-(x^2 - 3x) = x$

$-x^2 + 3x - x = 0$

$-x^2 + 2x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0$, $x = 2$

$|2^2 - 3(2)| = 2$

$|4 - 6| = 2$

$|-2| = 2$

$2 = 2$ ✓

Textbook Questions: page 389 - 391 #2, 4, 5, 6, 15