Math 2200

8.1 Solving Systems of Equations Graphically

Recall from Math 1201, a **System of Equations** is two or more equations with two or more variables. When graphed, these lines intersect at a point (x, y) or no point if they are parallel. In Level I the only systems of equations looked at were two lines. Here we will look at systems of equations involving a line and a parabola and two parabolas.

System of Linear-Quadratic Equations: A linear equation and a quadratic equation involving the same variables. The graph of this system involves a line and a parabola.

System of Quadratic-Quadratic Equations: two quadratic equations involving the same variables. The graph of this system involves two parabolas.

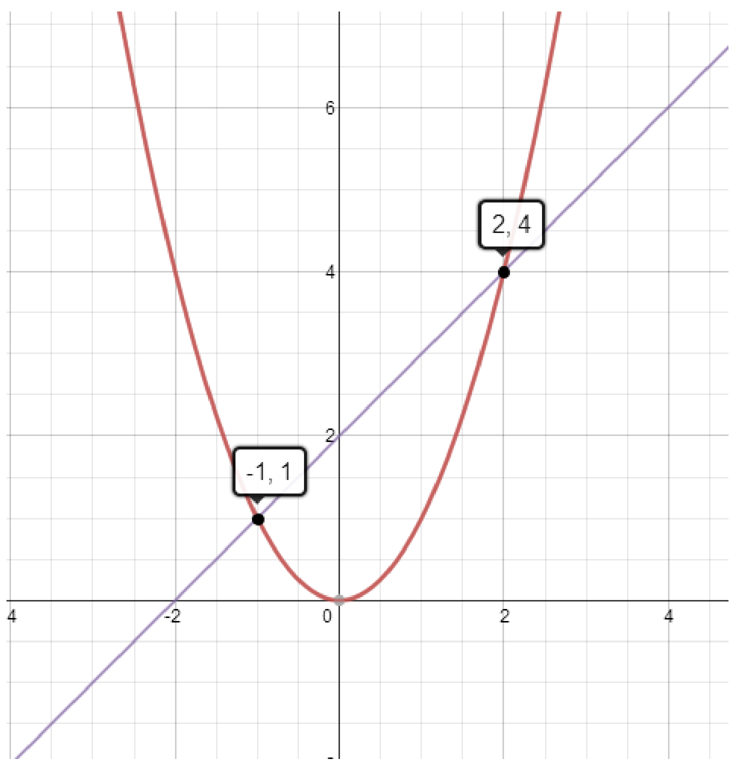
Any ordered pair (x, y) that satisfies both equations in a system of linear-quadratic or quadratic-quadratic equations is a **solution** of the system.

For example, the point $(2, 4)$ is a solution of the system:

$$y = x + 2$$
$$y = x^2$$

Since it satisfies both equations.

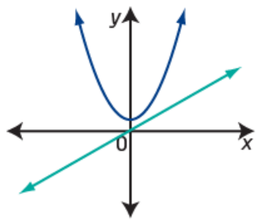
As you can see from the graph, $(2, 4)$ as well as the point $(-1, 1)$ are both solutions to the linear-quadratic systems of equations. In other words the graphs intersect at these two points.



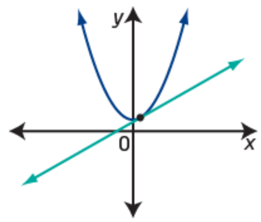
If a line intersects the parabola more than once, the line is referred to as a **secant line**.

If the line intersects the parabola at exactly one point, the line is called a **tangent line**.

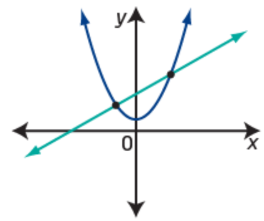
Number of Solutions of Linear-Quadratic and Quadratic-Quadratic Equations



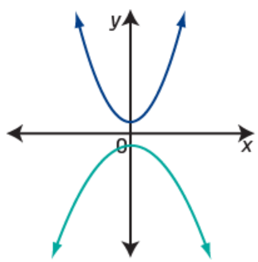
No point of
intersection
No solution



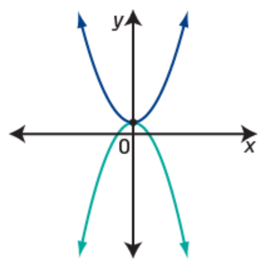
One point of
intersection
One solution



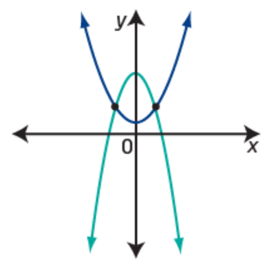
Two points of
intersection
Two solutions



No point of
intersection
No solution



One point of
intersection
One solution



Two points of
intersection
Two solutions

Coincident Systems
Here is a good place to point out that a quadratic - quadratic system can also have an infinite number of solutions. These are called a **coincident** systems. It's when one parabola lies over another.

Similarly to equivalent rational expressions, a coincident system of quadratic – quadratic equations is a situation where one quadratic equation is a multiple of another.

For example,

$$x^2 + 2x + 3y + 4 = 0$$
$$2x^2 + 4x + 6y + 8 = 0$$

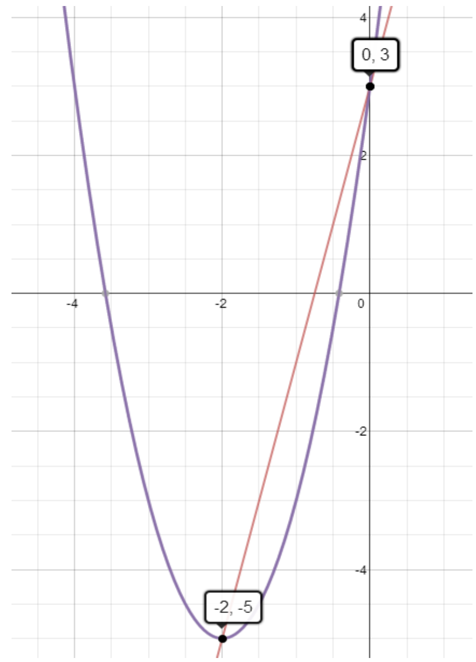
would be an example of a coincident system.

Example 1:
Solve the following system of equations graphically:

$$4x - y + 3 = 0$$
$$2x^2 + 8x - y + 3 = 0$$

Using https://www.desmos.com/calculator, or a TI-80 series graphing calculator we can graph these two functions and look for any possible intersection points:

$(-2,-5), (0,3)$

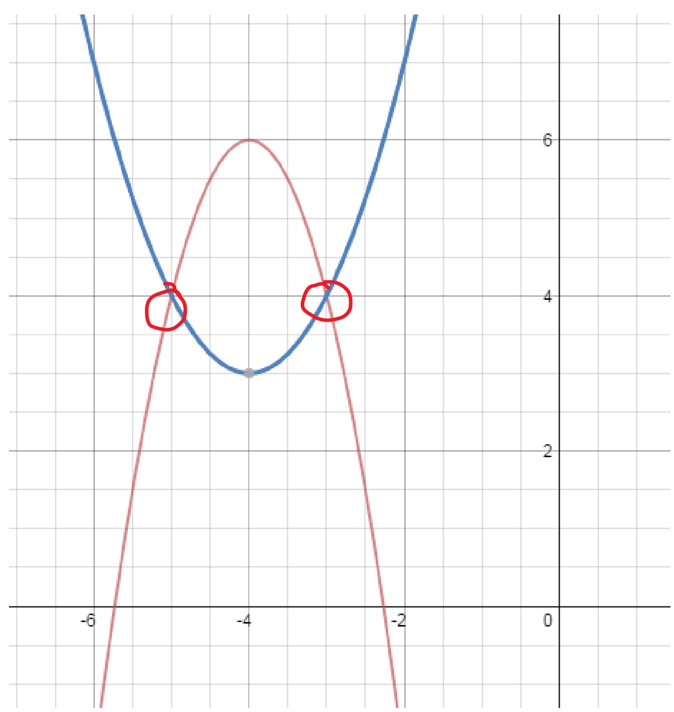


Example 2:

Solve the system graphically:

$$2x^2 + 16x + y = -26$$
$$x^2 + 8x - y = -19$$

$(-5,4), (-3,4)$



Working With Systems of Equations Logically
You can find out a lot about a system of equations without actually solving it. The following questions will help us logically figure out characteristics about some particular systems.

Example 3:
How can you tell by observation which of the following systems has no solution and which one has an infinite number of solutions?

(A)
$$\begin{aligned} (1)\quad -2x^2 + 3x - y + 4 &= 0 \\ (2)\quad -4x^2 + 6x - 2y + 8 &= 0 \end{aligned}$$

Coincident system because equation (2) is a multiple of equation (1)
$\therefore$ infinite number of solutions.

(B)
$$\begin{aligned} y - 5 &= 0 \rightarrow y = 5 \\ y &= -(x+1)^2 - 3 \end{aligned}$$

Vertex $(-1, -3)$

Since max value of -3 is below the line $y = 5$, there is no solution.

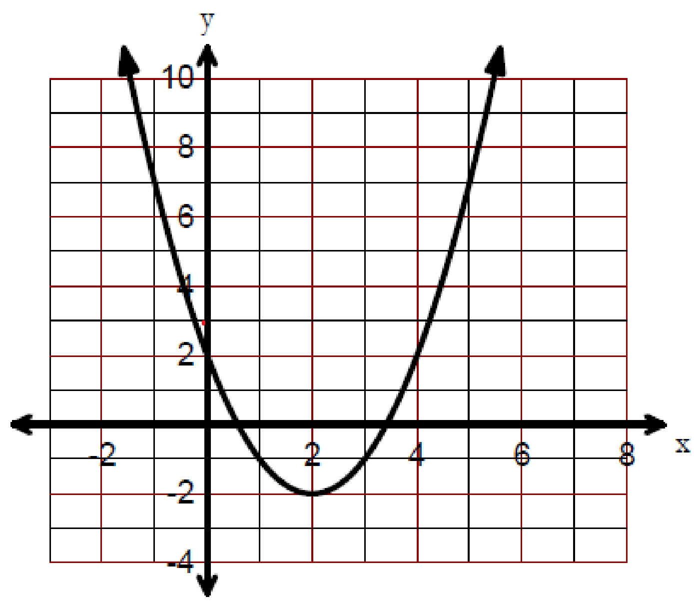
Example 4:
Do the following lines intersect the parabola shown below at zero, one or two points?

i. $y = \frac{1}{2}x + 3$
2 solutions

ii. $y = \frac{1}{2}x - 4$
no solutions

iii. $y = -2x$
no solution

iv. $x - 3 = 0$
1 solution



Textbook Questions: page 435 - 438 #2, 3, 4, 5, 7, 8, 12, 13, 14