Math 2200

8.2A Solving Systems of Equations Algebraically

Recall from the previous section that the systems of equations we are concerned with can have 0, 1 or two solutions. In this section we will apply the algebraic methods of substitution or elimination to solve these systems.

Example 1:
Solve the following system of equations:

Linear – Quadratic

$$\text{①}\quad 5x - y = 10$$
$$\text{②}\quad x^2 + x - 2y = 0$$

* Always solve for y.

Method 1: Substitution

Solve ① for y

$5x - 10 = y$

Sub ① into ②

$x^2 + x - 2(5x - 10) = 0$

$x^2 + x - 10x + 20 = 0$

$x^2 - 9x + 20 = 0$

$(x - 4)(x - 5) = 0$

$x = 4, \; x = 5$

$x = 4$

$y = 5(4) - 10 = 20 - 10 = 10$

$(4, 10)$

$x = 5$

$y = 5(5) - 10 = 25 - 10 = 15$

$(5, 15)$

* Eliminate y

Method 2: Elimination

$$\text{(2)} \quad x^2 + x - 2y = 0$$

$$\text{(1)} \quad 5x - y = 10$$

$$\text{(2)} + [-2\text{(1)}]$$

$$\begin{array}{rl} & x^2 + x - \cancel{2y} = 0 \\ + & \quad -10x + \cancel{2y} = -20 \\ \hline & x^2 - 9x = -20 \end{array}$$

$$x^2 - 9x + 20 = 0$$

See previous solution.

Your turn:

Example 2:
Solve the following system of equations:

$$\begin{aligned} 1 \quad 3x + y &= -9 \\ 2 \quad 4x^2 - x + y &= -9 \end{aligned}$$

Method 1: Substitution

Solve 1 for y.

$y = -3x - 9$

Sub 1 into 2

$4x^2 - x + (-3x - 9) = -9$

$4x^2 - 4x = 0$

$4x(x - 1) = 0$

$\frac{4x}{4} = \frac{0}{4}$, $x = 1$

$x = 0$

$x = 0$

$y = -3(0) - 9 = -9$

$(0, -9)$

$x = 1$

$y = -3(1) - 9 = -3 - 9 = -12$

$(1, -12)$

Method 2: Elimination

$$
\begin{array}{rrr}
\text{(2)} & 4x^2 - x + \cancel{y} & = -9 \\
- \text{(1)} & 3x + \cancel{y} & = -9 \\
\hline
 & 4x^2 - 4x & = 0
\end{array}
$$

See previous solution.

Example 3:
Solve the following system of equations:

Quadratic-Quadratic

$$\text{①}\quad 3x^2 - x - y - 2 = 0$$
$$\text{②}\quad 6x^2 + 4x - y = 4$$

Solve ① for y.

$3x^2 - x - 2 = y$

Sub ① into ②

$6x^2 + 4x - (3x^2 - x - 2) = 4$

$6x^2 + 4x - 3x^2 + x + 2 - 4 = 0$

$3x^2 + 5x - 2 = 0$

$\underline{6}$
1, 6
2, 3

$3x^2 + 6x - x - 2 = 0$

$3x(x+2) - (x+2) = 0$

$(x+2)(3x-1) = 0$

$x = -2, \; x = \frac{1}{3}$

$x = -2$

$y = 3(-2)^2 - (-2) - 2$

$y = 12 + 2 - 2$

$y = 12$

$(-2, 12)$

$x = \frac{1}{3}$

$y = 3\left(\frac{1}{3}\right)^2 - \frac{1}{3} - 2$

$y = \frac{1}{3} - \frac{1}{3} - 2$

$y = -2$

$\left(\frac{1}{3}, -2\right)$

Example 4:
Solve the following system of equations:

$$6x^2 - x - \cancel{y} = -1$$
$$-\quad 4x^2 - 4x - \cancel{y} = -6$$

$$2x^2 + 3x = 5$$

$$2x^2 - 3x - 5 = 0 \qquad \frac{10}{1,10 \quad 2,5}$$

$$2x^2 - 2x + 5x - 5 = 0$$

$$2x(x-1) + 5(x-1) = 0$$

$$(x-1)(2x+5) = 0$$

$$x = 1, \quad x = -\frac{5}{2}$$

$x = 1$

$$6(1)^2 - (1) - y = -1$$
$$5 + 1 = y$$
$$6 = y$$
$$(1, 6)$$

$x = -\frac{5}{2}$

$$6\left(-\frac{5}{2}\right)^2 - \left(-\frac{5}{2}\right) - y = -1$$
$$6\left(\frac{25}{4}\right) + \frac{5}{2} + 1 = y$$
$$\frac{75}{2} + \frac{5}{2} + 1 = y$$
$$\frac{80}{2} + 1 = y$$
$$40 + 1 = y$$
$$41 = y$$
$$\left(-\frac{5}{2}, 41\right)$$

Example 5:
Andrew was asked to solve the system:

$$x^2 - x + y = -2$$
$$2x^2 - 4x + 3y = 0$$

The beginning of his solution is shown below. Finish the solution and verify your answer. Identify and correct any error(s) you or Andrew may have made.

Andrew's Solution:
Multiply the first equation by −3 and add the second equation.

$$x^2 - x + y = -2 \xrightarrow{\times(-3)} -3x^2 + 3x - 3y = -2 \quad 6$$
$$2x^2 - 4x + 3y = 0 \longrightarrow \underline{\;2x^2 - 4x + 3y = 0\;}$$
$$-x^2 - x \qquad = -2$$

Now, solve $-x^2 - x = -2$.

$$-3x^2 + 3x \cancel{-3y} = 6$$
$$+ \quad \underline{2x^2 - 4x \cancel{+3y} = 0}$$
$$-x^2 - x = 6$$
$$-x^2 - x - 6 = 0$$
$$x^2 + x + 6 = 0 \quad \text{unfactorable}$$

$$D = b^2 - 4ac = (1)^2 - 4(1)(6) = 1 - 24 = -23$$

$\therefore$ No solution.

Creating a System of Equations from a Graph

You already know how to create an equation from a graph. Creating a system uses the same skills and techniques, only repeated for each graph.

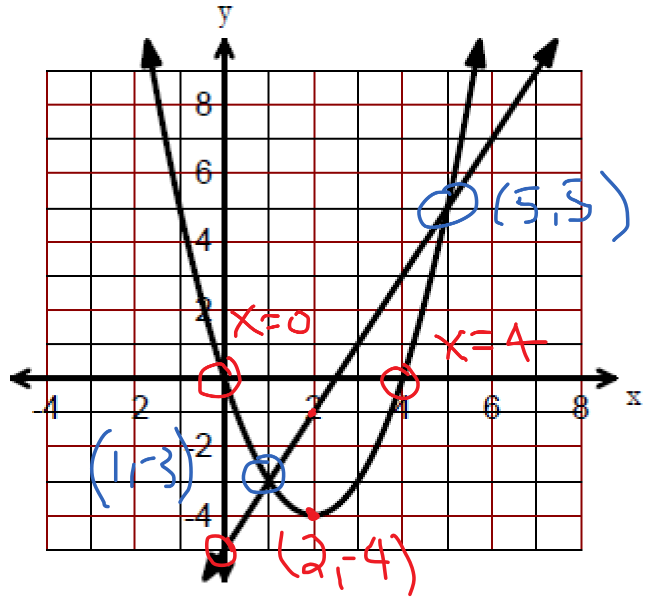
Example 6:

Create and solve the system of equations represented by the graphs below. Verify that the solutions obtained algebraically match those found graphically.

(A) $y = mx + b$ $\quad b = -5$

$m = \frac{4}{2} = 2$

① $y = 2x - 5$



$y = a(x-b)(x-c)$

$-4 = a(2-0)(2-4)$

$\frac{-4}{-4} = \frac{-4a}{-4}$

$a = 1$

$y = x(x-4)$

② $y = x^2 - 4x$

Comparison Method

$y = y$

$2x - 5 = x^2 - 4x$

$0 = x^2 - 4x - 2x + 5$

$0 = x^2 - 6x + 5$

$0 = (x-1)(x-5)$

$x = 1, x = 5$

$x = 1$

$y = 2(1) - 5 = -3$

$(1, -3)$

$x = 5$

$y = 2(5) - 5 = 5$

$(5, 5)$

(B)

$y = a(x-2)^2 + 0$

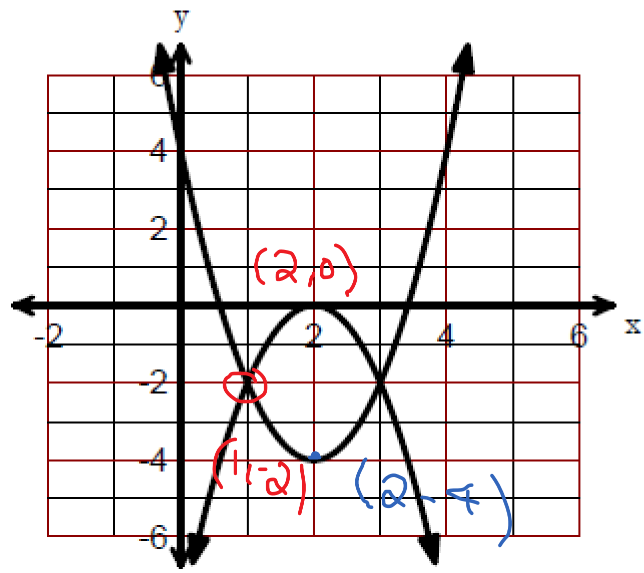
$-2 = a(1-2)^2 + 0$

$-2 = a$

$y = -2(x-2)^2$

$y = -2(x^2 - 4x + 4)$

$y = -2x^2 + 8x - 8$



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$y = a(x-2)^2 - 4$

$-2 = a(1-2)^2 - 4$

$-2 + 4 = a$

$a = 2$

$y = 2(x-2)^2 - 4$

$y = 2(x^2 - 4x + 4) - 4$

$y = 2x^2 - 8x + 8 - 4$

$y = 2x^2 - 8x + 4$

$y = y$

$-2x^2 + 8x - 8 = 2x^2 - 8x + 4$

$0 = 2x^2 + 2x^2 - 8x - 8x + 8 + 4$

$\frac{0}{4} = \frac{4x^2 - 16x + 12}{4}$

$0 = x^2 - 4x + 3$

$0 = (x-1)(x-3)$

$x = 1, \ x = 3$

$x = 1$

$y = -2(1)^2 + 8(1) - 8$

$y = -2$

$(1, -2)$

$x = 3$

$y = -2(3)^2 + 8(3) - 8$

$= -18 + 24 - 8$

$= -2$

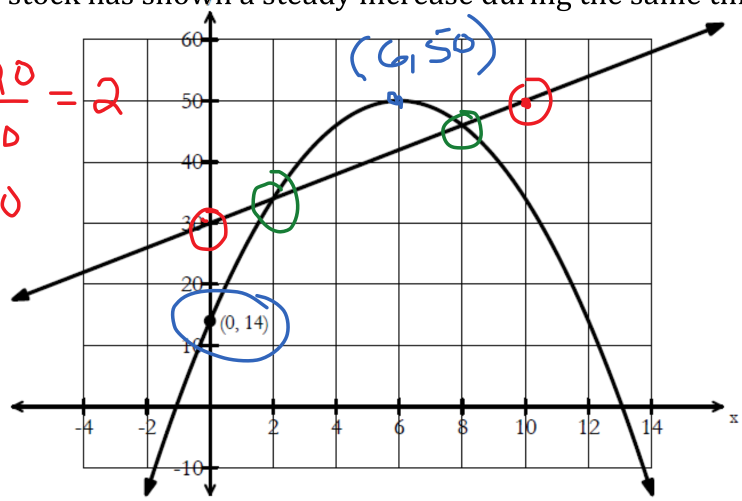
$(3, -2)$

Word Problems

Example 7:
The price C, in dollars per share, of a high-tech stock has fluctuated over a twelve-year period and is represented by the parabola shown. The price C, in dollars per share, of a second high-tech stock has shown a steady increase during the same time period.

$m = \frac{20}{10} = 2$

$b = 30$



(A) Determine the system of equations that models the price over time.

$y = 2x + 30$

$y = a(x-6)^2 + 50$

$14 = a(0-6)^2 + 50$

$14 - 50 = 36a$

$\frac{-36}{36} = \frac{36a}{36}$

$a = -1$

$y = -(x-6)^2 + 50$

$y = -(x^2 - 12x + 36) + 50$

$y = -x^2 + 12x - 36 + 50$

$y = -x^2 + 12x + 14$

(B) Solve the system.

$$Y = Y$$

$$2x + 30 = -x^2 + 12x + 14$$

$$x^2 + 2x - 12x + 30 - 14 = 0$$

$$x^2 - 10x + 16 = 0$$

$$(x-2)(x-8) = 0$$

$$x = 2,\ x = 8$$

(C) Determine the values where the two prices are the same.

$$x = 2$$

$$y = 2(2) + 30 = 34$$

$$x = 8$$

$$y = 2(8) + 30 = 46$$

Stocks are same price at \$34 and \$46.

Example 8:

Determine two integers such that the sum of the smaller number and twice the larger number is 46. Also, when the square of the smaller number is decreased by three times the larger, the result is 93. x & y

(A) Write the system of equations that relates to the problem.

① $x + 2y = 46$

② $x^2 - 3y = 93$

(B) Solve the system of equations using elimination or substitution.

$3① + 2②$

$$\begin{array}{rl} & 3x + \cancel{6y} = 138 \\ + & 2x^2 \quad - \cancel{6y} = 186 \\ \hline & 2x^2 + 3x = 324 \end{array}$$

$2x^2 + 3x - 324 = 0$

$x = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-324)}}{2(2)}$

$x = \dfrac{-3 \pm \sqrt{2601}}{4}$

$x = \dfrac{-3 \pm 51}{4}$

$x = \dfrac{-3-51}{4} = \dfrac{-54}{4} = \cancel{-13.5}$

$x = \dfrac{-3+51}{4} = \dfrac{48}{4} = \underline{12}$

$12 + 2y = 46$

$2y = 46 - 12$

$\dfrac{2y}{2} = \dfrac{34}{2}$

$y = \underline{17}$

Example 9:
Determine two integers that have the following relationships: Fourteen more than twice the first integer gives the second integer. The second integer increased by one is the square of the first integer.

x & y

(A) Write the system of equations that relates to the problem.

① $14 + 2x = y$

② $y + 1 = x^2$

(B) Solve the system of equations using elimination or substitution.

Sub ① into ②

$14 + 2x + 1 = x^2$

$0 = x^2 - 2x - 15$

$0 = (x+3)(x-5)$

$x = -3, x = 5$

$x = -3$

$y = 2(-3) + 14 = -6 + 14 = 8$

$x = 5$

$y = 2(5) + 14 = 10 + 14 = 24$

Textbook Questions: page 451 - 455 #3, 4, 5, 7, 10, 20