Math 2200

8.2B Solving Systems of Equations Algebraically

Determining if a Point is a Solution to a System of Equations

This a very simple process. Simply plug the point into both equations. If it satisfies **both** equations, the point is a solution to that system.

Example 1:

Sam solved the system:

$$\begin{aligned} &(1)\quad 2x - y = 9 \\ &(2)\quad y = x^2 - 4x \end{aligned}$$

x y

His solution is $(3, -3)$. Is his solution correct?

(1) $2(3) - (-3) = 9$

$6 + 3 = 9$

$9 = 9$ ✓

(2) $-3 = (3)^2 - 4(3)$

$-3 = 9 - 12$

$-3 = -3$ ✓

$\therefore (3, -3)$ is a solution.

Predicting the Number of Solutions

Recall the discriminant from Chapter 4.

$$D = b^2 - 4ac$$

We used the discriminant to determine the number of roots a quadratic function had.

$D > 0$, two roots

$D = 0$, one root

$D < 0$, no roots

We can use the discriminant to determine the number of solutions in a linear-quadratic and a quadratic-quadratic system of equations. The process is the same as solving a system of equations except once you have the final quadratic, simply find the discriminant.

Example 2:

How many solutions does the following system of equations have?

$y = y$

(1) $y = 3x + 5$

(2) $y = 3x^2 - 2x - 4$

$3x+5 = 3x^2 - 2x - 4$

$0 = 3x^2 - 2x - 3x - 4 - 5$

$0 = 3x^2 - 5x - 9$

$D = b^2 - 4ac$

$D = (-5)^2 - 4(3)(-9)$

$D = 133 > 0$

$\therefore$ There are two solutions.

Example 3:

How many solutions does the following system of equations have?

$y = 2x - 7$

$y = x^2 + 2x$

$y = y$

$x^2 \cancel{+2x} = \cancel{2x} - 7$

$x^2 + 7 = 0$

$D = b^2 - 4ac$

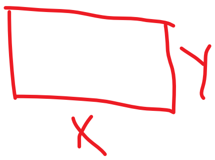
$D = 0^2 - 4(1)(7)$

$D = -28 < 0$

$\therefore$ No solution.

More Applications Involving Systems of Equations

Example 4:
A rectangular field has a perimeter of 500 m and an area of 14400 m^2. Find the length of the sides.



$$\frac{2x+2y}{2}=\frac{500}{2}$$

① $x+y=250$

Solve ① for y.

$$y=-x+250$$

Sub ① into ②

② $x\cdot y=14400$

$$x(-x+250)=14400$$

$$-x^2+250x-14400=0$$

$$x^2-250x+14400=0$$

$$x=\frac{-(-250)\pm\sqrt{(-250)^2-4(1)(14400)}}{2(1)}$$

$$x=\frac{250\pm\sqrt{4900}}{2}$$

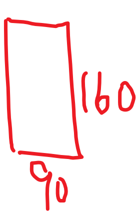
$$x=\frac{250\pm 70}{2}$$

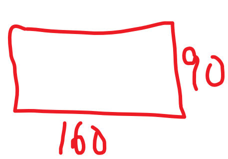
$$x=\frac{250-70}{2}, \quad x=\frac{250+70}{2}$$

$$x=90 \qquad x=160$$

$$y=-90+250 \qquad y=-160+250$$

$$y=160 \qquad y=90$$





Example 5:

Find a system of equations that represents two natural numbers that differ by 4 and whose squares have a sum of 136? x; y

$x - y = 4$

or

① $x - 4 = y$

Sub ① into ②

② $x^2 + y^2 = 136$

$x^2 + (x-4)^2 = 136$

$x^2 + x^2 - 8x + 16 - 136 = 0$

$\frac{2x^2 - 8x - 120}{2} = \frac{0}{2}$

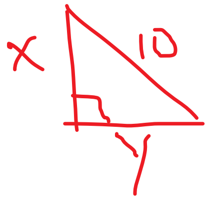
$x^2 - 4x - 60 = 0$

$(x+6)(x-10) = 0$

$\cancel{x = -6}$, $x = \underline{10}$

$y = 10 - 4 = \underline{6}$

Example 6:
A right triangle has a hypotenuse 10 cm long. If the perimeter is 22 cm, find the lengths of the other two sides.



$x + y + 10 = 22$

$x + y = 22 - 10$

(1) $x + y = 12$

Solve (1) for y

$y = -x + 12$

Sub (1) into (2)

$x^2 + y^2 = 10^2$

(2) $x^2 + y^2 = 100$

$x^2 + (-x + 12)^2 = 100$

$x^2 + x^2 - 24x + 144 - 100 = 0$

$$\frac{2x^2 - 24x + 44}{2} = \frac{0}{2}$$

$x^2 - 12x + 22 = 0$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(22)}}{2(1)}$$

$$x = \frac{12 \pm \sqrt{56}}{2}$$

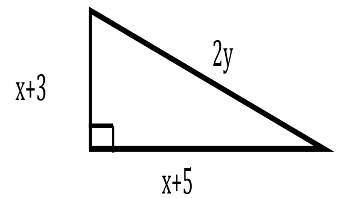
$$x = \frac{12 \pm 7.48}{2}$$

$$x = \frac{12 - 7.48}{2}, \quad x = \frac{12 + 7.48}{2}$$

$x = 2.25$, $x = 9.75$

$y = 12 - 2.25$
$= 9.75$, $y = 12 - 9.75$
$y = 2.25$

Example 7:
The right triangle shown has a perimeter of 24 cm and an area of $(2y+14)$ cm^2. Algebraically determine the value(s) of x and y.



$x+3+x+5+2y=24$

$2x+2y+8=24$

$2x+2y=24-8$

$\frac{2x+2y}{2}=\frac{16}{2}$

① $x+y=8$

Solve ① for y

$y=-x+8$

Sub ① into ②

$A=\frac{1}{2}b \cdot h$

$2y+14=\frac{1}{2}(x+5)(x+3)$

$2(2y+14)=\frac{1}{\cancel{2}}(x^2+8x+15)\cdot\cancel{2}$

② $4y+28=x^2+8x+15$

$4(-x+8)+28=x^2+8x+15$

$-4x+32+28=x^2+8x+15$

$0=x^2+8x+4x+15-60$

$0=x^2+12x-45$

$0=(x-3)(x+15)$

$x=3$, $\cancel{x=-15}$

$y=-3+8$

$y=5$

Example 8:
A sky diver jumped from a tower and fell freely for several seconds before releasing her parachute. Her height, h, in metres, above the ground at any time is given by $h = -4.9t^2 + 5000$ before she released her parachute, and $h = -4t + 4000$ after she released her parachute.

(A) If t represents time in seconds, how long after jumping did she release her parachute?

$h = h$

$-4.9t^2 + 5000 = -4t + 4000$

$0 = 4.9t^2 - 4t - 1000$

$t = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(4.9)(-1000)}}{2(4.9)}$

$t = \dfrac{4 \pm \sqrt{19616}}{9.8}$

$t = \dfrac{4 \pm 140.1}{9.8}$

$t = \dfrac{4 - 140.1}{9.8} = \cancel{-13.9}\,s$

$t = \dfrac{4 + 140.1}{9.8} = 14.7\,s.$

(B) How high was she above the ground at that time?

$h = -4(14.7) + 4000 = 3941.2\,m$

Textbook Questions: page 451 - 455 #1, 2, 6, 8, 9, 10, 11, 13, 19