Math 2200

9.1 Linear Inequalities

In Grade 9, the concept of inequalities was introduced for one variable.

Example 1:
Solve:

(A) $2x - 3 \geq 9$

$2x \geq 9+3$

$\frac{2x}{2} \geq \frac{12}{2}$

$x \geq 6$

(B) $-4x + 5 \leq -15$

$-4x \leq -15-5$

$\frac{-4x}{-4} \leq \frac{-20}{-4}$

$x \geq 5$

Linear Inequalities in Two Variables

A linear inequality in two variables may be in one of the following four forms:

- $Ax + By < C$
- $Ax + By \leq C$
- $Ax + By > C$
- $Ax + By \geq C$

Where A, B and C are real numbers.

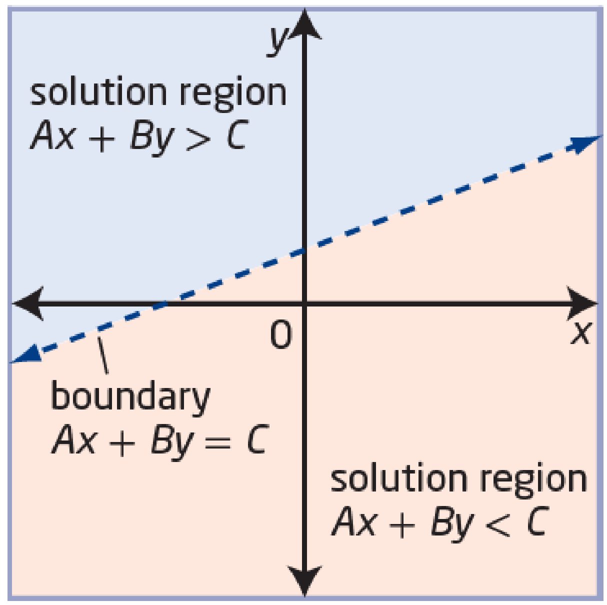
An inequality in the two variables x and y describes a region on the Cartesian plane. The ordered pair (x, y) is a solution to a linear inequality if the inequality is true when the values x and y are substituted into the inequality. The sets of points that satisfy a linear inequality can be called the solution set or **solution region**.

The line related to the linear equality $Ax + By = C$, or **boundary**, divides the Cartesian plane into two solution regions.

- For one solution region, $Ax + By > C$ is true.
- For the other solution region, $Ax + By < C$ is true.

boundary

- a line or curve that separates the Cartesian plane into two regions
- may or may not be part of the solution region
- drawn as a solid line and included in the solution region if the inequality involves $\leq$ or $\geq$
- drawn as a dashed line and not included in the solution region if the inequality involves $<$ or $>$



Example 2:

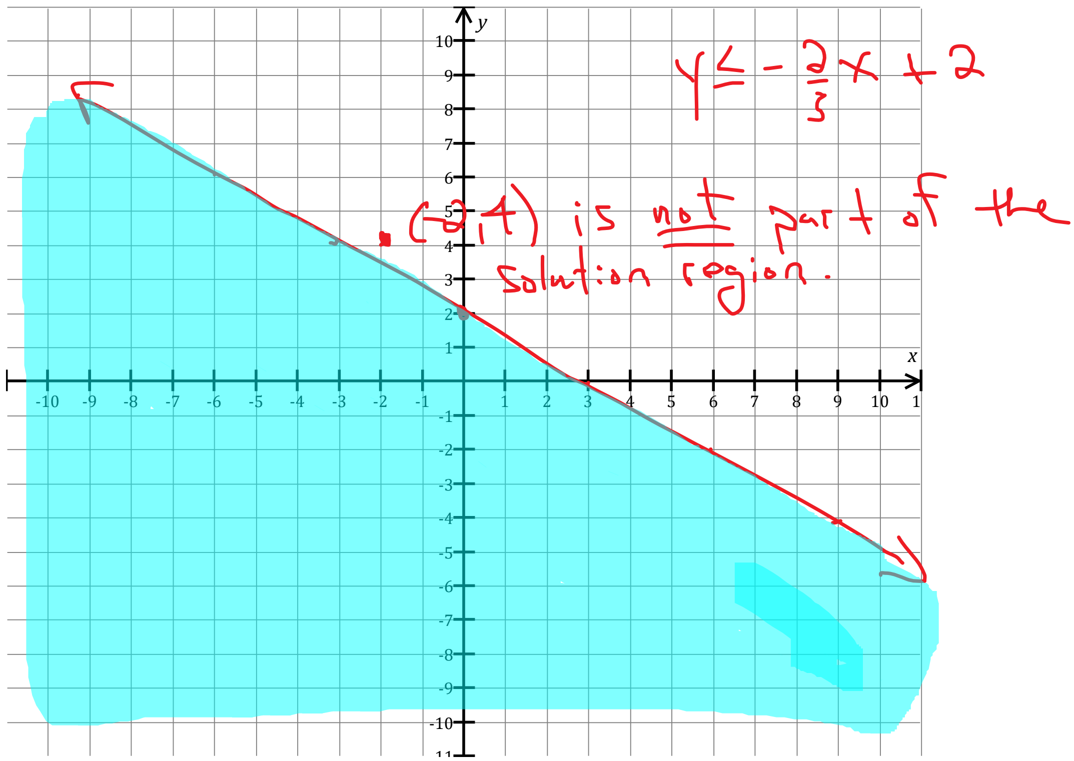
(A) Graph $2x + 3y \leq 6$

Solve as you would any equation, in $y=mx+b$ form.

$3y \leq -2x + 6$

$\frac{3y}{3} \leq \frac{-2x+6}{3}$

$y \leq -\frac{2}{3}x + 2$ Boundary line: $y = -\frac{2}{3}x + 2$



x y

(B) Determine if the point $(-2, 4)$ is part of the solution.

$2x + 3y \le 6$

$2(-2) + 3(4) \le 6$

$-4 + 12 \le 6$

$8 \not\le 6$

$\therefore$ (-2,4) is not part of the solution region.

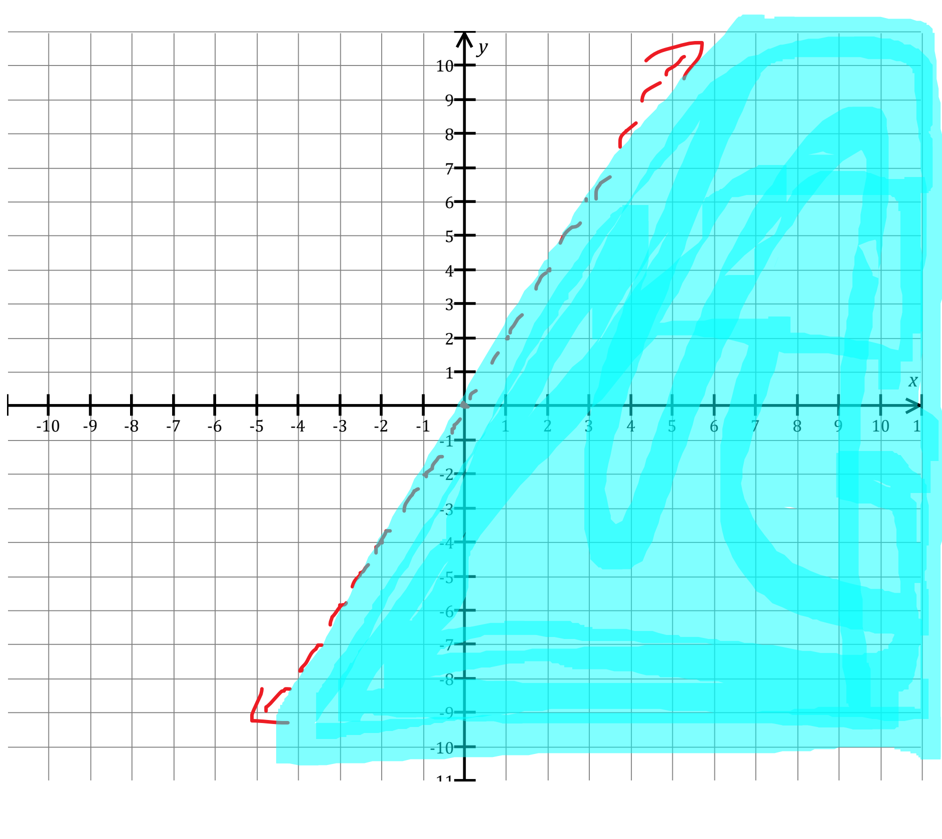
Example 3:

Graph: $10x - 5y > 0$.

$-5y > -10x$

$\frac{-5y}{-5} > \frac{-10x}{-5}$

$y < 2x$



Notice that when the inequality is changed to slope-intercept or $y = mx + b$ form, it's easy to see that you shade the y-axis above or below the boundary depending on if the inequality is greater than or less than.

Example 4:
Complete the table:

Inequality	shade above or below	broken or solid line
$y \leq -2x + 8$	below	solid
$x + y > 2$	above	broken

$y > -x + 2$

Example 5:
The solution region for the inequality $5x - 3y > 10$ is above the line since it contains a "greater than sign". Do you agree or disagree with this statement. Explain your reasoning.

$5x - 3y > 10$

$\frac{-3y}{-3} > \frac{-5x + 10}{-3}$

$y < \frac{5}{3}x - \frac{10}{3}$

Sign changes to "less than" when we divide by a negative. Therefore we shade below.

Example 6:
A student was asked to graph $3x - 2y > 12$. His solution is shown below. Identify and correct his errors.

- didn't switch sign when dividing by a negative.
- boundary line should be broken.

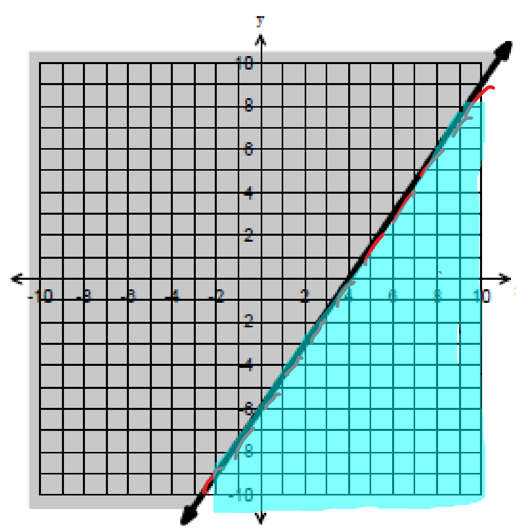
Student Solution:

$3x - 2y > 12$

$-2y > -3x + 12$

$y > \frac{3}{2}x - 6$

$<$



Word Problems

Inequalities are applicable to real life contexts. Problems that can be expressed as an inequality in two variables require two unknown quantities under certain constraints. As always, translate the word problem into mathematics – this time an inequality. Define the variables that are being used to represent the unknown quantities. The shaded feasible region represents all possible combinations for the two quantities.

Example 7:
With two minutes left in a basketball game, your team is 12 points behind. What are two different numbers of 2-point and 3-point shots your team could score to earn at least 12 points? Graph this situation.

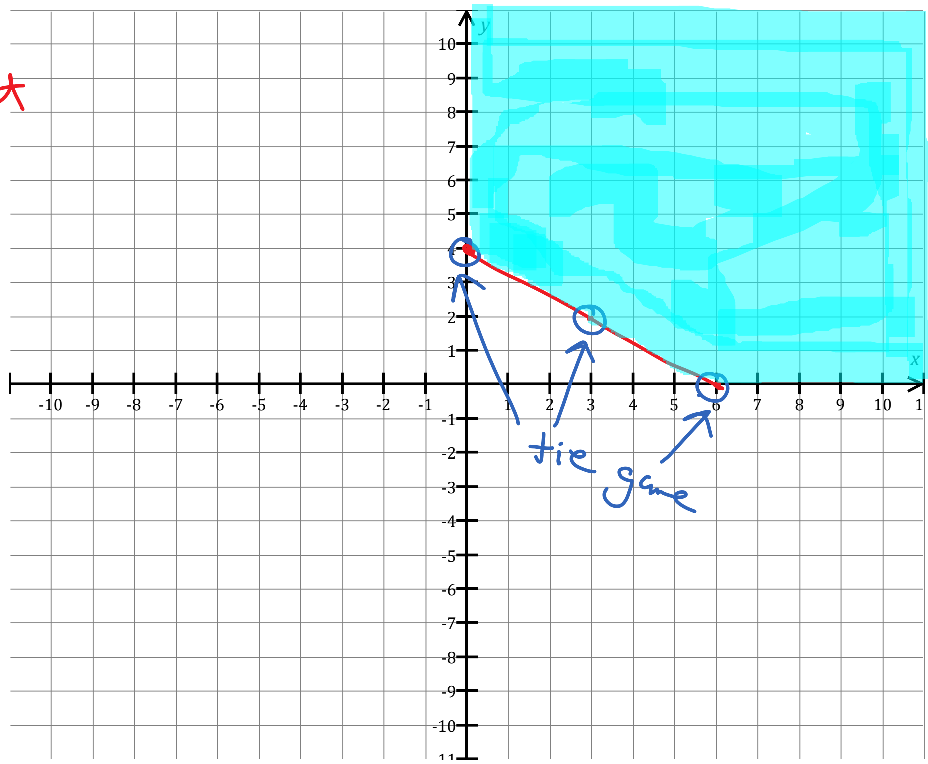
x: # 2 point basket
y: # 3 point basket

$2x + 3y \geq 12$

$3y \geq -2x + 12$

$\frac{3y}{3} \geq \frac{-2x}{3} + \frac{12}{3}$

$y \geq -\frac{2}{3}x + 4$



Example 8:

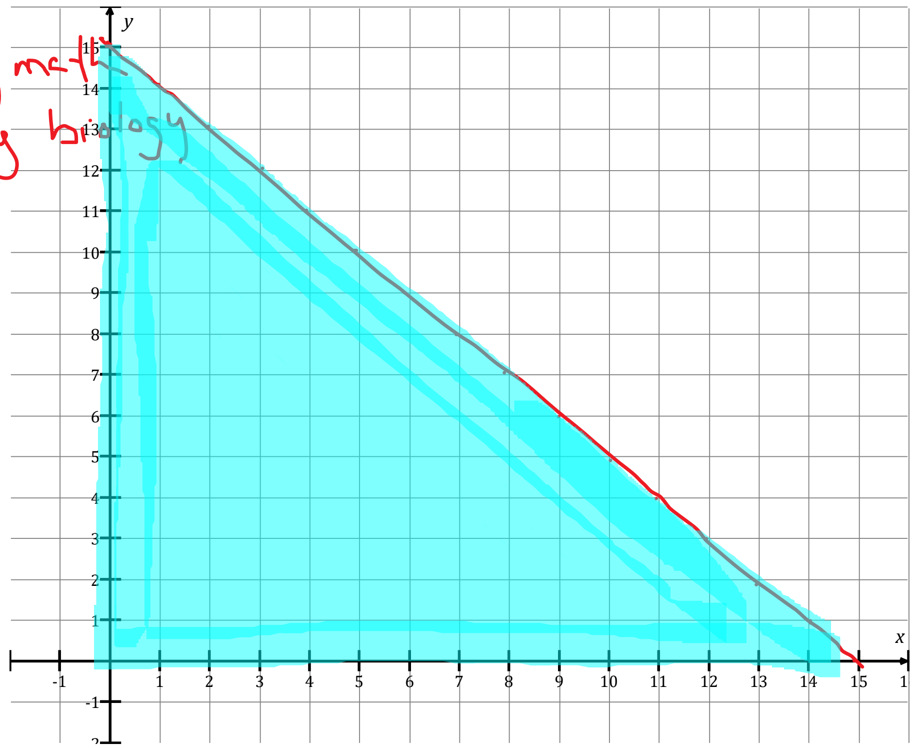
Susan plans to spend a maximum of 15 hours reviewing Math and Biology in preparation for examinations. Draw a graph showing how much time she could spend studying each subject.

X: hours studying math

Y: hours studying biology

$x + y \leq 15$

$y \leq -x + 15$



Example 9:
A contractor has at most one hundred tonnes of soil to be moved using two trucks. One truck has a 4 ton capacity and the other has a 5 ton capacity. Make a graph to show the various combinations of loads the two trucks could carry to complete the job.

X: # 4 ton loads

Y: # 5 ton loads

$4x + 5y \leq 100$

$\frac{5y}{5} \leq \frac{-4x + 100}{5}$

$y \leq -\frac{4}{5}x + 20$

Textbook Questions: page 472 - 473 #1, 3, 4, 8, 9, 11, 12