Math 2200

9.2 Quadratic Inequalities in One Variable

Quadratic Inequalities in one variable can have one of the four following forms:

- $ax^2 + bx + c < 0$
- $ax^2 + bx + c \leq 0$
- $ax^2 + bx + c > 0$
- $ax^2 + bx + c \geq 0$

Quadratic Inequalities can be solved graphically or algebraically. The solution to a quadratic inequality in one variable can have no values, one value or an infinite number of values. The solution can be represented as a set of Domain values written in set notation.

Solving Quadratic Inequalities in One Variable by Graphing

To determine the solution of the inequality, graph the corresponding function and look for the values of x for which the graph of $f(x)$ lies on, above or below the x-axis, depending on which boundary region is desired.

Example 1:

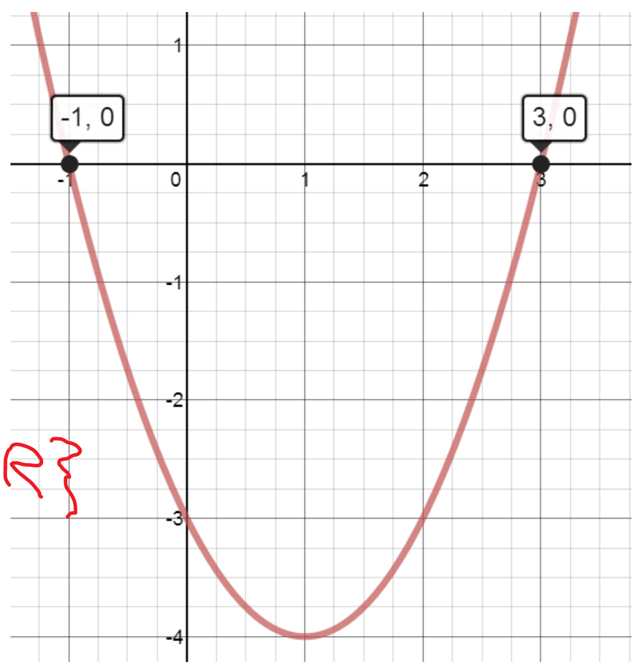
(A) Solve $x^2 - 2x - 3 \leq 0$

Using https://www.desmos.com/calculator, we get:

$y = x^2 - 2x - 3$

$y = 0$ (x-axis)

$\{x \mid -1 \leq x \leq 3, x \in R\}$



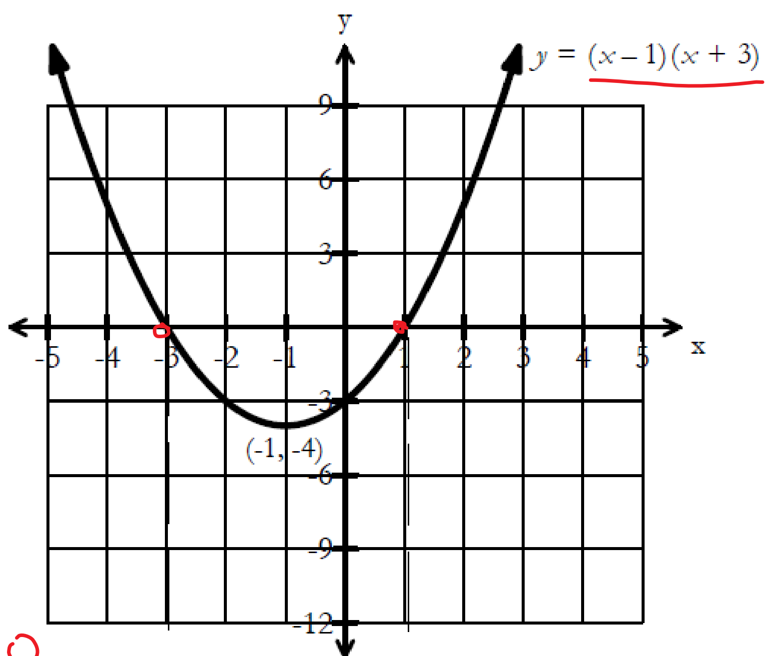
(B) Solve $x^2 - 2x - 3 \geq 0$

$\{x \mid x \leq -1, x \geq 3, x \in R\}$

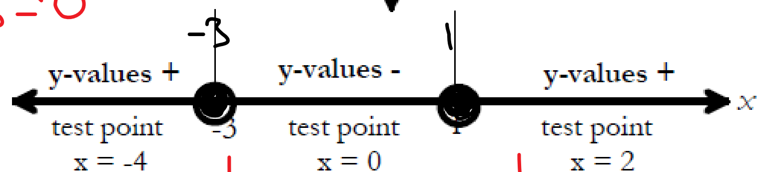
Solving Quadratic Inequalities in One Variable Algebraically

Solve the related equation to find the roots. Then, use a number line and test points to determine the intervals that satisfy the inequality. Be sure to use hollow dots for $<$ or $>$ and solid dots for $\leq$ or $\geq$.

Example 2:
Solve algebraically: $x^2 + 2x - 3 \leq 0$



$x^2+2x-3 \leq 0$



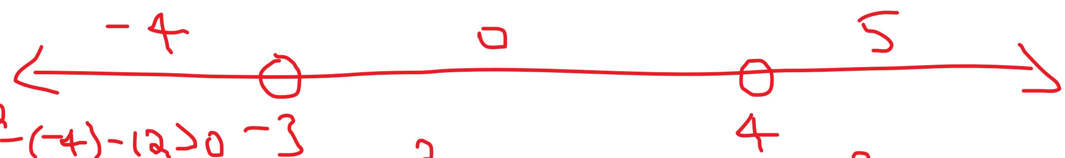
$(-4)^2+2(-4)-3 \leq 0$	$(0)^2+2(0)-3 \leq 0$	$(2)^2+2(2)-3 \leq 0$
$16-8-3 \leq 0$	$-3 \leq 0$ ✓	$4+4-3 \leq 0$
$5 \nleq 0$ ✗		$5 \nleq 0$ ✗

$\{x \mid -3 \leq x \leq 1, x \in R\}$

Example 3:
Solve algebraically: $x^2 - x - 12 > 0$

$(x+3)(x-4)=0$

$x=-3,\ x=4$



$(-4)^2-(-4)-12>0$
$16+4-12>0$
$8>0$ ✓

$(0)^2-0-12>0$
$-12 \not> 0$ ✗

$(5)^2-5-12>0$
$25-5-12>0$
$8>0$ ✓

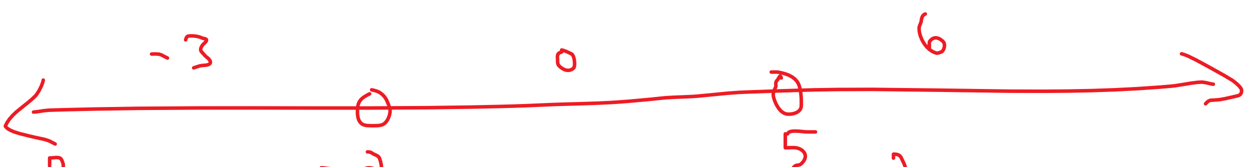
$\{x \mid x<-3,\ x>4,\ x\in R\}$

Example 4:
Solve algebraically: $-x^2 + 3x + 10 < 0$

$x^2-3x-10=0$

$(x+2)(x-5)=0$

$x=-2,\ x=5$



$-(-3)^2+3(-3)+10<0$
$-9-9+10<0$
$-8<0$ ✓

$-(0)^2+3(0)+10<0$
$10 \not< 0$ ✗

$-(6)^2+3(6)+10<0$
$-36+18+10<0$
$-8<0$ ✓

$\{x \mid x<-2,\ x>5,\ x\in R\}$

Solving a Quadratic Inequality with Minimum or Maximum Constraint

We can compare inequalities and find solutions to these problems as we did in absolute value functions.

Example 5:
Given $A = x^2 - 7$ and $B = -4x + 5$, for what values of x is $A < B$?

$A < B$

$x^2 - 7 < -4x + 5$

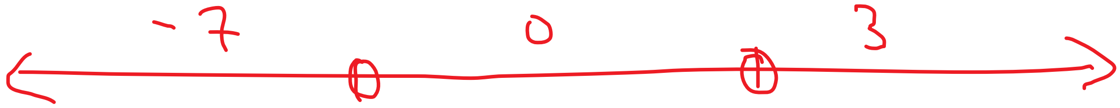
$x^2 + 4x - 7 - 5 < 0$

$x^2 + 4x - 12 < 0$

$x^2 + 4x - 12 = 0$

$(x-2)(x+6) = 0$

$x = 2,\ x = -6$



$(-7)^2 + 4(-7) - 12 < 0$

$49 - 28 - 12 < 0$

$9 \not< 0$ ✗

$(0)^2 + 4(0) - 12 < 0$

$-12 < 0$ ✓

$(3)^2 + 4(3) - 12 < 0$

$9 + 12 - 12 < 0$

$9 \not< 0$ ✗

$A < B$ for $\{x \mid -6 < x < 2, x \in R\}$

Example 6:
When a projectile is fired into the air, its height h, in metres, t seconds later is given by the equation $h(t) = 11t - 3t^2$. When is the projectile at least 6 m above the ground?

at least 6m: ≥ 6

$11t - 3t^2 \geq 6$

$-3t^2 + 11t - 6 \geq 0$

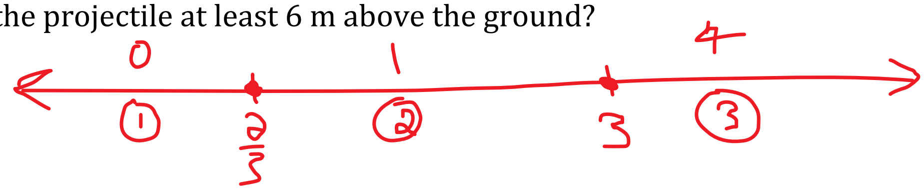
$3t^2 - 11t + 6 = 0$ $\quad \frac{18}{2\ 9}$

$3t^2 - 9t - 2t + 6 = 0$

$3t(t-3) - 2(t-3) = 0$

$(t-3)(3t-2) = 0$

$t = 3,\ t = \frac{2}{3}$



① $-3(0)^2 + 11(0) - 6 \geq 0$

$-6 \not\geq 0$ ✗

② $-3(1)^2 + 11(1) - 6 \geq 0$

$2 \geq 0$ ✓

③ $-3(4)^2 + 11(4) - 6 \geq 0$

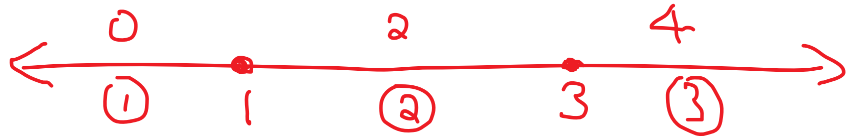
$-10 \not\geq 0$ ✗

$\{t \mid \frac{2}{3} \leq t \leq 3, t \in R\}$

Projectile is at or above 6m from $\frac{2}{3}$s to 3s.

Example 7:
When a baseball is hit by a batter, the height of the ball, $h(t)$, at time t, is determined by the equation $h(t) = -16t^2 + 64t + 4$. For which interval of time is the height of the ball greater than or equal to 52 feet?

$-16t^2 + 64t + 4 \geq 52$



$-16t^2 + 64t + 4 - 52 = 0$

$\frac{-16t^2 + 64t - 48}{-16} = \frac{0}{-16}$

$t^2 - 4t + 3 = 0$

$(t - 1)(t - 3)$

$t = 1, t = 3$

① $-16(0)^2 + 64(0) - 48 \geq 0$

$-48 \ngeq 0$ ✗

② $-16(2)^2 + 64(2) - 48 \geq 0$

$16 \geq 0$ ✓

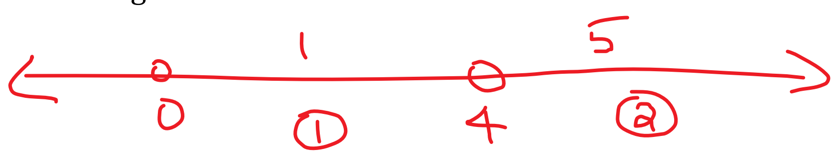
③ $-16(4)^2 + 64(4) - 48 \geq 0$

$-48 \ngeq 0$ ✗

$\therefore \{t \mid 1 \leq t \leq 3, t \in R\}$

Example 8:
The surface area, A, of a cylinder with radius r is given by the formula $A = 2r^2 - 5r$. What possible radii would result in an area that is greater than 12 cm^2 ?

$2r^2 - 5r > 12$



$2r^2 - 5r - 12 > 0$

$2r^2 - 8r + 3r - 12 = 0$

$2r(r - 4) + 3(r - 4) = 0$

$(r - 4)(2r + 3) = 0$

$r = 4, \cancel{r = -\frac{3}{2}}$

① $2(1)^2 - 5(1) > 12$

$-3 \ngtr 12$ ✗

② $2(5)^2 - 5(5) > 12$

$25 > 12$ ✓

$\{r \mid r > 4, r \in R\}$

A radius larger than 4cm would give an area greater than 12cm^2.

Textbook Questions: page 484 - 485 #1, 2, 3, 4, 5 (use roots and test points), 7, 8, 9