Math 2200

9.3 Quadratic Inequalities in Two Variables

Quadratic Inequalities in two variables can have one of the four following forms:

- $y < ax^2 + bx + c$
- $y \leq ax^2 + bx + c$
- $y > ax^2 + bx + c$
- $y \geq ax^2 + bx + c$

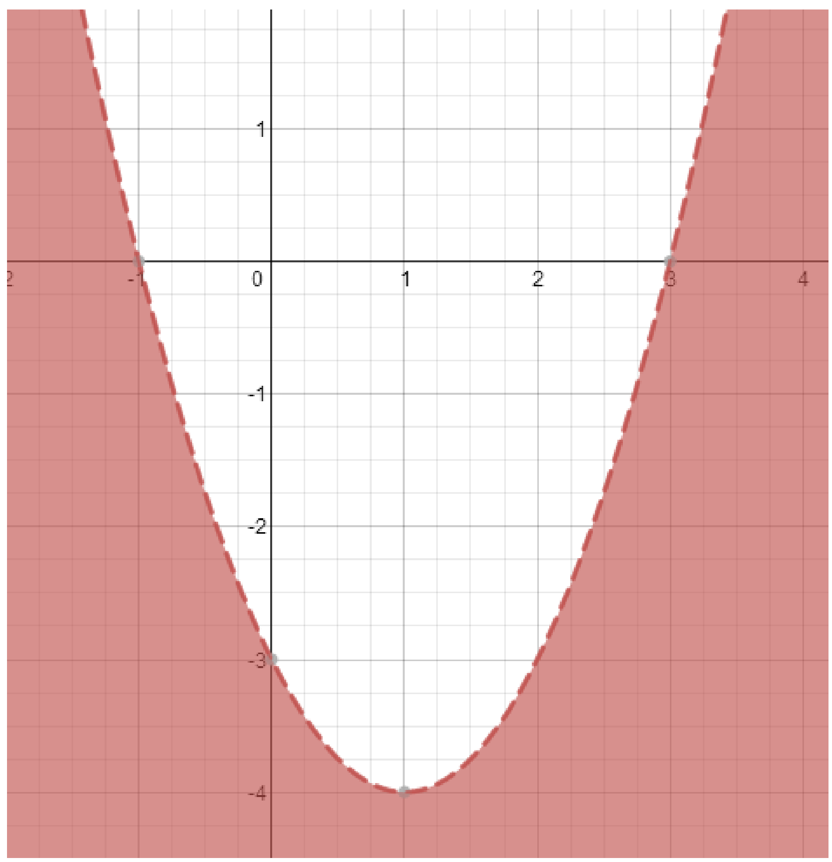
A quadratic inequality in two variables represents a region of the Cartesian plane with a parabola as the boundary. The graph of a quadratic inequality is the set of points (x, y)that are solutions to the inequality.

Example 1:
Graph $y < x^2 - 2x - 3$

Using https://www.desmos.com/calculator

We can use test points but it's much easier to look at where the area contains the y-axis.

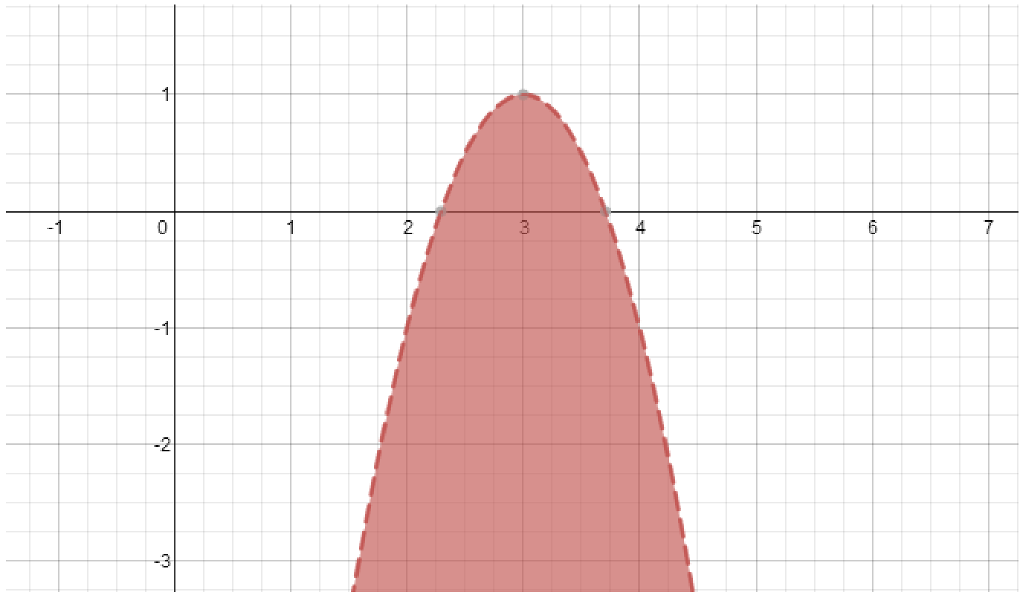
Since it's "$y <$", where is the y-axis below the boundary?



Example 2:

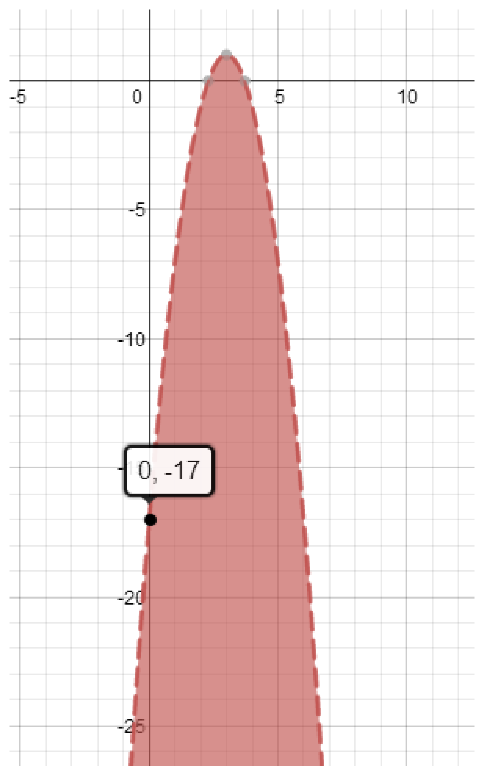
(A) Graph $y < -2(x-3)^2 + 1$

Notice that you can't see the y-intercept in this graph.



If we zoom out we can eventually find the y-intercept, but this isn't practical.

It's best just to learn how to visualize that, eventually the y-intercept is under the curve and then shade accordingly.



(B) Determine if the point $(\overset{x}{2}, \overset{y}{-4})$ is a solution to the inequality.

$-4 < -2(2-3)^2 + 1$ $\therefore$ $(2, -4)$ is a solution to the inequality.

$-4 < -1$ ✓

Example 3:
Sketch the graph of $y < -2(x-1)^2 + 4$.

Sketch boundary line:

$y = -2(x-1)^2 + 4$

vertex: $(1, 4)$

$y = -2(x^2 - 2x + 1) + 4$

$y = -2x^2 + 4x - 2 + 4$

$y = -2x^2 + 4x + 2$

y-int: $(0, 2)$

$$\frac{-2x^2 + 4x + 2}{-2} = \frac{0}{2}$$

$x^2 - 2x - 1 = 0$

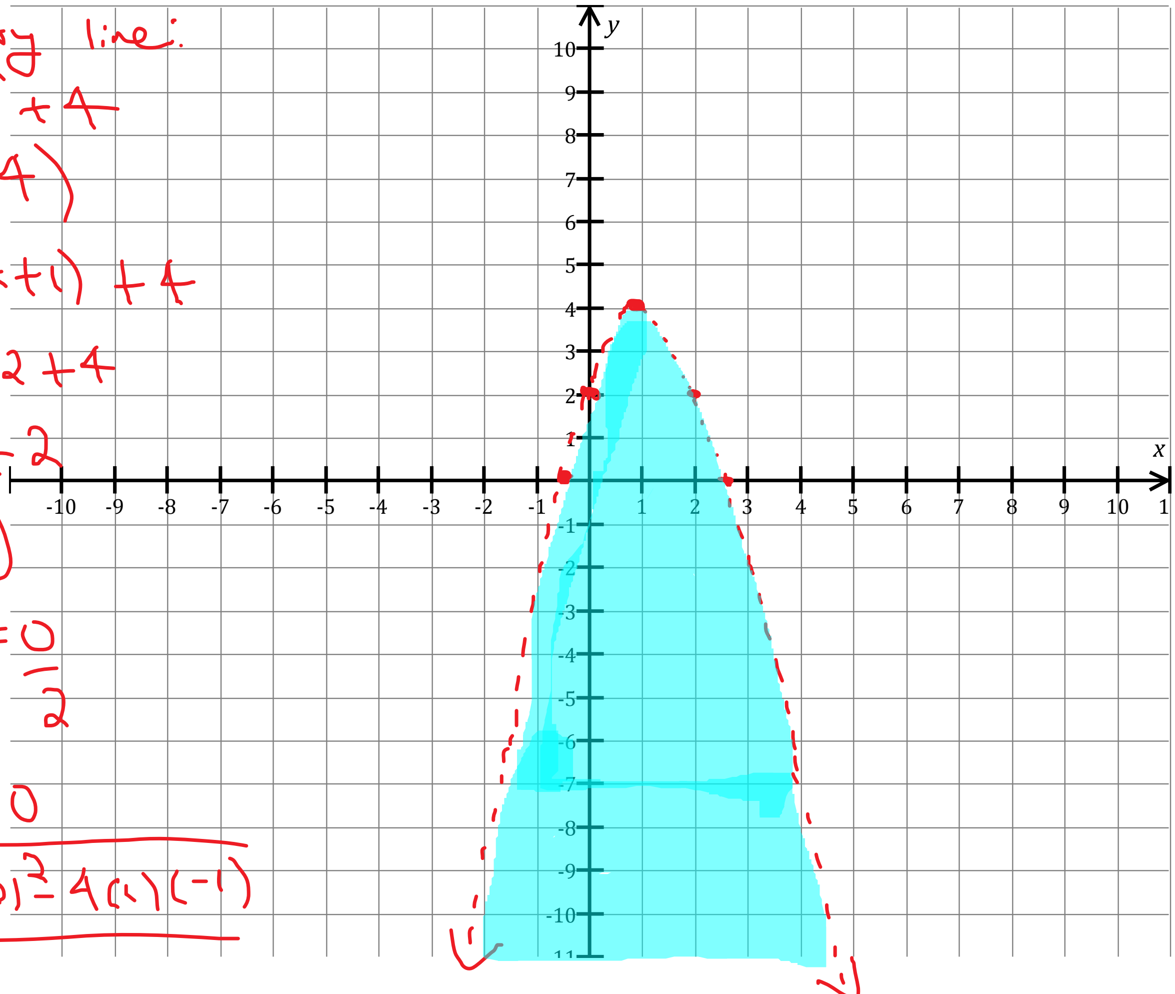
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm 2.8}{2}$$

$$x = \frac{2 + 2.8}{2}, \quad x = \frac{2 - 2.8}{2}$$

$x = 2.4 \qquad x = -0.4$



or test points $(0,0)$

$y < -2(x-1)^2 + 4$

$0 < -2(0-1)^2 + 4$

$0 < -2 + 4$

$0 < 2$ ✓

Example 4:
Sketch the graph of $y \geq x^2 - 4x - 5$.

y-int: $(0,-5)$

$x^2 - 4x - 5 = 0$

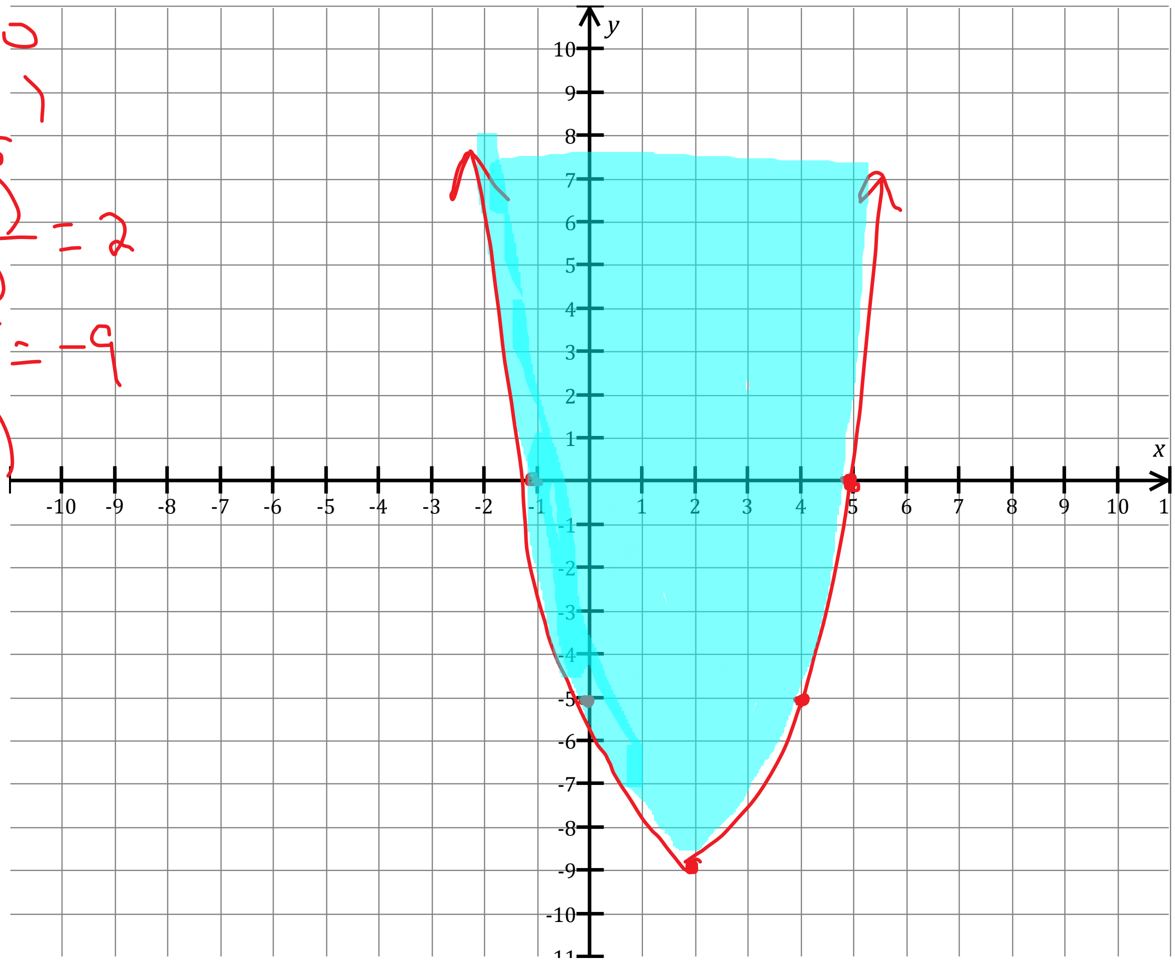
$(x+1)(x-5)$

$x=-1, x=5$

$p = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$

$q = 2^2 - 4(2) - 5 = -9$

vertex $(2,-9)$



Example 5:

Sketch the graph of $y < \frac{1}{3}(x-3)(x+3)$.

x-ints: $x = -3$, $x = 3$

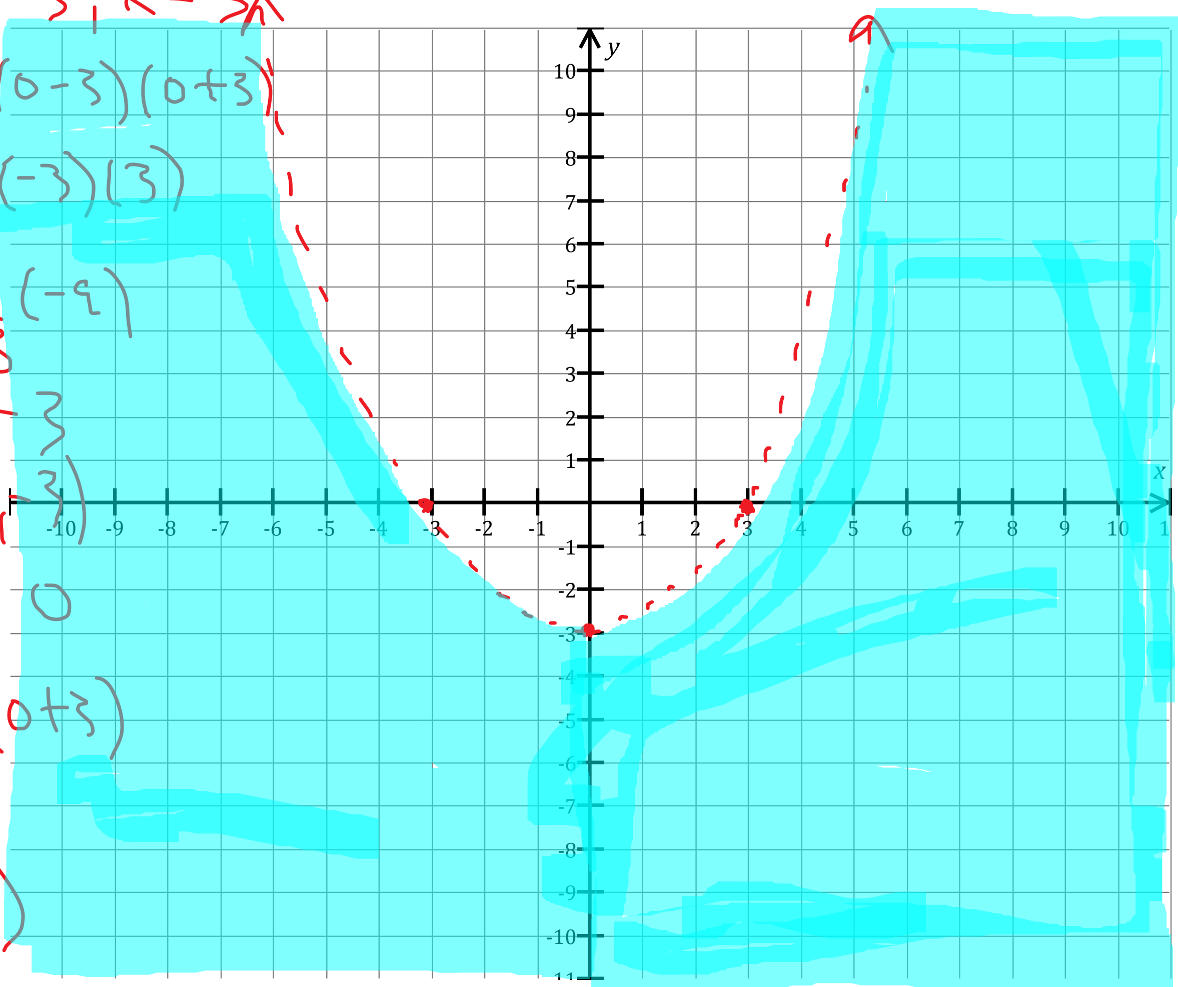
y-int: $y = \frac{1}{3}(0-3)(0+3)$

$= \frac{1}{3}(-3)(3)$

$= \frac{1}{3}(-9)$

$= -3$

$(0, -3)$



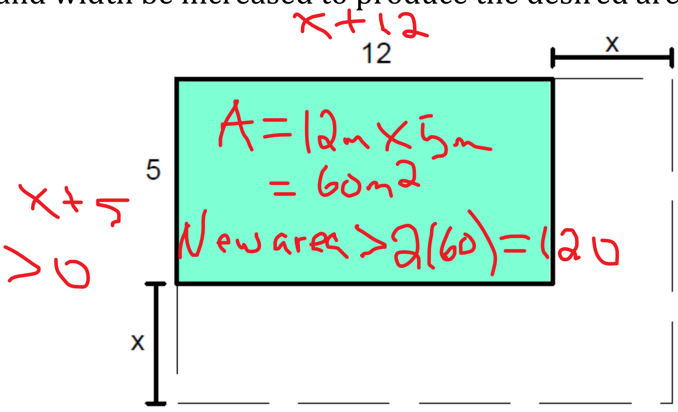
$p = \frac{-3+3}{2} = 0$

$q = \frac{1}{3}(0-3)(0+3)$

$= -3$

Vertex $(0, -3)$

Example 6:
The base of a rectangular bin currently has dimensions 12 m by 5 m. The base is to be enlarged by an equal amount on the width and length so that the area is more than doubled. How much should the length and width be increased to produce the desired area?



$$l \cdot w = A$$

$$(x+5)(x+12) > 120$$

$$x^2 + 12x + 5x + 60 - 120 > 0$$

$$x^2 + 17x - 60 > 0$$

$$(x+20)(x-3) > 0$$

$$x + 20 > 0, \quad x - 3 > 0$$

$$\cancel{x > -20}, \quad \boxed{x > 3}$$

The length and width should be increased by more than 3m.

Example 7:

Determine the inequality associated with the following graph:

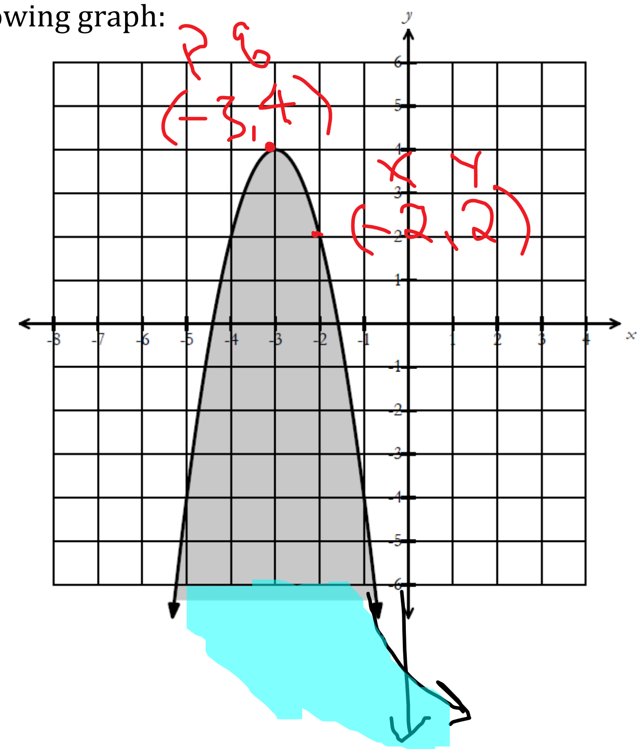
$$y = a(x-p)^2 + q$$

$$2 = a[-2-(-3)]^2 + 4$$

$$2 - 4 = a$$

$$a = -2$$

$$y \leq -2(x+3)^2 + 4$$



Example 8:
What inequality makes the following graph?

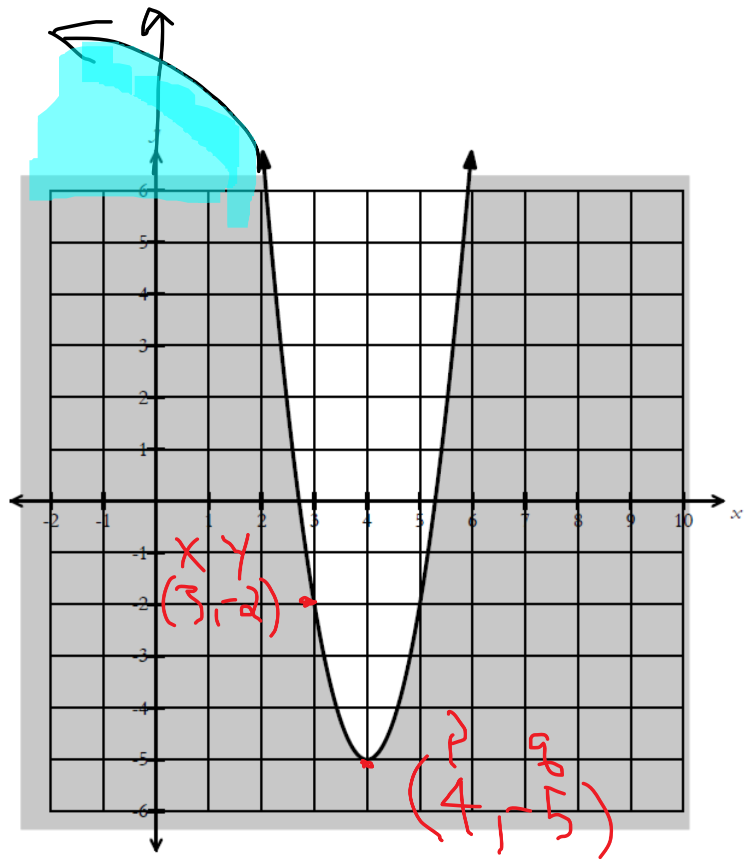
$y = a(x-p)^2 + q$

$-2 = a(3-4)^2 - 5$

$-2 + 5 = a$

$a = 3$

$y \leq 3(x-4)^2 - 5$



Textbook Questions: page 496 - 500 #1, 3, 5, 6, 7, 8, 9