Math 2201 **Date:**_______________

1.3 Using Reasoning to Find a Counterexample to a Conjecture

When students make a conjecture, they will attempt to either prove their conjecture by constructing a valid argument or they may disprove their conjecture by finding a **counterexample**. One counterexample is sufficient to disprove a conjecture.

Example 1:
To introduce this topic, students could be asked to come up with counterexamples for statements such as the following:

(A) Any piece of furniture having four legs is a table.

False. A chair has 4 legs.

(B) If something has a knob on it, it's a faucet.

False. A door has a knob.

(C) If the grass is wet, it is raining.

False. Sprinkler may be on.

(D) All students like sports.

False. Kamryn.

Use strategies, such as drawing pictures or using measuring instruments, when looking for a counterexample. After a counterexample is found, students should revise their conjecture to include the new evidence.

It is important to realize that not finding a counterexample does not prove the conjecture. You need to try ALL possible cases to prove a conjecture.

A mathematician named Goldbach is famous for discovering a conjecture which has not yet been proved to be true or false. Goldbach conjectured that every even number greater than 2 can be written as the sum of two prime numbers. For example, 4 = 2 + 2, 6 = 3 + 3, and 8 = 3 + 5. No one has ever proved this conjecture to be true or found a counterexample to show that it is false, so the conjecture remains just that.

Example 2:
Provide a counterexample for each of the following conjectures:

(A) If a number is divisible by 2, then it is divisible by 4.

$\frac{10}{4} = 2.5$ False.

(B) If $x + 4 > 0$, then x is positive.

$-3+4 > 0$ False.

$1 > 0$

(C) For all positive numbers n, $\frac{1}{n} < n$.

$n = 2$ $\frac{1}{2} < 2$

$\frac{1}{1} \not< 1$ False.

$n = 3$ $\frac{1}{3} < 3$

$n = 4$ $\frac{1}{4} < 4$

Example 3:
Respond to the following:

(A) For all real numbers x, the expression x^2 is greater than or equal to x. Do you agree? Justify your answer.

No.

$x = 1 \quad 1^2 \geq 1$
$1 \geq 1$ ✓

$x = 0 \quad 0^2 \geq 0$
$0 \geq 0$ ✓

$x = 2 \quad 2^2 \geq 2$
$4 \geq 2$ ✓

$x = 0.5 \quad (0.5)^2 \geq 0.5$
$0.25 \ngeq 0.5$

$x = -2 \quad (-2)^2 \geq -2$
$4 \geq -2$

False.

(B) If $x > 0$, then $\sqrt{x} < x$. Do you agree? Justify your answer.

No

$x = 1 \quad \sqrt{1} < 1$
$1 \not< 1$ False.

Textbook Questions: page 22 - 23 #1a, b, c, f, g, 2, 3, 4, 6, 9, 11, 14.