Math 2201 **Date:__________________**

1.4 Proving Conjectures: Deductive Reasoning

To come up with a conjecture using inductive reasoning, we make up several specific examples, and we look for a pattern to help us come up with the conjecture. The problem with this process is that even though the conjecture may hold true for the small number of examples that we looked at, it may NOT be true for all possible examples. Thus, we need to find a way to determine whether a conjecture is true for ALL possible cases.

It would not be reasonable to examine all specific examples for a specific situation, so a more general method was developed to test conjectures. It is called **deductive reasoning**.

Deductive Reasoning: involves drawing a specific conclusion through logical reasoning by starting with general statements that are known to be valid. With deductive reasoning, instead of using specific numbers as examples, we use a variable to represent all possible numbers in question.

Let's begin by looking at examples involving the **transitive property**. Consider the following example:

- All natural numbers are whole numbers.
- All whole numbers are integers.
- 3 is a natural number.
- What can be deduced about the number 3?

· 3 is a whole number
· 3 is an integer

Things to Note about Deductive Reasoning
There are common types of numbers that we may have to represent using variables when we prove conjectures deductively. Here are a few:

- any integer: n
- consecutive integers: $n, n+1, n+2, n+3, \ldots$
- an even number: $2n$
- an odd number: $2n+1$

Example 1:
Ask students to prove the following conjectures deductively:

(A) Mammals have fur or hair. Lions are classified as mammals. What can be deduced about lions?

Lions have fur or hair.

(B) Prove that the sum of a 2-digit number and the number formed by reversing its digits will always be divisible by 11.

Inductive:

$46+64=110$

$\frac{110}{11}=10$

$49+94=143$

$\frac{143}{11}=13$

Deductive:

- Let $10n+m$ be any two digit number
- then $10m+n$ is the reverse

$10n+m+10m+n=11n+11m$

$\frac{\cancel{11}n+\cancel{11}m}{\cancel{11}}=n+m$

(C) Prove that the difference between an odd integer and an even integer is odd.

Inductive:

$3-2=1$

$5-2=3$

$7-2=5$

$7-4=3$

$9-4=5$

Deductive:

Let $2n$ be any even number

Let $2m+1$ be any odd number

$2m+1-2n$

$=2m-2n+1$

$=2(m-n)+1$

Since any number doubled plus 1 is odd, any odd number subtracted by an even is odd.

Comparison of Inductive and Deductive Reasoning
It is important to compare inductive and deductive reasoning through the use of examples. The example below will highlight the differences.

The sum of four consecutive integers is equal to the sum of the first and last integers multiplied by two.

Inductive Reasoning	Deductive Reasoning
1, 2, 3, 4 $1+2+3+4=10$ $2(1+4)=10$	let the numbers be x, $x+1$, $x+2$, $x+3$ $x + (x+1) + (x+2) + (x+3) = 4x+6$ $2[x + (x+3)] = 2(2x + 3) = 4x+6$

$2+3+4+5=14$ $(2+5)\times 2=14$

The following table summarizes the differences between inductive and deductive reasoning:

Inductive Reasoning	Deductive Reasoning
Begins with experiences or a number of observations.	Begins with statements, laws, or rules that are considered true.
An assumption is made that the pattern or trend will continue. The result is a conjecture.	The result is a conclusion reached from previously known facts.
Conjectures may or may not be true. One counterexample proves the conjecture false.	Conclusion must be true if all previous statements are true.
Used to make educated guesses based on observations and patterns.	Used to draw conclusions that logically flow from the hypothesis.

Further summarizing the table:

Come up with a conjecture through ***inductive reasoning*** (make a general statement by finding patterns in specific examples).

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Determine whether a conjecture is valid through ***deductive reasoning*** (use general facts to come up with a specific statement - use variables instead of just numbers).

Example 2:
Prove inductively and deductively the sum of six consecutive positive numbers is a multiple of three.

Inductive:

1, 2, 3, 4, 5, 6

$1+2+3+4+5+6=21$

$\frac{21}{3}=7$

Deductive:

$x, x+1, x+2, x+3, x+4, x+5$

$x+x+1+x+2+x+3+x+4+x+5=6x+15$

$\frac{6x+15}{3}=2x+5$

Example 3:
John provided inductively, that whenever you add five consecutive integers, the sum is always 5 times the median of the numbers.

$$1+2+3+4+5=15 \qquad 3 \cdot 5 = 15$$
$$(-15)+(-14)+(-13)+(-12)+(-11)=-65 \qquad -13 \cdot 5 = -65$$
$$(-3)+(-2)+(-1)+0+1=-5 \qquad -1 \cdot 5 = -5$$

Prove his conjecture deductively.

$x, x+1, x+2, x+3, x+4$

Middle number

$x + x + 1 + x + 2 + x + 3 + x + 4 = 5x + 10$

$5(x+2) = 5x + 10$

Example 4:
Brian conjectured that adding two consecutive odd numbers will always equal an even number:

$$3+5=8$$
$$5+7=12$$
$$17+19=36$$

Did he use inductive or deductive reasoning? Explain your reasoning.

He used specific examples instead of using variables to make a generalization for all cases.

Example 5:
Nathan is doing a number trick, which involves a series of steps outlined below. Prove inductively and deductively that the trick does indeed work.

Instructions:

- Pick a number
- Double the number
- Add 20
- Divide by 2
- Subtract the original number
- The result is 10

Inductive:

5
10
30
15
10

Deductive:

$$\frac{2n+20}{2} - n$$
$$= n + 10 - n$$
$$= 10$$

Example 6:
Use both inductive and deductive reasoning to show that the sum of two odd integers is an even number.

Inductive:

$7 + 9 = 16$

$5 + 3 = 8$

$-3 + 9 = 6$

Deductive:

Let $2n+1$ be any odd number.
Let $2m+1$ be another odd number.

$$2n+1+2m+1$$
$$= 2n+2m+2$$
$$= 2(n+m+1)$$

Any number doubled is even.

Textbook Questions: page 31 - 32 #1, 4, 5, 7, 10