

## 1.5 Proofs That Are Not Valid

It is beneficial to be able to analyze proofs that contain errors. To reinforce understanding of inductive and deductive reasoning, you will identify errors in a given proof, explain why those errors might have occurred and how they can be corrected. Some typical errors include the following:

- Proofs that begin with a false statement:

### Example 1:

All high school students like Facebook. Rebecca is a high school student. Therefore, Rebecca likes Facebook.

We do not know if all high school students like Facebook. Therefore we can't say for sure that Rebecca likes Facebook.

- Algebraic errors

### Example 2:

<p>Shelby was trying to prove this number trick:</p> <ul style="list-style-type: none"> <li>• pick a number</li> <li>• double your number</li> <li>• add 20</li> <li>• divide by 2</li> <li>• subtract the original number</li> <li>• the result is 10</li> </ul>	<p>Shelby wrote the following:</p> <p>Let <math>n</math> be your number</p> $2n$ $2n + 20$ $\frac{2n + 20}{2}$ $= n + 10$ <p><math>n + 20 - n</math></p> $20$
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Identify and correct the error in Shelby's work.

$$\frac{2n + 20}{2} - n$$

$$= n + 10 - n$$

$$= 10$$

- Division by zero

**Example 3:**

Pedro claims he can prove that  $2 = 5$ . His work is shown below: Suppose  $a = b$

$$-3a = -3b$$

$$-3a + 5a = -3b + 5a$$

$$2a = -3b + 5a$$

$$2a - 2b = -3b + 5a - 2b$$

$$2a - 2b = 5a - 5b$$

$$2(a - b) = 5(a - b)$$

$$\frac{2(a-b)}{(a-b)} = \frac{5(a-b)}{(a-b)}$$

$$2 = 5$$

Step 1: Multiplying by -3

Step 2: Add  $5a$  to both sides

Step 3: Simplify

Step 4: Subtract  $2b$  from each side

Step 5: Simplify

Step 6: Factor

Step 7: Divide by  $(a - b)$

$$a - b = 0$$

↑ you can't divide by 0.

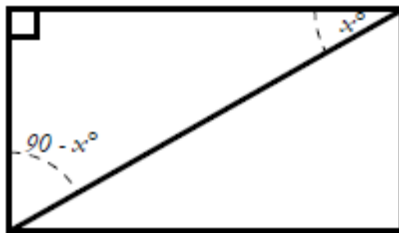
Is Pedro correct?

No.  $a - b = 0$ .

- Circular reasoning

**Example 4:**

An argument is circular if its conclusion is among its premises. Darren claims he can prove that the sum of the interior angles in a triangle is  $180^\circ$ .



Here is his proof: I constructed a rectangle. Next, I drew a diagonal. I knew that all of the angles in a rectangle are  $90^\circ$ . I labelled one of the other angles in the triangle  $x$ . Therefore, the other angle must be  $180^\circ - 90^\circ - x = 90^\circ - x$ . Then,  $90^\circ + x + (90^\circ - x) = 180^\circ$ .

Can't assume a result that follows from what you are trying to prove. He assumed that the sum of three angles equals  $180^\circ$  in a proof where he is trying to prove the sum of three angles is  $180^\circ$ .

**Example 5:**

Richard was given the following situation:

Ms. Lilly is a teacher at Kay High School. Jessica is a student at Kay High School. Write a conjecture about the relationship between Ms. Lilly and Jessica.

Richard represented his conclusion in the Venn Diagram below and concluded that Jessica is a student of Ms. Lilly's. Identify Richard's error and construct a new diagram to represent this situation.

