1.6 Reasoning to Solve Problems

Here you will be exposed to problem solving situations that require the use of inductive and/or deductive reasoning. You will explore some situations where you are asked to first show inductively that a pattern exists and then prove it deductively. It is important to recognize that inductive and deductive reasoning are not separate entities, they work together. Consider the following example:

Tyler was investigating patterns on the hundreds chart. He was asked to choose any four numbers that form a 2 × 2 square on the chart. He chose the following:

4	5
14	15

He should be able to use inductive reasoning to make a conjecture about the sum of each diagonal and then use deductive reasoning to prove his conjecture is always true.

Inductive:

$4 + 15 = 19$

$5 + 14 = 19$

Conjecture: the sum of the diagonals of a square of four blocks on a hundreds chart will be equal.

Deductive:

n	$n+1$
$n+10$	$n+11$

$n + n + 11 = 2n + 11$

$n + 1 + n + 10 = 2n + 11$

QED

Example 1:
Emma was given this math trick:

- Choose a number.
- Multiply by 6.
- Add 4.
- Divide by 2.
- Subtract 2.

Emma was asked to use inductive reasoning to make a conjecture about the relationship between the starting and ending numbers, and then use deductive reasoning to prove that her conjecture is always true.

Inductive:

8	12
48	72
52	76
26	38
24	36

Conjecture: the answer is always three times the original number

Deductive: n

$$\frac{6n+4}{2} - 2$$

$$= 3n + 2 - 2$$

$$= 3n$$

QED

Example 2:
Ten men meet for a bowling tournament and each shakes the hand of every other man. Determine the number of handshakes that occurred. Explain the strategy used to arrive at the answer.

1st guy: shakes 9 hands

2nd guy: shake 8 other hands

3rd guy: shaks 7 other hands

$\vdots$

10nth guy: shakes 1 other hand

$$9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$$

Example 3:
Look at a monthly calendar and pick any 3 squares in a row-across, down or diagonal. Using inductive reasoning, and make a conjecture about the middle number, then use deductive reasoning to prove their conjecture.

Inductive:

$$10 + 26 = 2(18)$$
$$12 + 24 = 2(18)$$
$$17 + 19 = 2(18)$$
$$11 + 25 = 2(18)$$

Conjecture: the sum of the corners is twice the middle number.

Deductive: $n - m + n + m$
$$= 2n$$

SUN	MON	TUE	WED	THU	FRI	SAT
2016 MAY						
1	2	3	4	5	6	7
8	9	10 $n-m$	11	12	13	14
15	16	17	18 n	19	20	21
22	23	24	25	26 $n+m$	27	28
29	30	31				

Your turn:
Solve each problem using reasoning:

(A)   If you have a 5-L and 3-L bottle and plenty of water, how can you get 4-L of water in the 5-L bottle?

- Fill up 3-L bottle
- pour in 5-L bottle
- Fill up 3L bottle again.
- Pour into the 5-L bottle until 5-L bottle is full.
- Dump the 5-L bottle
- Pour remaining 1L in 3-L bottle in 5-L bottle
- Fill up 3-L bottle again
- Pour 3-L into the 5-L bottle.

(B)   Ted, Ken, Allyson, and Janie (two married couples) each have a favourite sport: swimming, running, biking, and golf. Given the following clues, determine who likes which sport.
- Ted dislikes golf.
- Each woman's favourite sport is featured in a triathlon.
- Ken nor his wife enjoy running.
- Allyson bought her husband a new bike for his birthday to use in his favourite sport.
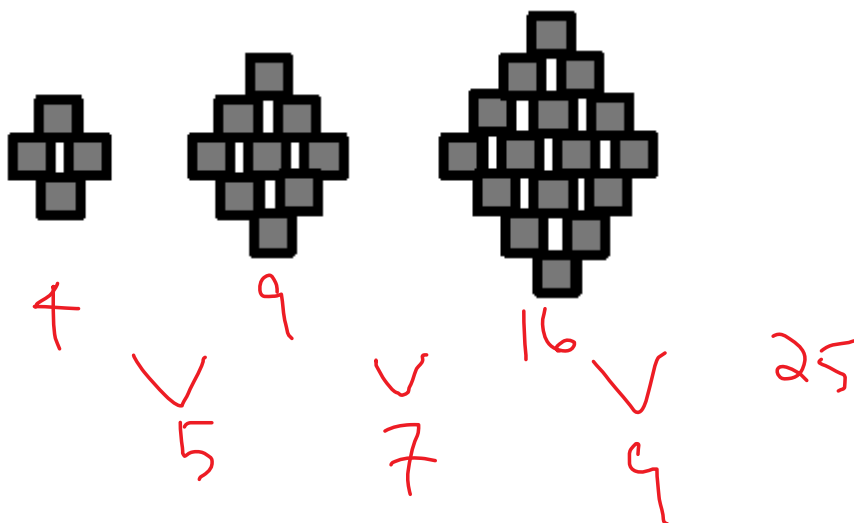
Ken → golfs   Ted - Allyson

Ted → bikes   Ken - Janie

Allyson → runs

Janie → swims

(C)   Determine the number of squares that would be in the next, 4th, figure.

4   9   16   25

      5   7   9

or

Count : 25