Math 2201   Date:_____
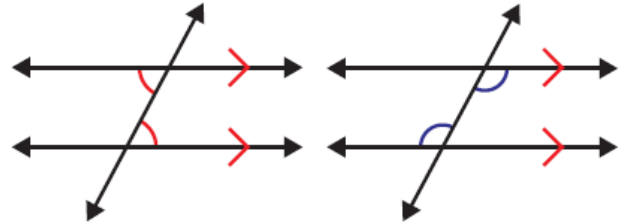
2.2A Exploring Parallel Lines

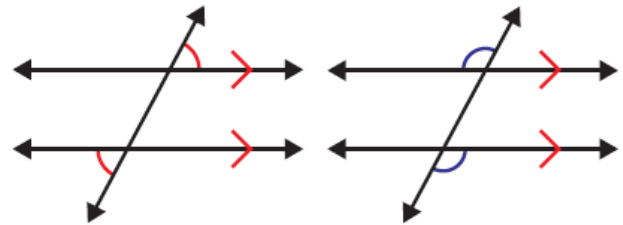Before we continue, we need some more terminology.

Alternate Interior Angles
Two non-adjacent interior angles on opposite
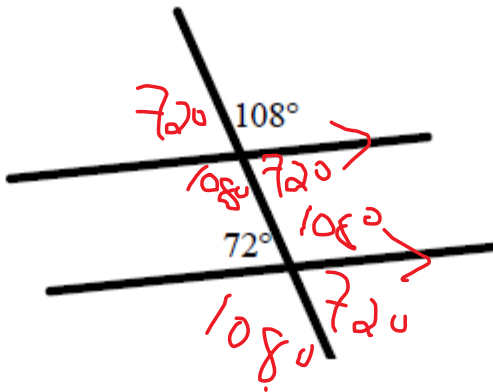sides of a transversal.

Alternate Exterior Angles
Two exterior angles formed between two
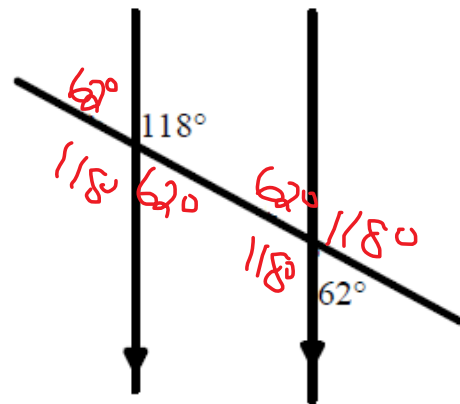lines and a transversal, on opposite sides
of the transversal.

Example 1:
Use alternate interior and alternate exterior angle properties to find the missing angles:

(A)   (B)

72° 108°
108° 72°
72°
108°
108° 72°
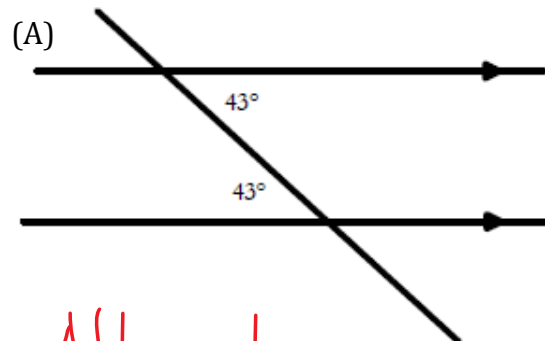
62°
118°
118° 62°
62° 118°
118°
62°

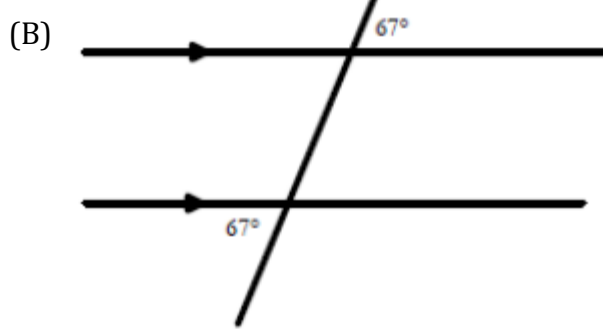In Summary, when a transversal cuts two parallel lines:

- Alternate interior angles are equal.
- Alternate exterior angles are equal.

Example 2:
For each diagram, indicate the property used to show equal angles:

(A)

43°

43°

Alternate Interior

(B)

67°

67°

Alternate Exterior

Constructing Parallel Lines
You will now learn to construct parallel lines and verify the angle relationships that were developed earlier. There are a variety of strategies, such as a compass, protractor, or paper folding and protractor, to ensure lines are parallel. Consider the following:

Using a Protractor

Step 1: Draw a line AB. Place a point P above the line.

Step 2: Draw a line through P and passing through AB. Label the intersection point Q.

Step 3: Using a protractor measure $\angle PQB$. Use your protractor to then form an angle having the same measure at P.
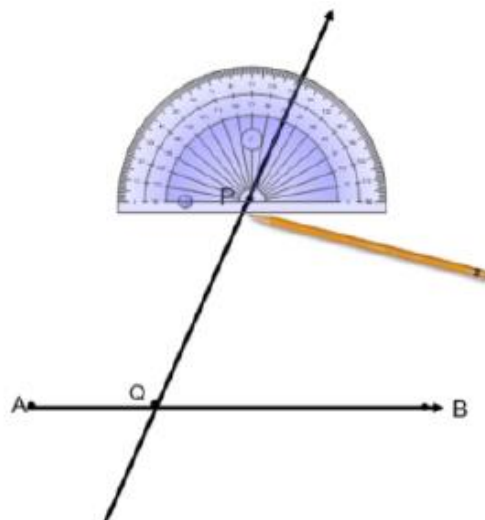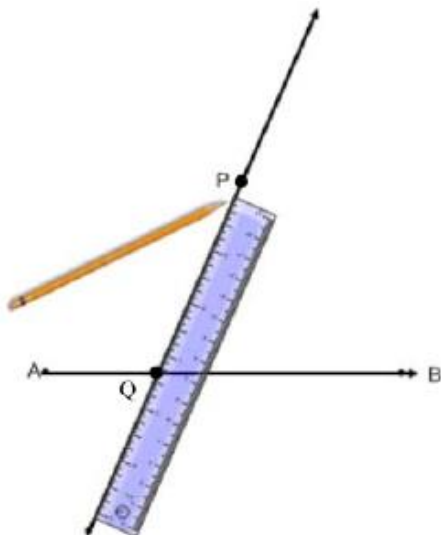
Paper Folding and a Protractor.

Step 1: Take a blank sheet of paper and fold it in half.

Step 2: Fold it in half again.

Step 3: Unfold the paper and trace any 2 of the fold lines, using a ruler and protractor.

Step 4: Construct a transversal.

Verify that the lines are parallel by measuring the angles they have created; **corresponding**, **vertically opposite**, **alternate interior**, **alternate exterior**, **interior** angles on the same side of the transversal.

How many angle relationships are necessary to measure to prove the lines are parallel?

Just 1. Depends on which relationship.

Example 3:
John is building a fence in his backyard and plans to construct the fence parallel to his house. Using the diagram shown below, where would John build the fence?
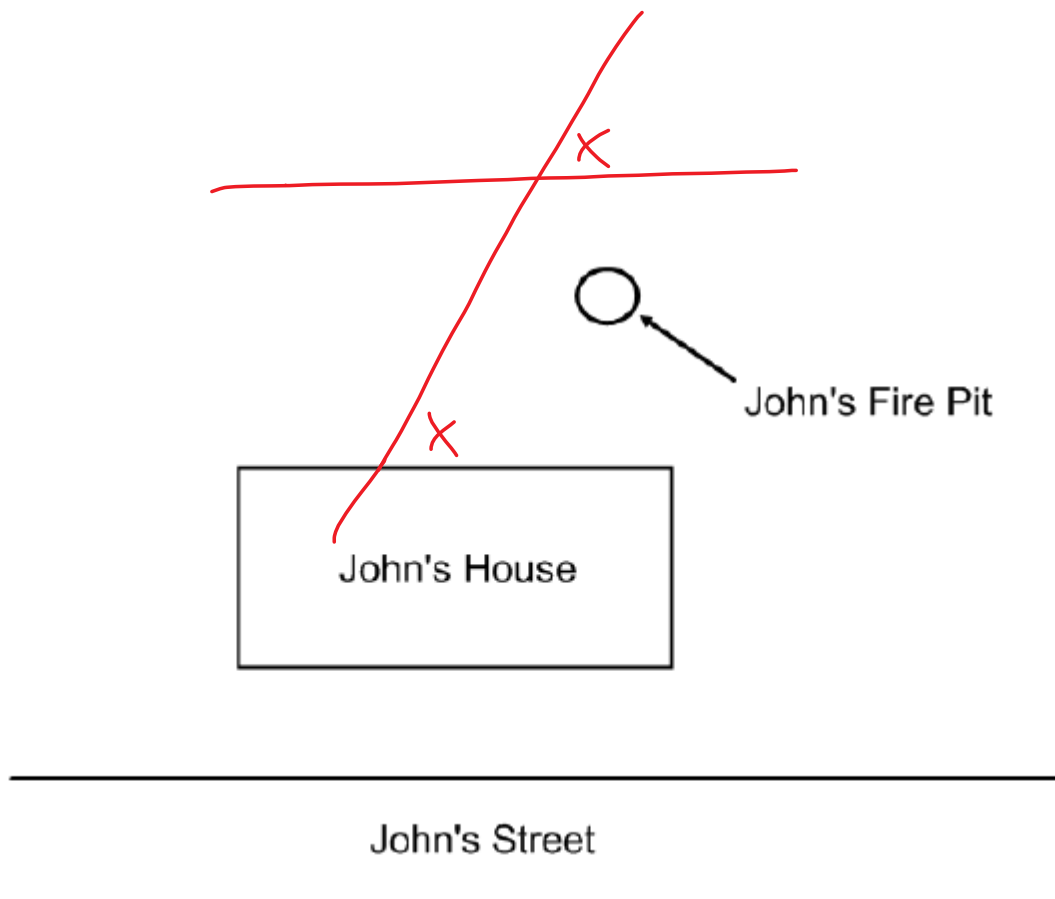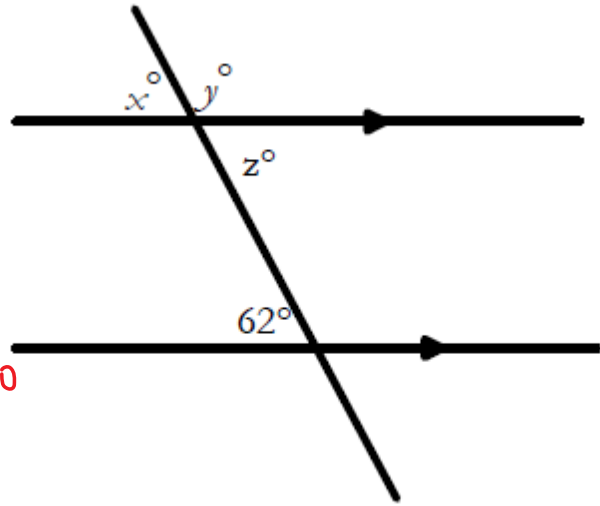
Example 5:
Find the measure of ∠x, ∠y and ∠z.

$$\angle z = 62°$$
$$\angle x = 62°$$
$$\angle y = 180° - 62° = 118°$$

$x°$ $y°$

$z°$

$62°$

Example 6:
Determine the value of x and then determine the angle measures.

$$2x - 10 = x + 15$$
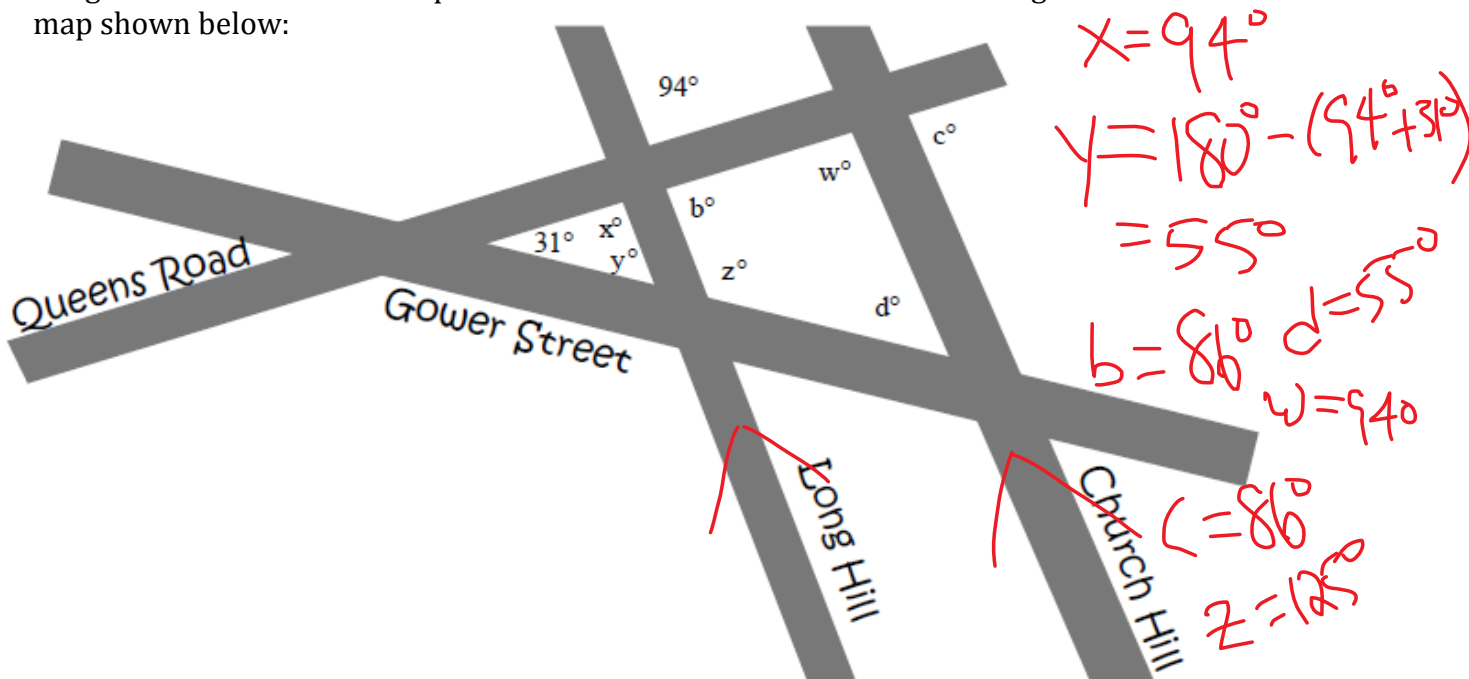$$2x - x = 15 + 10$$
$$x = 25$$

$$2(25) - 10$$
$$= 40°$$

$(2x - 10)°$

$(x + 15)°$

$40°$
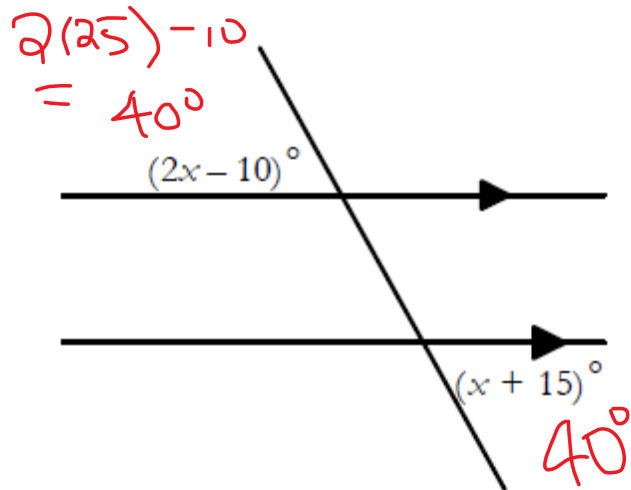
Example 7:
Long Hill and Church Hill are parallel to each other. Determine the missing measures on the map shown below:

94°

c°

w°

b°

31° x°
y°

z°

d°

Queens Road

Gower Street

Long Hill

Church Hill

$x = 94°$
$y = 180° - (94° + 31°)$
$= 55°$
$b = 86°$ $d = 55°$
$w = 94°$
$c = 86°$
$z = 125°$

Proving That Lines Are Parallel
You work backwards and determine if lines are parallel when given angle measurements.

Two lines are parallel if they're cut by a transversal such that:

- Two corresponding angles are congruent.
- Two alternate interior angles are congruent.
- Two alternate exterior angles are congruent.
- Two same-side interior angles are supplementary.
- Two same-side exterior angles are supplementary.

Example 8:
Determine whether the following lines are parallel. Justify your answer.

$180° - 124° = 56°$

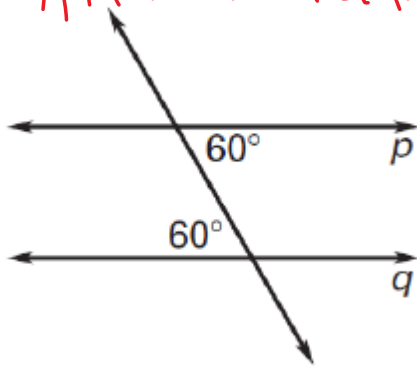∴ Lines are parallel
because two same side
exterior angles are supplementary.

124°

56°
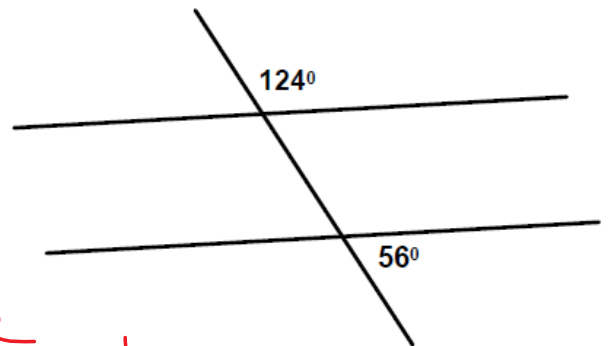
Example 9:
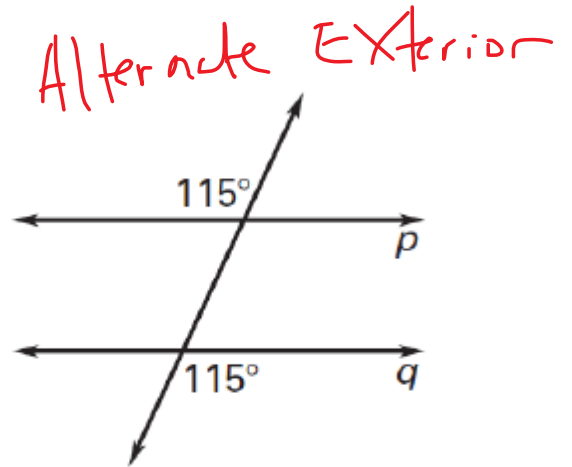State the property that prove lines p and q are parallel?
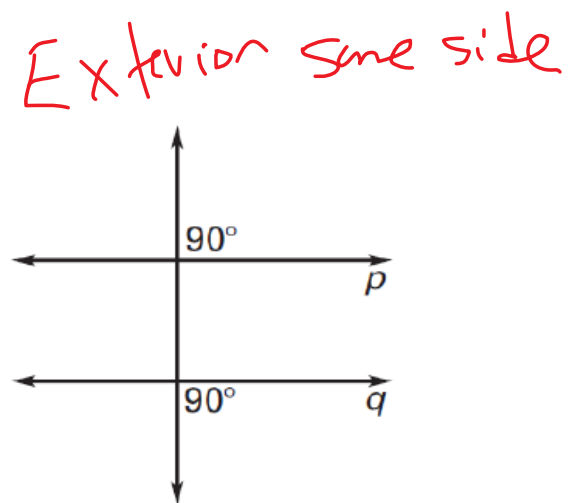
(A) *Alternate Interior*

60° p

60° q

Corresponding Angles

(B) *Alternate Exterior*

115° p

115° q

Exterior same side

(C)

$x°$ p

$x°$ q

(D)

90° p

90° q
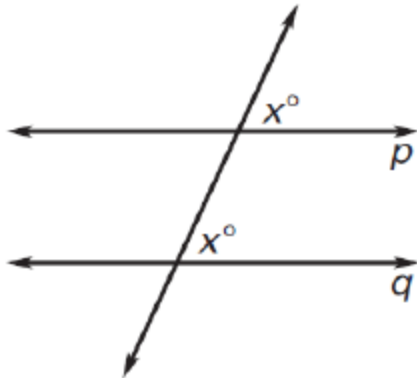
Example 10:
Find the value of x that makes $m \parallel n$.

(A)

$2x°$ m

80° n

$$\frac{2x}{2} = \frac{80}{2}$$

$$x = 40°$$

(B)

m

$(3x + 15)°$ n

75°
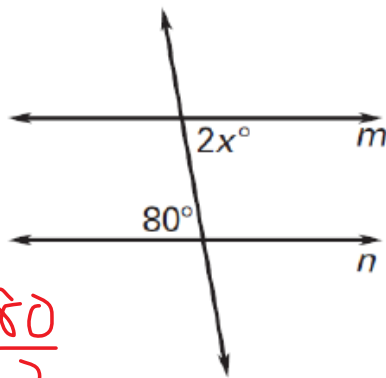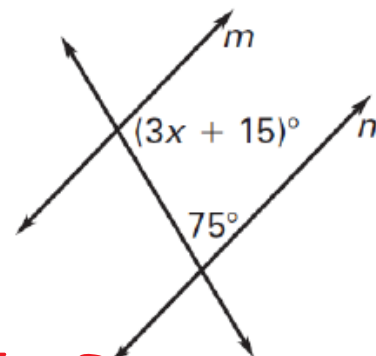
$$3x + 15 + 75 = 180°$$
$$3x + 90 = 180°$$
$$3x = 180 - 90$$
$$3x = 90°$$

$$x = \frac{90°}{3}$$

$$x = 30°$$

Common Errors

It is beneficial to analyze solutions that contain errors, explain why errors might have occurred and how they can be corrected. This reinforces the angle relationships that have been developed throughout this unit.

A common error occurs when you incorrectly identify pairs of angles, leading to an incorrect measurement. For example, identifying same side interior angles as equal rather than supplementary.
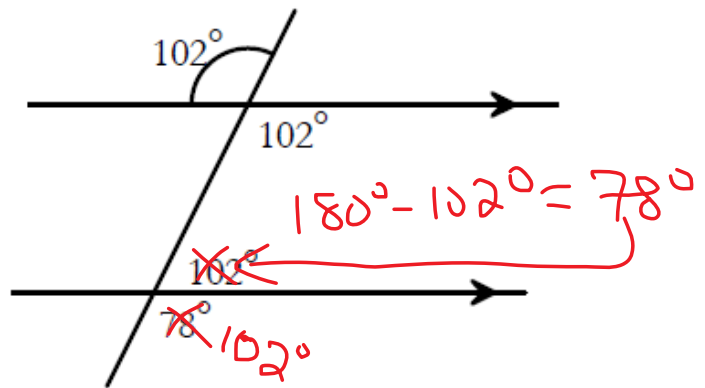
Example 11:
Identify and correct any errors in the following solution:

Find the values of w, x and y.
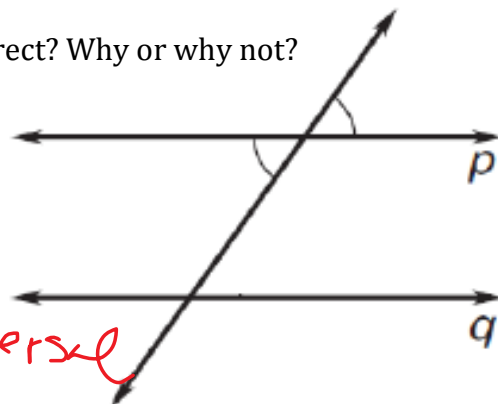
Solution:

$$180° - 102° = 78°$$
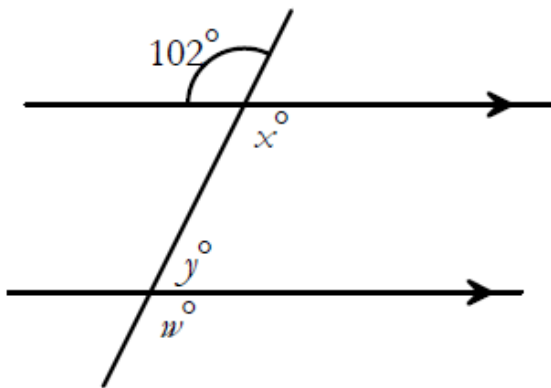
Another common student error occurs when you identify lines as being parallel based only on vertically opposite angles that are equal.

Example 13:
Wally says the lines p and q are parallel. Is he correct? Why or why not?

No. We have no measurement to relate angles formed by the transversal and q to p.

Your turn:

1. Suppose Prince Philip Drive and Elizabeth Avenue follow a straight line path and intersect Allandale Road at angles of 98° and 96° as shown in the map below. If the streets were to continue in a straight line, would their paths ever cross? Explain your reasoning.
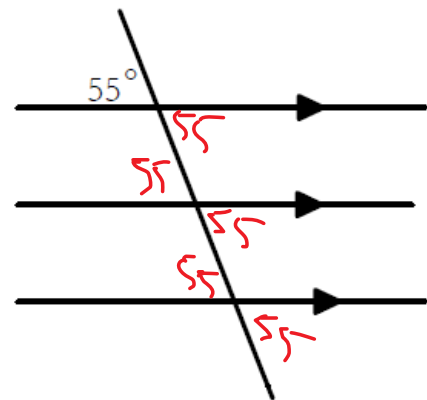
98°

Prince Philip Drive

96°

Elizabeth Avenue

They will cross to the west.

Allandale Road
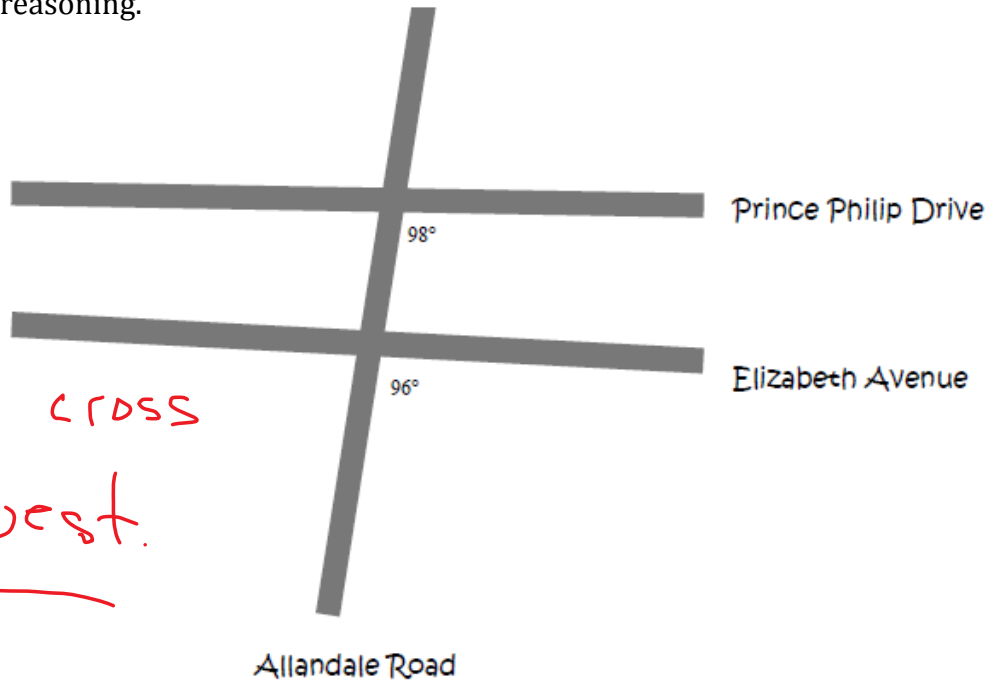
2. What is the value of a?

130°
a

130° / 50°
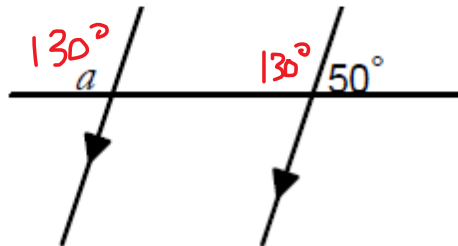
3. How many angles in the diagram measure 55° ?

6

55°

55
55
55
55
55

4. Identify and correct the error.

∠w = 120°

∠y = 60°

$w = 60°$

$y = 120°$

60°

w

y
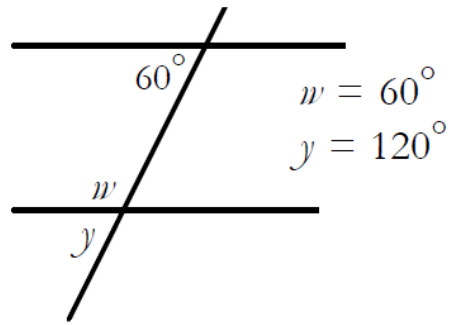
5. Solve for the missing angles in each diagram:

(A)

104

76⁰

A

B 76

C 104

76 D

E 104

F 104

G 76

(B)

72⁰

108

y+2⁰

(C)

2(10)+10 = 30°

2x + 10⁰

x + 20⁰   30⁰

2x+10 = x+20

2x−x = 20−10

x = 10
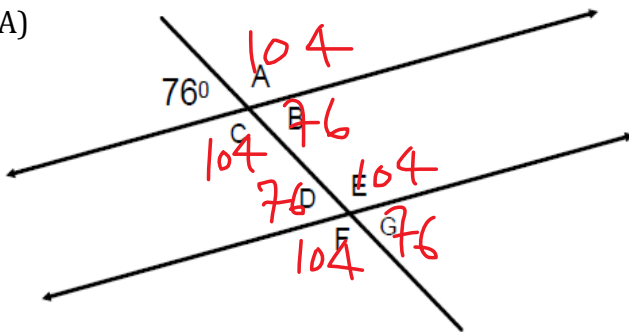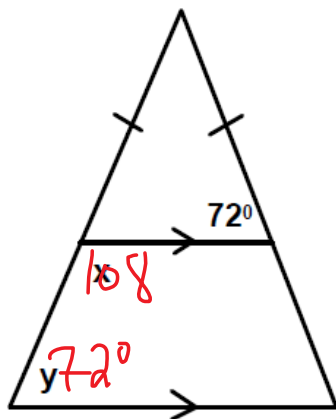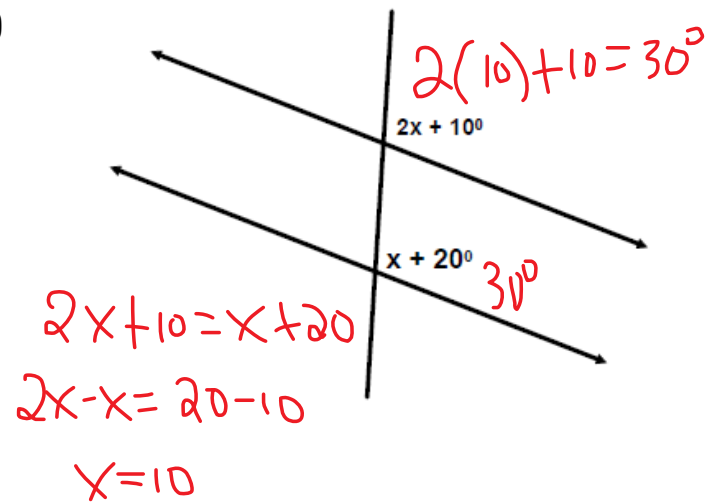
6.   Does the given angle measure proves that the lines are parallel. Justify your answer.

Yes. Alternate interior angles.

50°

50°

Textbook Questions: page 78 - 82 #1, 2, 3, 4, 8, 10, 12, 15, 16, 18, 19, 20