Math 2201   Date:_____

2.2B Proving Properties of Angles Formed by Transversals and Parallel Lines

You have explored the angle relationships when two parallel lines are cut by a tranversal. You will now use this knowledge of corresponding angles, vertically opposite angles and supplementary angles to formally prove the other relationships, such as alternate interior angles.

The Transitive Property
If two angles are equal to the same thing, then they are equal to each other.

$$\text{If } A = B \text{ and } B = C,$$

$$\text{then } A = C.$$

A two-column proof and a paragraph proof are two of the most common strategies used to construct proofs involving properties of angles formed by transversals and parallel lines.

Let's make a conjecture that involves alternate exterior angles formed by parallel lines and a transversal.

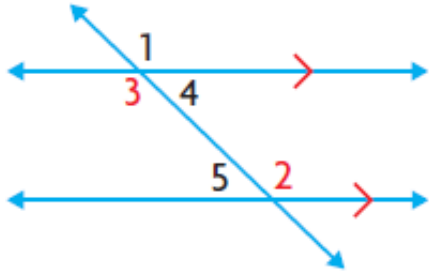Conjecture: alternate exterior angles are equal

We'll use the following to prove this conjecture:
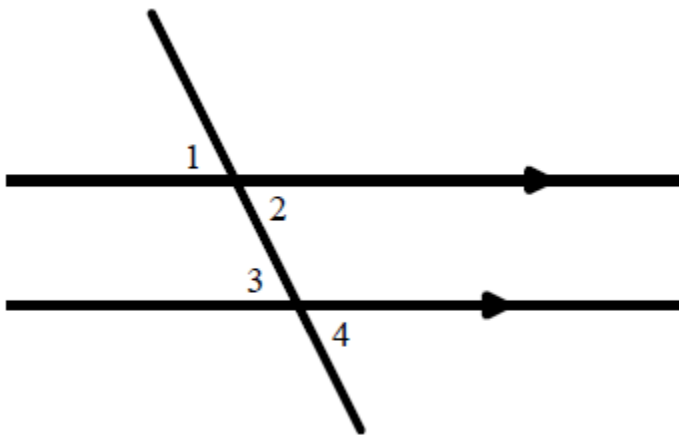
Statement	Justification
$\angle 1 = \angle 2$	vertically opposite angles
$\angle 2 = \angle 4$	corresponding angles
$\angle 1 = \angle 4$	transitive property

Example 1:
Prove that alternate interior angles, formed when a transversal intersects a pair of parallel lines, are equal.

Statement	Justification
$\angle 1 = \angle 3$	Vertically opposite
$\angle 1 = \angle 2$	Corresponding angles
$\angle 2 = \angle 3$	Transitive property

Example 2:
Prove that interior angles on the same side of a transversal, formed when a transversal intersects a pair of parallel lines, are supplementary.
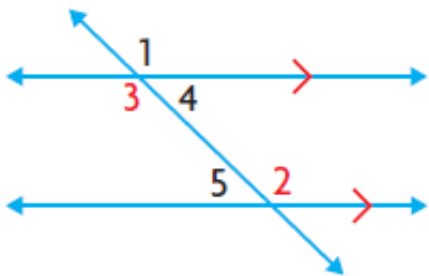
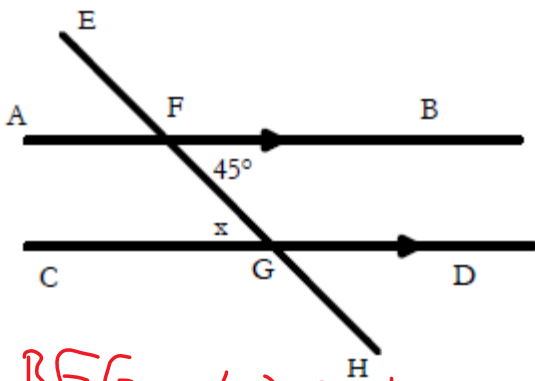| Statement	Justification
$\angle 1 = \angle 2$	Corresponding angles
$\angle 1 = \angle 3$	Vertically opposite
$\angle 2 + \angle 5 = 180°$	Angles form a straight line
$\angle 1 + \angle 5 = 180°$	Transitive property
$\angle 3 + \angle 5 = 180°$	Transitive property

Common Errors
It is beneficial to analyze solutions that contain errors, explain why errors might have occurred and how they can be corrected. This reinforces the angle relationships that have been developed throughout this unit.

Example 3:
Identify and correct the error for the following solution:

Determine the measure of x.

Statement	Justification
$\angle BFG = 45°$	given
$\angle BFG$ ✗ $\angle FGD$	interior angles on the same side of the tranversal are equal
$\angle FGD + \angle FGC = 180°$ $\angle FGC = 180° - \angle FGD$ $= 180° - 45°$ $= 135°$	supplementary angles (angles forming a straight line)

$\angle BFG = \angle DGH$ — Corresponding angles

$X = \angle DGH$ Vertically Opposite

Example 4:
One side of a cellphone tower will be built as shown. Use the angle measures to prove that braces CG, BF, and AE are parallel.

Statement	Justification
$\angle HGC = \angle GFB$	Given
$\overline{CG} \parallel \overline{BF}$	Corresponding angles
$\angle FBE = \angle BEA$	Given
$\overline{BF} \parallel \overline{AE}$	alternate interior angles
$\overline{CG} \parallel \overline{AE}$	transitive property
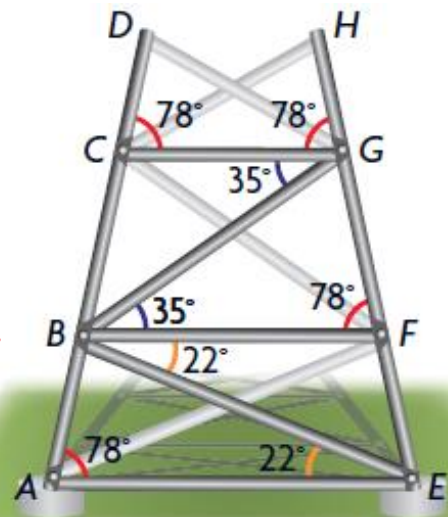
Textbook Questions: page 78 - 82 #1, 2, 3, 4, 8, 10, 12, 15, 16, 18, 19, 20