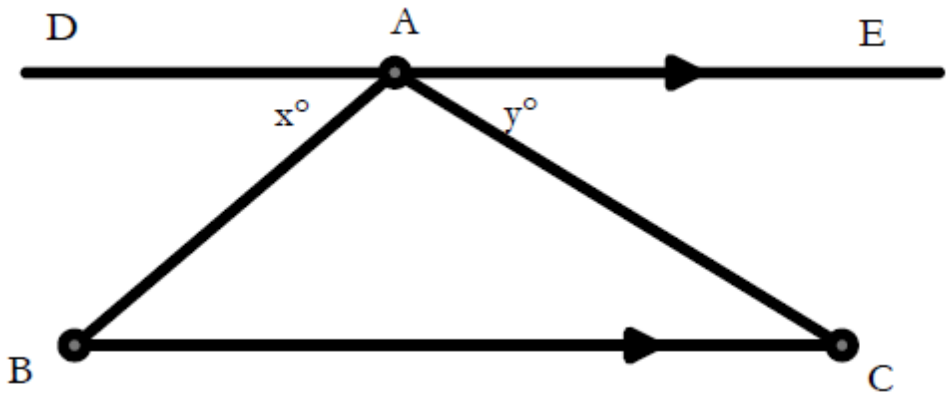
Math 2201 **Date:**_____

2.3 Angle Properties in Triangles

To begin, we will prove, using deductive reasoning, that the sum of the interior angles of any triangle is 180°. We will use the following diagram to illustrate our proof:

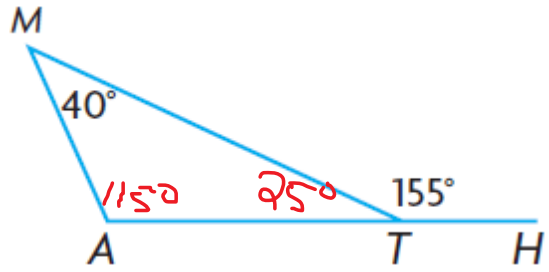
Property: the sum of the interior angles in any triangle is 180°.

To prove this property, we will use a set of parallel lines, a transversal and an additional line to create a triangle.



Statement	Reason
$\angle X = \angle ABC$	Alternate interior angles
$\angle Y = \angle ACB$	Alternate interior angles
$\angle X + \angle BAC + \angle Y = 180°$	Angles form a straight line
$\angle ABC + \angle BAC + \angle ACB = 180°$	Transitive property

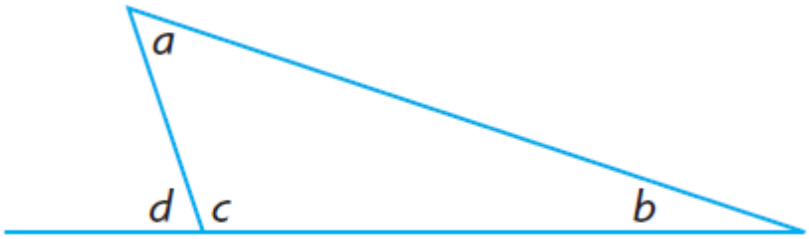
Example 1:
In the diagram, $\angle MTH$ is an **exterior angle** of $\triangle MAT$. Determine the measures of the unknown angles in $\triangle MAT$.



$\angle mTA = 180° - 155 = 25°$

$\angle mAT = 180° - (40° + 25°) = 115°$

Example 2:
Prove: $\angle d = \angle a + \angle b$



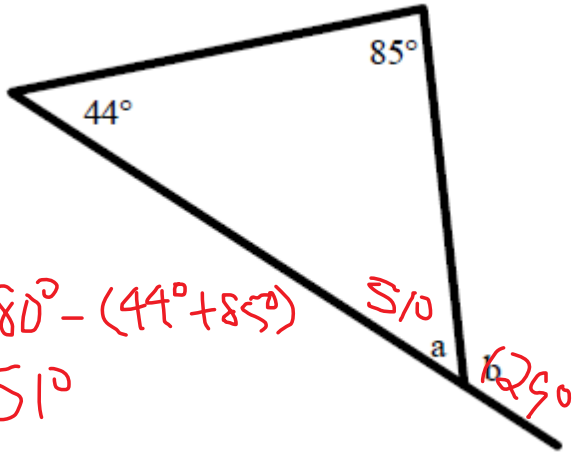
Statement	Reason
$\angle d + \angle c = 180°$	angles form a straight line
$\angle d = 180° - \angle c$	algebra
$\angle a + \angle b + \angle c = 180°$	3 angles of a triangle
$\angle a + \angle b = 180° - \angle c$	algebra
$\angle a + \angle b = \angle d$	transitive property

Finding Angles of Triangles
We can use the above property along with others that we have learned, to solve for missing angle measures.

Earlier in this chapter, you were exposed to diagrams where they were asked to determine missing angle measures formed by transversals and parallel lines. You will continue to find the measures of unknown angles but now within triangles.

Example 3:
Find the missing anglges in each triangle.

(A)

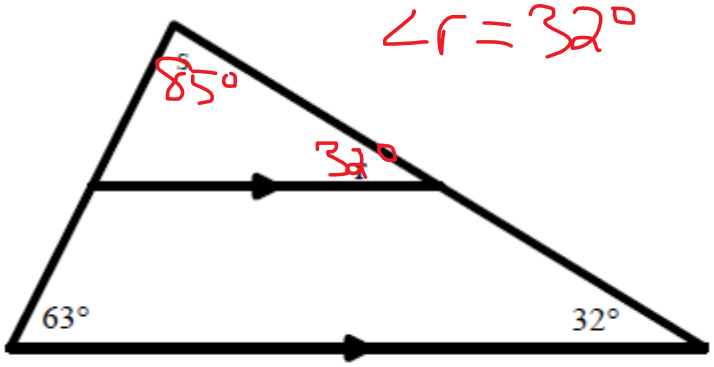


$\angle a = 180° - (44° + 85°)$

$\angle a = 51°$

$\angle b = 180° - 51° = 129°$

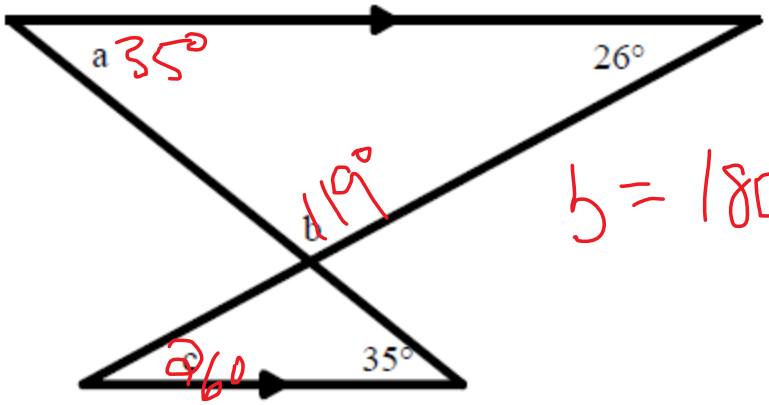
(B)



$\angle r = 32°$

$s = 180° - (32° + 63°) = 85°$

(C)



$b = 180° - (26° + 35°) = 119°$

Example 4:
In Δ ABC, for example, $\angle A = 6x - 30$, $\angle B = -x + 50$, and $\angle C = 3x$. Determine the measure of $\angle A$.

$\angle A+\angle B+\angle C=180^\circ$

$6x-30-x+50+3x=180$

$8x+20=180$

$8x=180-20$

$\frac{8x}{8}=\frac{160}{8}$

$x=20$

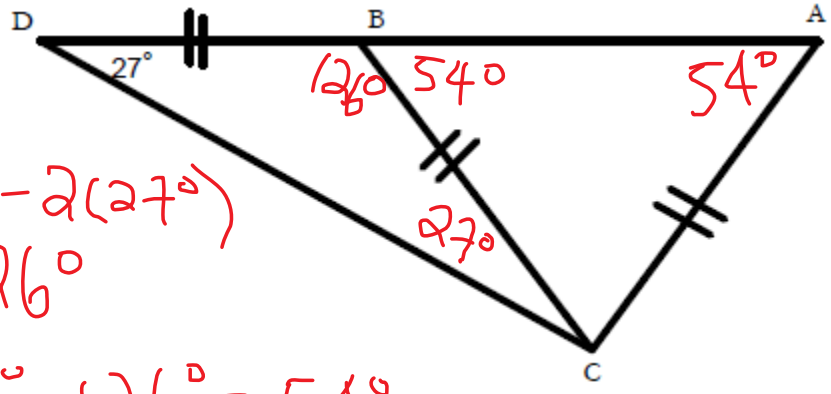
$\angle A=6x-30$

$=6(20)-30$

$=120-30$

$=90^\circ$

Example 5:
Determine the value of $\angle A$. Explain your reasoning.



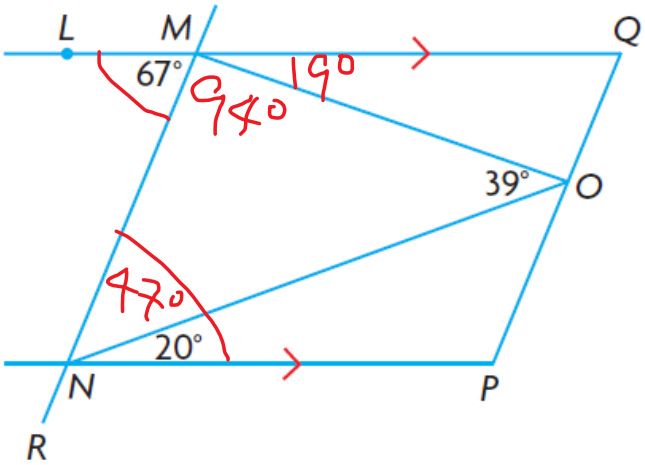
$\angle A=54^\circ$

$180^\circ-2(27^\circ)$

$=126^\circ$

$180^\circ-126^\circ=54^\circ$

Example 6:
Determine the measures of $\angle NMO$, $\angle MNO$, and $\angle QMO$.



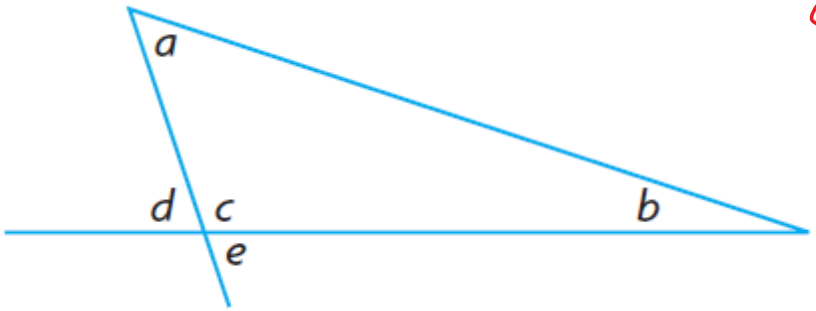
$\angle NMO=94^\circ$

$\angle MNO=47^\circ$

$\angle QMO=19^\circ$

Your turn:

1. Prove: $\angle e = \angle a + \angle b$



$a+b+c=180°$

$a+b=180°-c$

$d+c=180°$

$d=180°-c°$

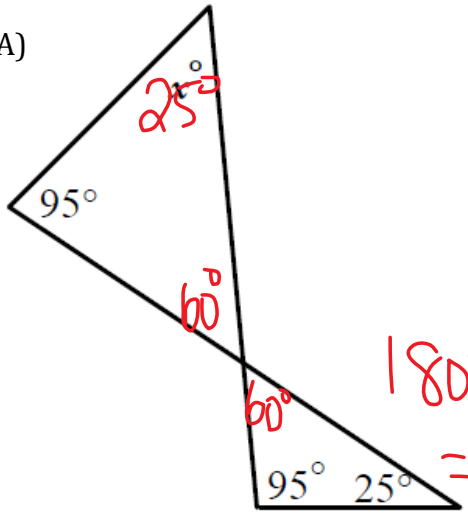
$d=a+b$

$e=d$

$e=a+b$

2. Determine the missing measures:

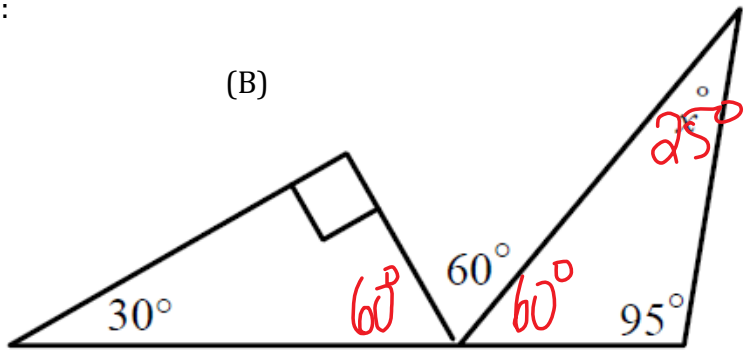
(A)



$180°-(95°+25°)$

$=60°$

(B)



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