Math 2201 Date:____________________

3.1 Review of Right Triangle Trigonometry and Acute Triangles

Review of Right Triangle Trigonometry

Recall from Math 1201

soh-cah-toa

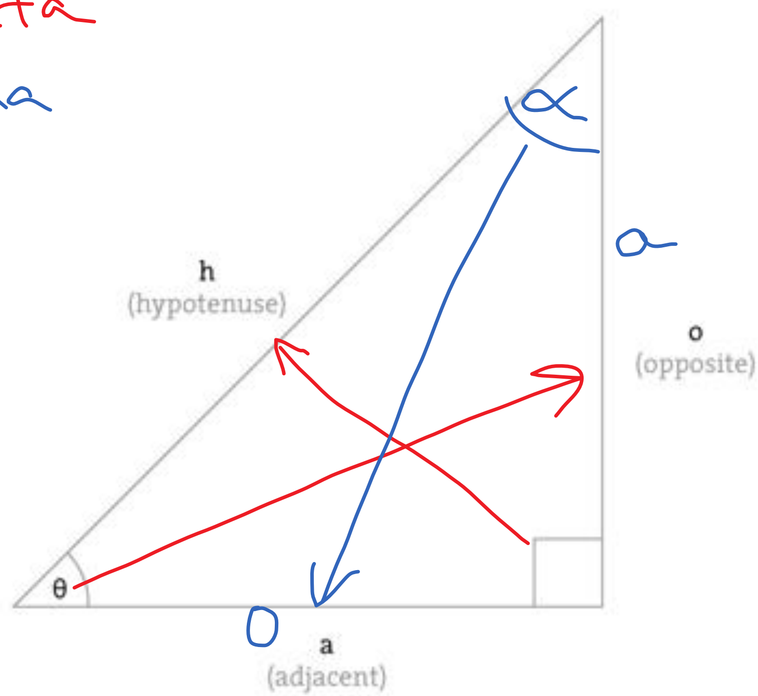
sine cosine tangent

$$\text{sine}(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{tangent}(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

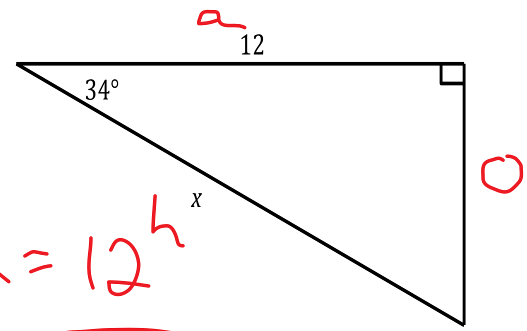
$$\text{cosine}(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \text{tangent}(\theta) = \frac{\text{sine}(\theta)}{\text{cosine}(\theta)}$$

θ : theta

α : alpha



Example 1:
Find the length of side x.



~~soh~~ cah ~~toa~~

$$\cos\theta = \frac{a}{h}$$

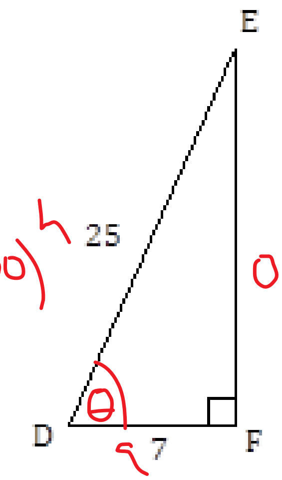
$$\cos 34^\circ = \frac{12}{x}$$

$$\frac{0.8290}{1} = \frac{12}{x}$$

$$\frac{0.8290x}{0.8290} = \frac{12}{0.8290}$$

$$x = 14.5$$

Example 2:
Find the measure of $\angle D$.



~~soh~~ cah ~~toa~~

$$\cos\theta = \frac{a}{h}$$

$$\cos\theta = \frac{7}{25}$$

$$\cos\theta = 0.2800$$

$$\theta = \cos^{-1}(0.2800)$$

$$\theta = 74^\circ$$

Example 3:
Determine $\angle A$.

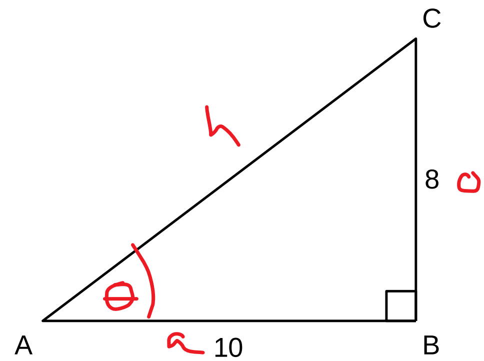
~~soh~~ ~~cah~~ **toa**

$$\tan\theta = \frac{o}{a}$$

$$\tan\theta = \frac{8}{10}$$

$$\theta = \tan^{-1}(0.8000)$$

$$\theta = 39^\circ$$



Example 4:
Determine the length of side d to the nearest tenth of a centimetre.

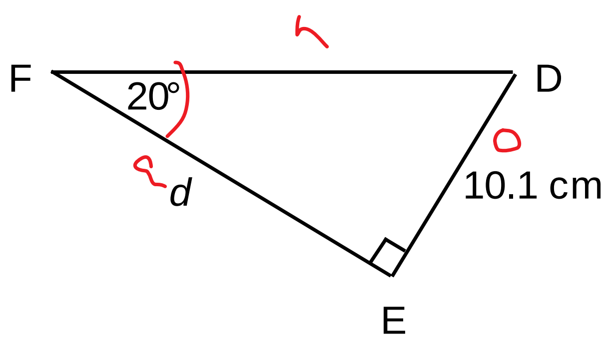
$$\tan\theta = \frac{o}{a}$$

$$\tan 20^\circ = \frac{10.1}{d}$$

$$\frac{0.3640}{1} = \frac{10.1}{d}$$

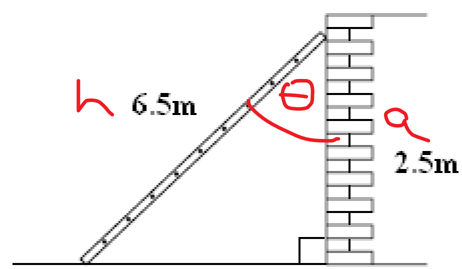
$$\frac{0.3640d}{0.3640} = \frac{10.1}{0.3640}$$

$$d = 27.7\text{ cm}$$



Example 5:
A ladder is 6.5m long and is placed against a wall so that it reaches 2.5m up the wall as shown in the diagram below. What is the value of the angle, to the nearest degree, that the ladder makes with the wall?

~~soh~~ **cah** ~~toa~~



$\cos\theta = \frac{a}{h}$

$\cos\theta = \frac{2.5}{6.5}$

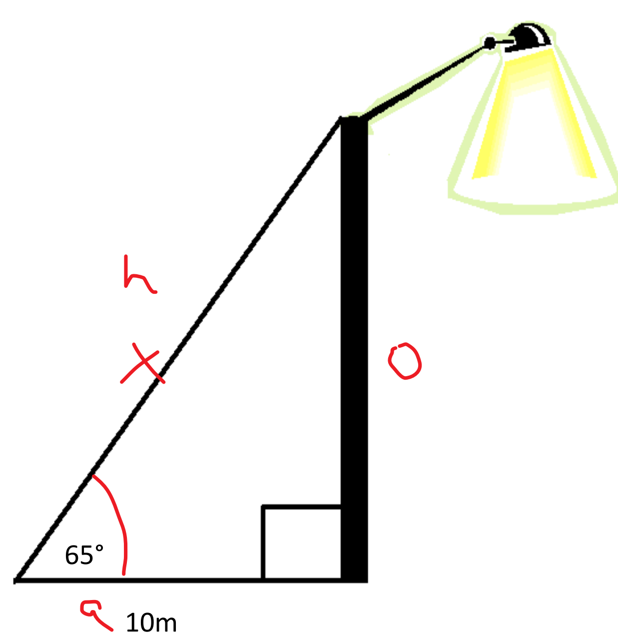
$\cos\theta = 0.3846$

$\theta = \cos^{-1}(0.3846)$

$\theta = 67^\circ$

Example 6:
A light pole has a support wire attached to its top. The wire is attached to the ground 10m away and makes a 65° with the ground. If the pole makes a 90° with the ground, what is the length of the wire, in metres?

~~soh~~ **cah** ~~toa~~



$\cos\theta = \frac{a}{h}$

$\cos 65^\circ = \frac{10}{x}$

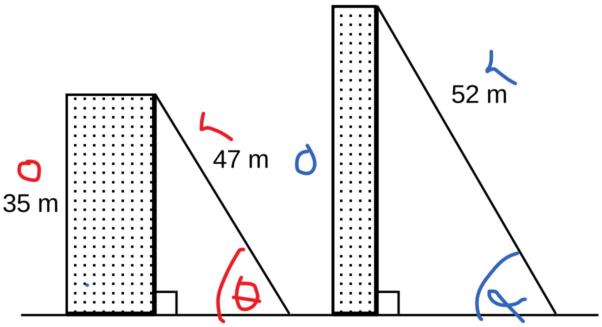
$\frac{0.4226}{1} = \frac{10}{x}$

$\frac{0.4226x}{0.4226} = \frac{10}{0.4226}$

$x = 24\text{ m}$

Example 7:

Guy wires are attached to buildings as shown. A student says the angles of inclination of the wires are the same. Is the student correct? Justify your answer.



θ:

soh cah toa

$\sin\theta = \frac{o}{h}$

$\sin\theta = \frac{35}{47}$

$\sin\theta = 0.7447$

$\theta = \sin^{-1}(0.7447)$

$\theta = 48^\circ$

α:

soh cah toa

$\cos\alpha = \frac{37}{52}$

$\cos\alpha = 0.7115$

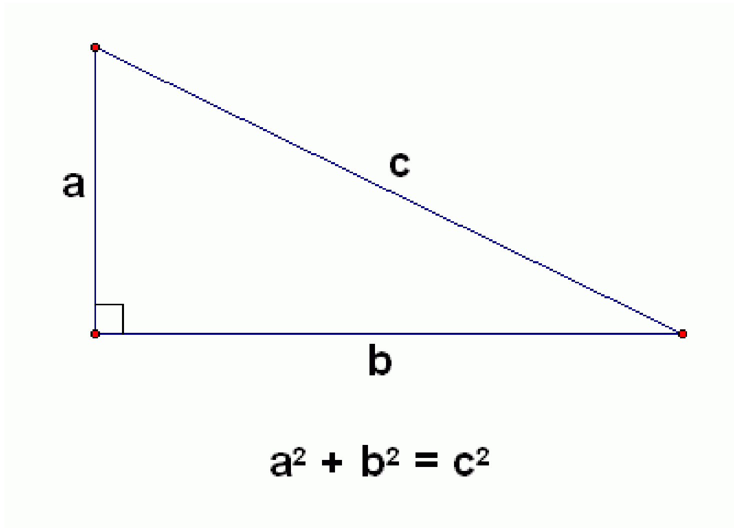
$\alpha = \cos^{-1}(0.7115)$

$\alpha = 45^\circ$

No, the angles are not equal.

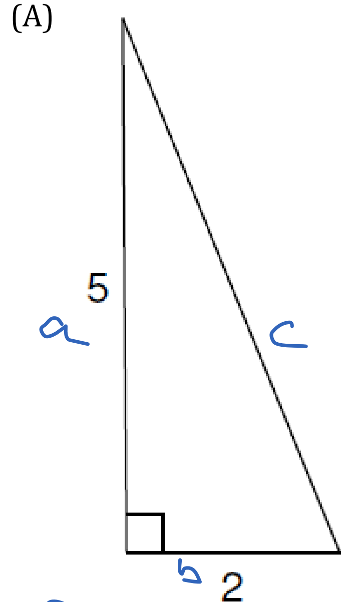
Pythagorean Theorem

One of the most important properties in right-triangle trigonometry is the relationship between the two sides of a right triangle and the hypotenuse.



Example 7:
Find the missing side:

(A)



$a^2+b^2=c^2$

$5^2+2^2=c^2$

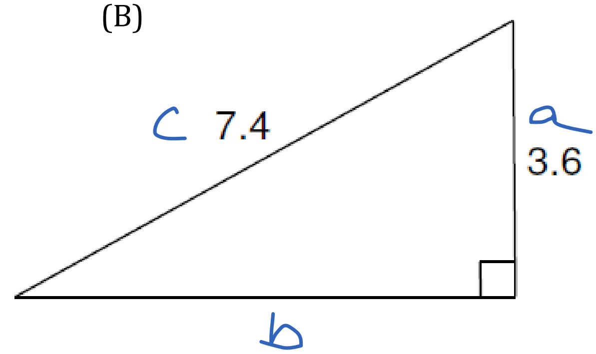
$25+4=c^2$

$29=c^2$

$\sqrt{29}=\sqrt{c^2}$

$\rightarrow c=5.4$

(B)



$(3.6)^2+b^2=(7.4)^2$

$12.96+b^2=54.76$

$b^2=54.76-12.96$

$b^2=41.8$

$\sqrt{b^2}=\sqrt{41.8}$

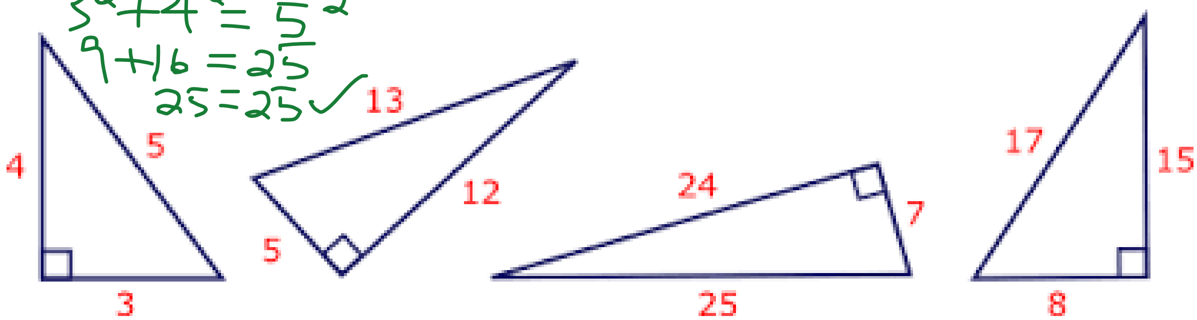
$b=6.5$

Pythagorean Triples

There are some right triangles where all three sides are whole numbers. These are called Pythagorean Triples or Triads. Keep in mind that the largest number is the hypotenuse.

The following are some common Pythagorean triples:

$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$ ✓



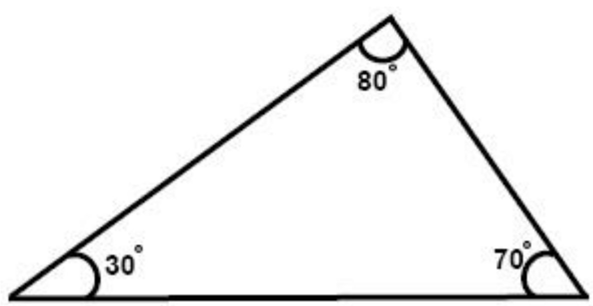
It's important to note that multiples of Pythagorean Triples are also Pythagorean Triples. For example, since $3-4-5$ is a Pythagorean Triple, $6-8-10$ and $9-12-15$ are also.

Acute Triangles

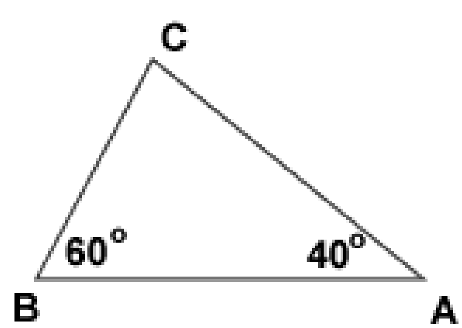
Acute Angle: an angle measuring less than 90°.

Acute Triangle: a triangle in which all angles measure less than 90°.

For example:



Example 8:
Find $\angle C$. Is the triangle acute?

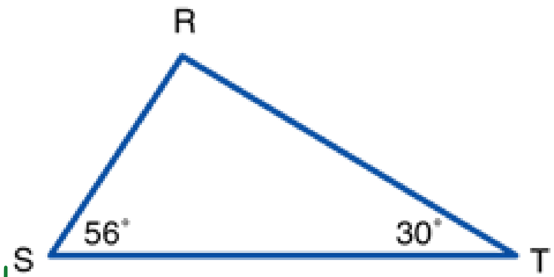


$\angle C = 180° - (60° + 40°)$
$= 180° - 100°$
$= 80°$

$\therefore \triangle ABC$ is acute.

Example 9:
Find $\angle R$. Is the triangle acute?

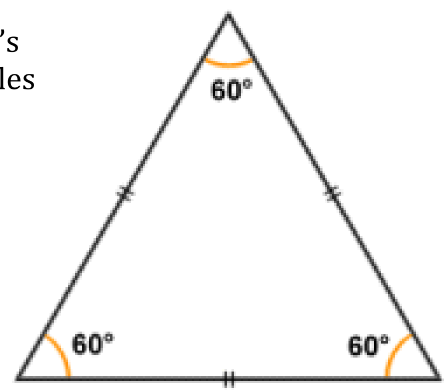
$R = 180^\circ - (56^\circ + 30^\circ)$
$= 180^\circ - 86^\circ$
$= 94^\circ$
$\triangle RST$ is NOT acute.



An acute triangle may have the same lengths of sides or different side lengths. It may also have two sides of same measure too. In simpler words, we can have an **equilateral acute** triangle, **isosceles acute** triangle as well as the typical **scalene acute** triangle. These internal angles of the triangle add up to 180^o.

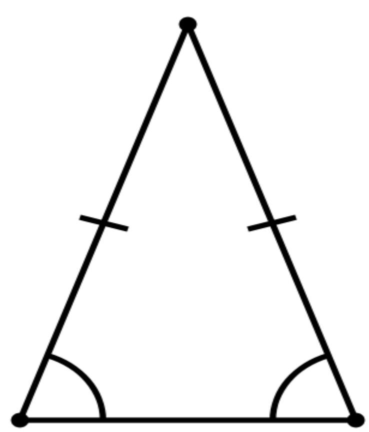
Acute Equilateral Triangle: a triangle where all angles measure to less than 90^o and all angles are equal in measure.

In actuality, all equilateral triangles are acute but it's important to note what this case looks like. All angles are 60^o.

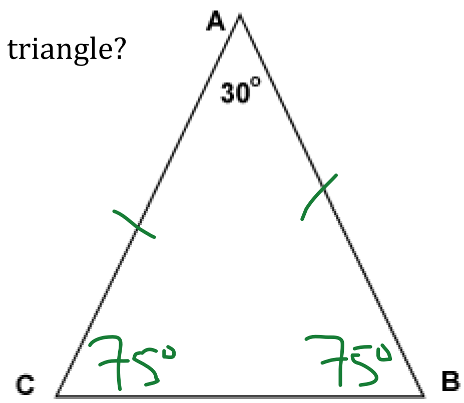


Acute Isosceles Triangle: An **isosceles triangle** that has exactly two equal sides, and therefore two equal base angles. The angle included by the legs is called the vertex angle and the angles that have the base as one of their sides are called the base angles.

For example:



Example 10:
What is the measure of the base angles for the following triangle?



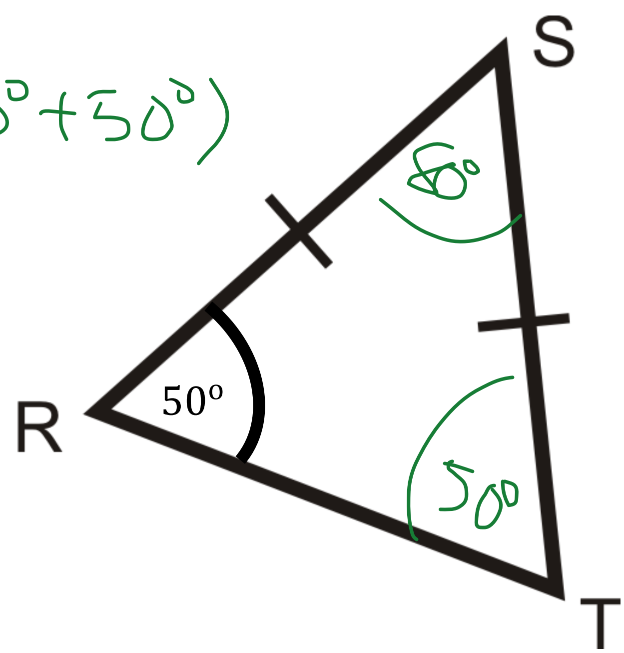
$\angle C = \angle B$

$\dfrac{(180° - 30°)}{2}$

$= \dfrac{150°}{2}$

$= 75°$

Example 11:
Find all missing angles.



$\angle S = 180° - (50° + 50°)$

$= 180° - 100°$

$= 80°$