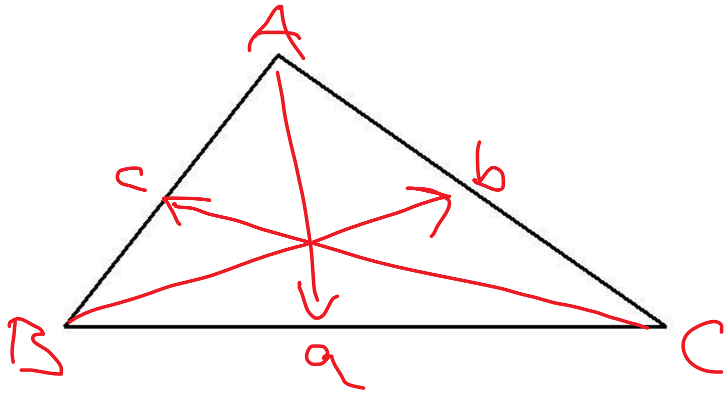
Math 2201 Date:____________________

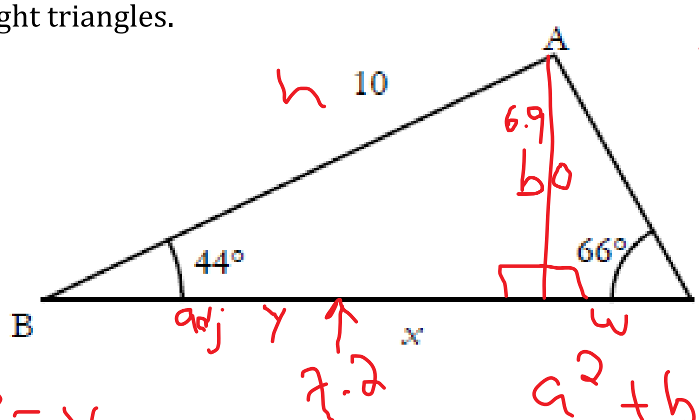
3.2 The Sine Law

Labelling a Triangle
To properly label a triangle, name all angles as capital letters. The side opposite each angle will then have the corresponding lower case letter.



The Sine Law

Before we get into the Sine Law, let's use some right triangle trigonometry techniques we reviewed in the previous lession to solve for x in the following diagram. Hint: divide the figure into two right triangles.



$x = y + w$

$\cos 44° = \frac{y}{10}$

$\frac{0.7193}{1} = \frac{y}{10}$

$y = 10(0.7193)$

$y = 7.2$

$a^2 + b^2 = c^2$

$(7.2)^2 + b^2 = 10^2$

$\sqrt{b^2} = \sqrt{48.16}$

$b = 6.9$

$$\tan 66^\circ = \frac{6.9}{w}$$

$$\frac{2.2460}{1} = \frac{6.9}{w}$$

$$\frac{2.2460w}{2.2460} = \frac{6.9}{2.2460}$$

$$w = 3.1$$

$$x = y + w$$

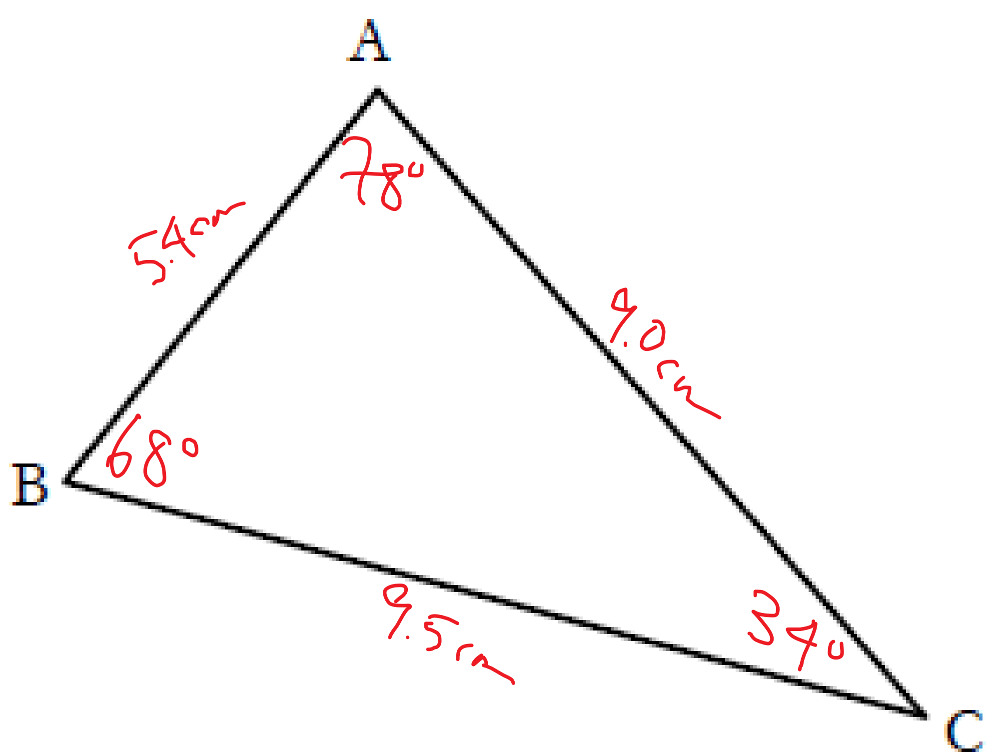
$$x = 7.2 + 3.1$$

$$x = 10.3$$

Using right-triangle trigonometry to solve non-right triangles is not very practical. While not difficult, it can be time consuming. Fortunately other, more efficient methods have been discovered.

Developing the Sine Law

Measure the side lengths and angles of the following triangle using a ruler and protractor.



Complete the following table:

	Measure		**Measure**	**Calculate**	
∠A	78°	side *a*	9.5	$\frac{\sin A}{a}$	$\frac{\sin 78^\circ}{9.5} = 0.1$
∠B	68°	side *b*	9.0	$\frac{\sin B}{b}$	$\frac{\sin 68^\circ}{9.0} = 0.1$
∠C	34°	side *c*	5.4	$\frac{\sin C}{c}$	$\frac{\sin 34^\circ}{5.4} = 0.1$

1. What conjecture can you make regarding the ratios calculated above?

They are the same. (0.1)

2. Would your conjecture be valid if you were to use the reciprocal of the ratios?

$\frac{9.5}{\sin 78^\circ} = 9.7 \quad \frac{9.0}{\sin 68^\circ} = 9.7 \quad \frac{5.4}{\sin 34^\circ} = 9.7$

The Sine Law

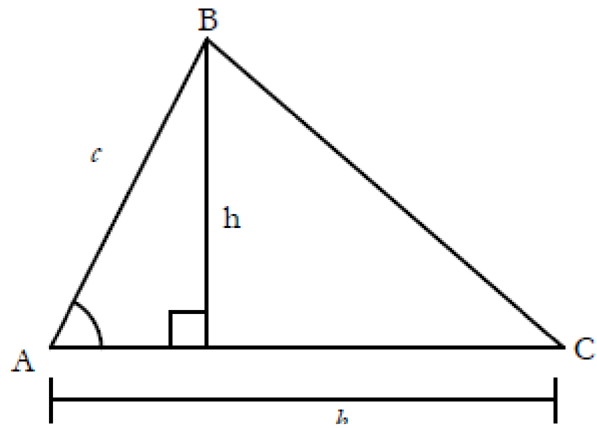
The Law of Sines can be derived using the area formula of a right triangle. Consider the following diagram:

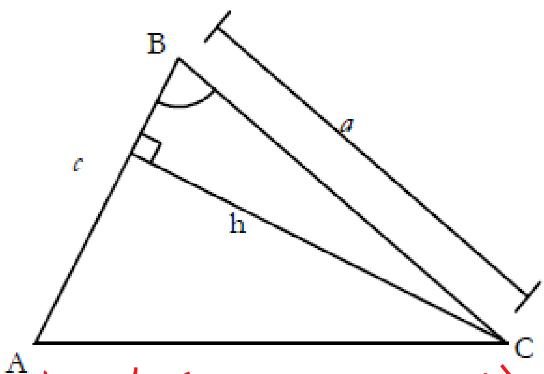
$A = \frac{1}{2}(\text{base})(\text{height})$

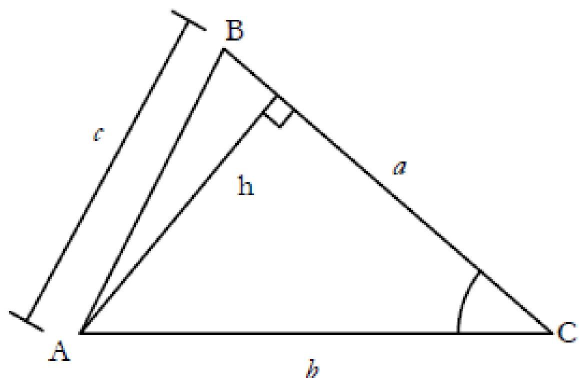
$\sin A = \frac{h}{c}$

$h = c \sin A$

$A = \frac{1}{2} bc \sin A$







$A = \frac{1}{2}(\text{base})(\text{height})$

$\sin B = \frac{h}{a}$

$h = a \sin B$

$A = \frac{1}{2} c \cdot a \sin B$

$A = \frac{1}{2} ac \sin B$

$\sin C = \frac{h}{a}$

$h = a \sin C$

$A = \frac{1}{2} b \cdot a \sin C$

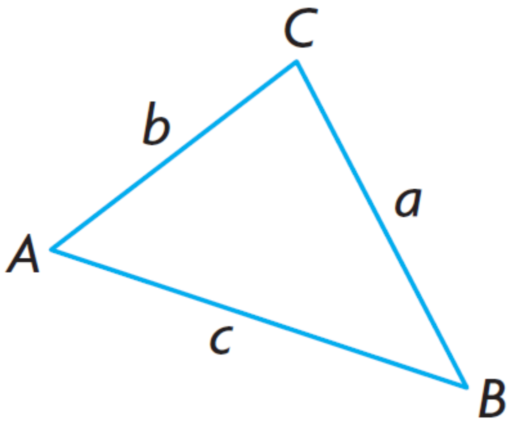
$A = \frac{1}{2} ab \sin C$

$A_1 = A_2 = A_3$

$$\frac{\cancel{\frac{1}{2}}\cancel{bc}\sin A}{\cancel{\frac{1}{2}}a\cancel{bc}} = \frac{\cancel{\frac{1}{2}}\cancel{ac}\sin B}{\cancel{\frac{1}{2}}\cancel{a}b\cancel{c}} = \frac{\cancel{\frac{1}{2}}\cancel{ab}\sin C}{\cancel{\frac{1}{2}}\cancel{ab}c}$$

$$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Let ΔABC be any acute triangle, where a, b and c represent the measures of the sides opposite ∠A, ∠B and ∠C, respectively. Then:



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ ← Find sides

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$ ← Find angles

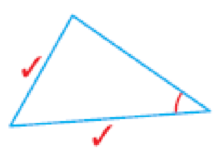
In Summary

Key Idea

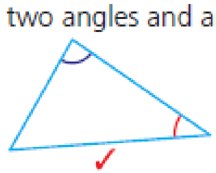
- The sine law can be used to determine unknown side lengths or angle measures in acute triangles.

Need to Know

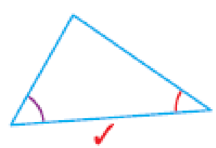
- You can use the sine law to solve a problem modelled by an acute triangle when you know:
  - two sides and the angle opposite a known side.



  - two angles and any side.



or



- If you know the measures of two angles in a triangle, you can determine the third angle because the angles must add to 180°.
- When determining side lengths, it is more convenient to use:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- When determining angles, it is more convenient to use:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 1:
Solve for the each variable:

(A) $\dfrac{x}{\sin 48^o} = \dfrac{7}{\sin 59^o}$

$\dfrac{x}{0.7431} = \dfrac{8.1664}{1}$

$x = (0.7431)(8.1664)$

$x = 6.1$

(B) $\dfrac{5}{\sin B} = \dfrac{11}{\sin 68^o}$ reciprocate

$\dfrac{\sin B}{5} = \dfrac{\sin 68^o}{11}$

$\dfrac{\sin B}{5} = \dfrac{0.0843}{1}$

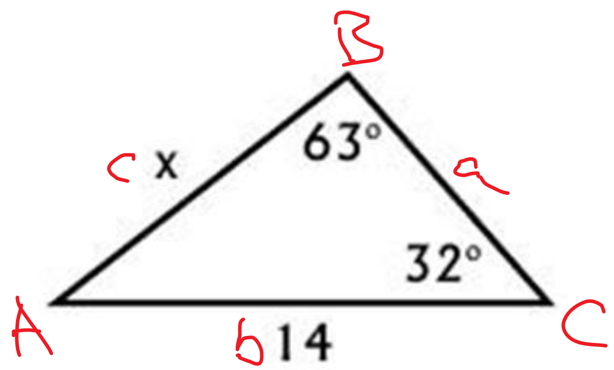
$\sin B = 5(0.0843)$

$\sin B = 0.4215 \rightarrow B = \sin^{-1}(0.4215)$

$B = 25^o$

Example 2:
Find the value for x.

(A) $\cancel{\dfrac{a}{\sin A}} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$



• need complete ratio

$\dfrac{14}{\sin 63^o} = \dfrac{x}{\sin 32^o}$

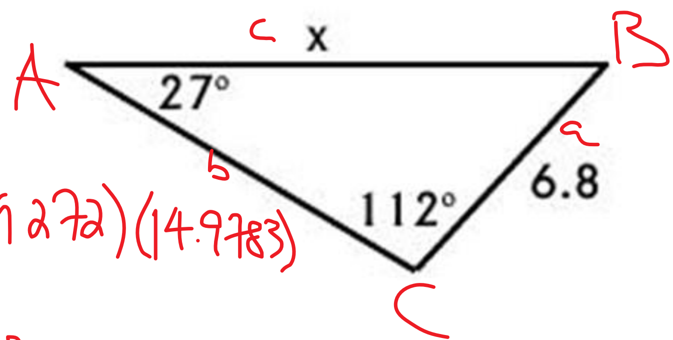
$\dfrac{15.7126}{1} = \dfrac{x}{0.5299}$

$x = (15.7126)(0.5299)$

$x = 8.3$

(B)

$$\frac{a}{\sin A} = \cancel{\frac{b}{\sin B}} = \frac{c}{\sin C}$$



$$\frac{6.8}{\sin 27°} = \frac{x}{\sin 112°}$$

$$\frac{14.9783}{1} = \frac{x}{0.9272}$$

$$x = (0.9272)(14.9783)$$

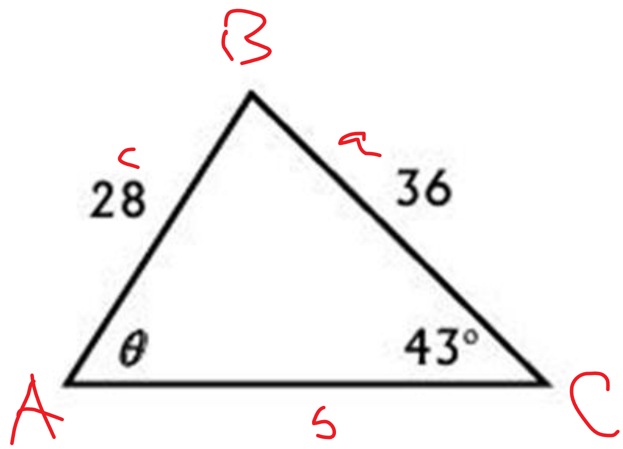
$$x = 13.8$$

Example 3:

Determine the measure of θ.

(A)

$$\frac{\sin A}{a} = \cancel{\frac{\sin B}{b}} = \frac{\sin C}{c}$$



$$\frac{\sin\theta}{36} = \frac{\sin 43°}{28}$$

$$\frac{\sin\theta}{36} = \frac{0.0244}{1}$$

$$\sin\theta = (0.0244)(36)$$

$$\sin\theta = 0.8784$$

$$\theta = \sin^{-1}(0.8784)$$

$$\theta = 61°$$

(B)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \cancel{\frac{\sin C}{c}}$$

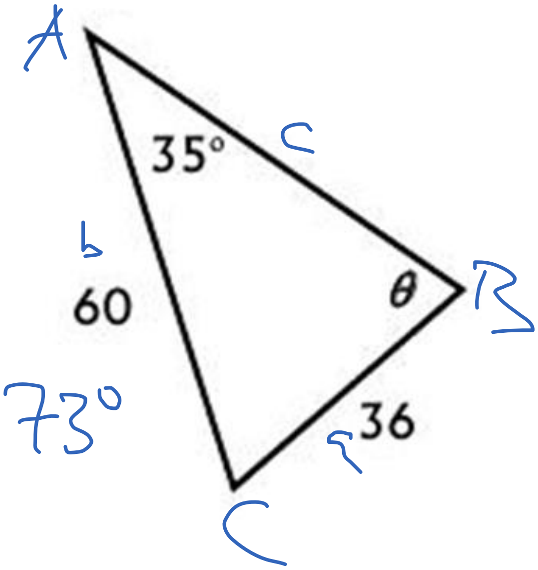
$$\frac{\sin 35^\circ}{36} = \frac{\sin\theta}{60}$$

$$\frac{0.0159}{1} = \frac{\sin\theta}{60}$$

$$\sin\theta = 0.9540$$

$$\theta = \sin^{-1}(0.9540)$$

$$\theta = 73^\circ$$



Example 4:
Find the length of side *b*.

$$\cancel{\frac{a}{\sin A}} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Need a complete ratio!
(angle with opposite side)

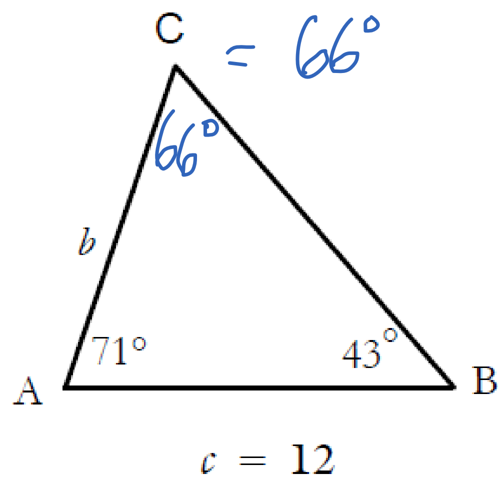
$$\frac{b}{\sin 43^\circ} = \frac{12}{\sin 66^\circ}$$

$$\frac{b}{0.6820} = \frac{13.1356}{1}$$

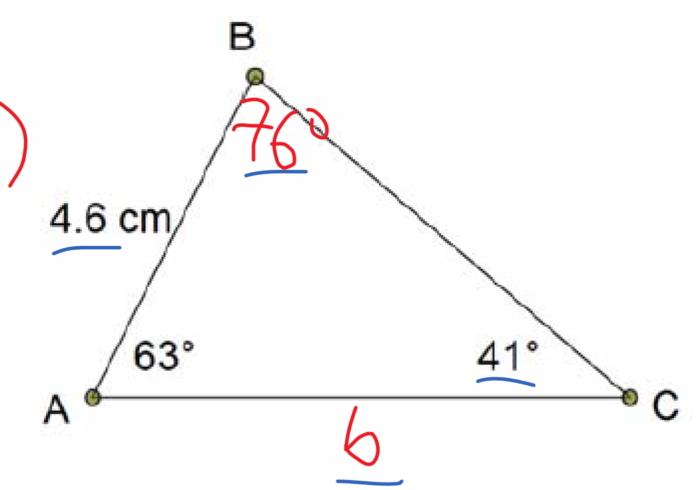
$$b = (0.6820)(13.1356)$$

$$b = 9.0$$

$$\angle C = 180^\circ - (71^\circ + 43^\circ) = 66^\circ$$



Example 5:
Find the length of $\overline{AC}$.



$\angle B = 180° - (63° + 41°)$

$\angle B = 76°$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

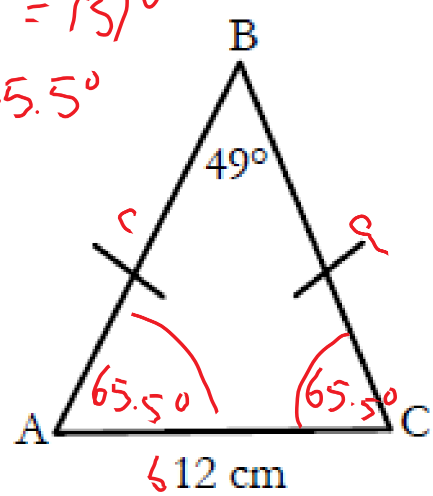
$$\sin 76 \cdot \frac{b}{\cancel{\sin 76°}} = \frac{4.6}{\sin 41°} \cdot \sin 76°$$

$$b = \frac{4.6 \sin 76°}{\sin 41°} = 6.8 \text{ cm}$$

Example 6:
Find the missing side lengths in the isosceles triangle below.

$180° - 49° = 131°$

$$\frac{131°}{2} = 65.5°$$



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

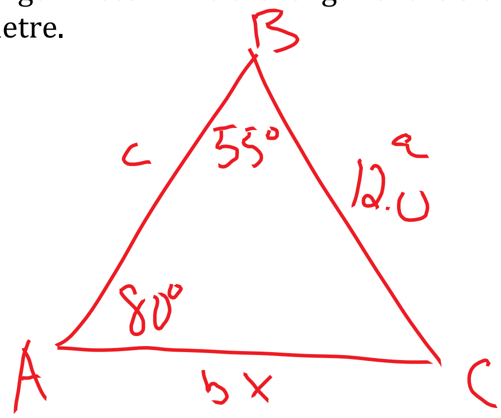
$$\frac{a}{\sin 65.5°} = \frac{12}{\sin 49°}$$

$$\frac{a}{0.9100} = \frac{15.9001}{1}$$

$a = 14.5 \text{ cm}$

$c = 14.5 \text{ cm}$

Example 7:
A triangle has angles measuring 80° and 55°. The side opposite the 80° angle is 12.0 m in length. Determine the length of the side opposite the 55° angle to the nearest tenth of a metre.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \cancel{\frac{c}{\sin C}}$$

$$\frac{12.0}{\sin 80°} = \frac{x}{\sin 55°}$$

$$\frac{12.1851}{1} = \frac{x}{0.8192}$$

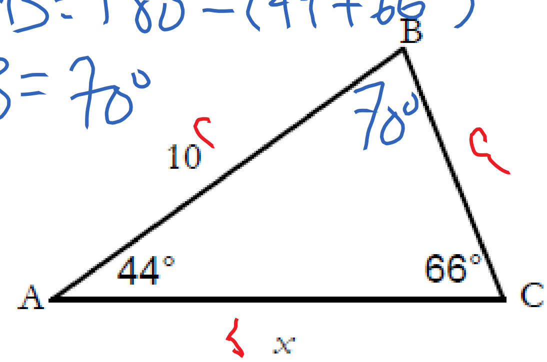
$$x = 10.0\text{m}$$

Example 8:
Two observers sight a plane at angles of elevation of 44°and 66°. If one observer is 10 km away from the plane, how far apart are the two observers from one another?

$$\cancel{\frac{a}{\sin A}} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\angle B = 180° - (44° + 66°)$$

$$\angle B = 70°$$



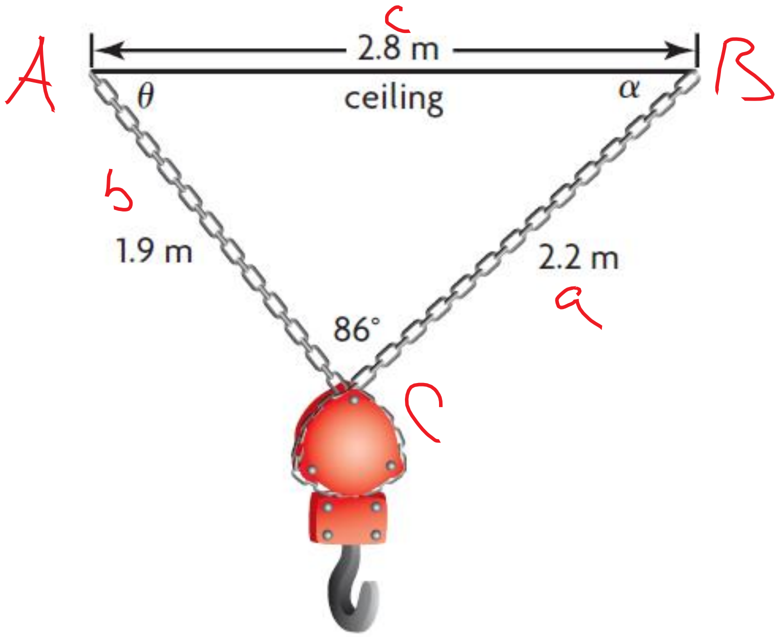
$$\frac{x}{\sin 70°} = \frac{10}{\sin 66°}$$

$$\frac{x}{0.9397} = \frac{10.9464}{1}$$

$$x = 10.3\text{km}$$

The observers are 10.3 km apart.

Example 9:
Toby uses chains attached to hooks on the ceiling and a winch to lift engines at his father's garage. The chains, the winch, and the ceiling are arranged as shown. Toby solved the triangle using the sine law to determine the angle that each chain makes with the ceiling to the nearest degree. He claims that $\theta = 40°$ and $\alpha = 54°$. Is he correct? Explain, and make any necessary corrections.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin\theta}{2.2} = \frac{\sin 86°}{2.8}$$

$$\frac{\sin\theta}{2.2} = \frac{0.3563}{1}$$

$$\sin\theta = 0.7838$$

$$\theta = \sin^{-1}(0.7838)$$

$$\theta = 51°$$

$$\alpha = 180° - (51° + 86°) = 43°$$

Toby is incorrect. θ is $51°$ and α is $43°$

Textbook Questions: page: 131 # 1, 2, 3, 4
page: 138 - 141; # 1, 2, 3, 4, 5, 6, 10, 11, 13, 15, 18