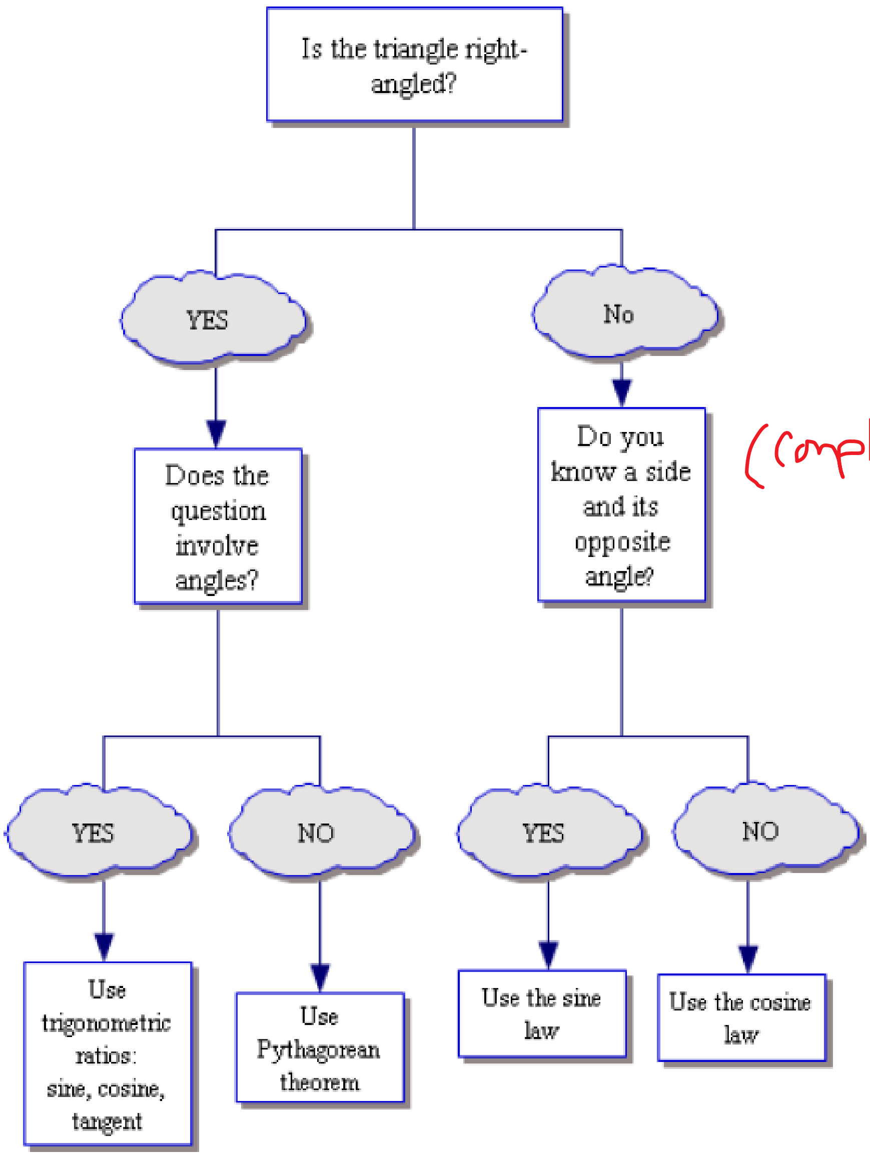
Math 2201 **Date:**____________________

3.4 Solving Problems Using Acute Triangles

By now you have been exposed to the Sine law, the Cosine law, the primary trigonometric ratios, the Pythagorean theorem and the sum of the angles in a triangle. You may need to use a strategy or a combination of strategies to solve problems represented by one or more than one triangle.

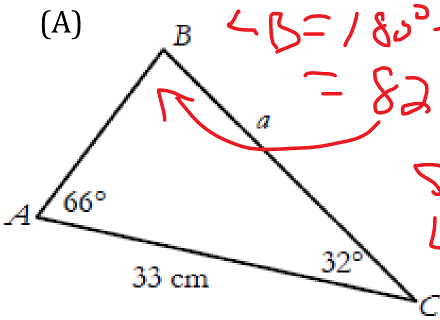
The following diagram my help you to get used to deciding which technique to use while you are learning to solve triangles.



(complete ratio)

Example 1
Which method would you use to find each missing side or angle?

(A)

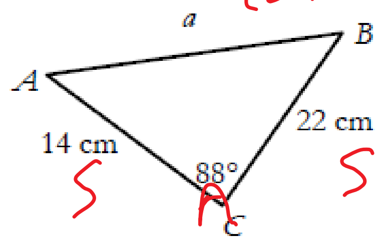


$\angle B = 180^\circ - (66^\circ + 32^\circ)$
$= 82^\circ$

Sine Law.

(B)

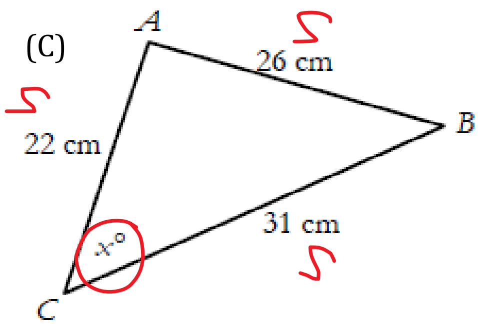
Cosine law



S A S

Cosine law

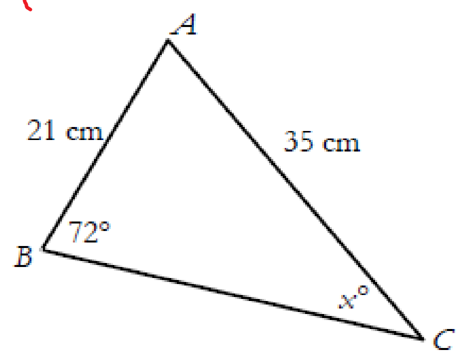
(C)



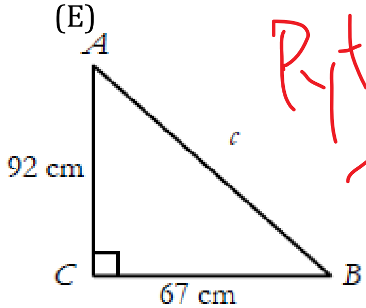
S S S

(D)

Sine law

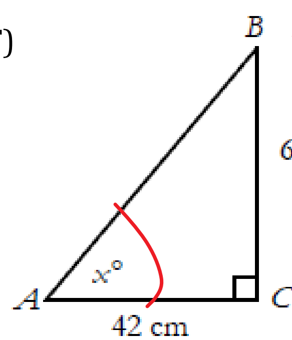


(E)



Pythagorean Theorem

(F)



~~tangent~~ tangent

o

a

(Soh cah toa)

Example 2

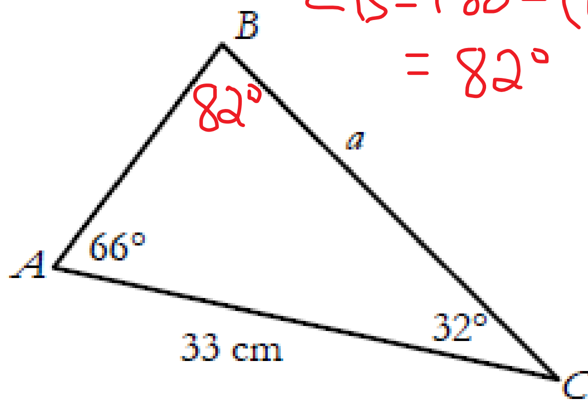
Find each missing side or angle:

(A) $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin 66^\circ} = \frac{33}{\sin 82^\circ}$

$\frac{a}{0.9135} = \frac{33.3243}{1}$

$a = 30\text{ cm}$



$\angle B = 180^\circ - (66^\circ + 32^\circ)$
$= 82^\circ$

(B)

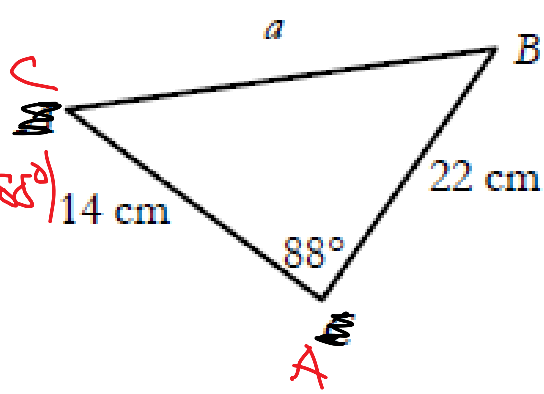
$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = (14)^2 + (22)^2 - 2(14)(22)\cos(88^\circ)$

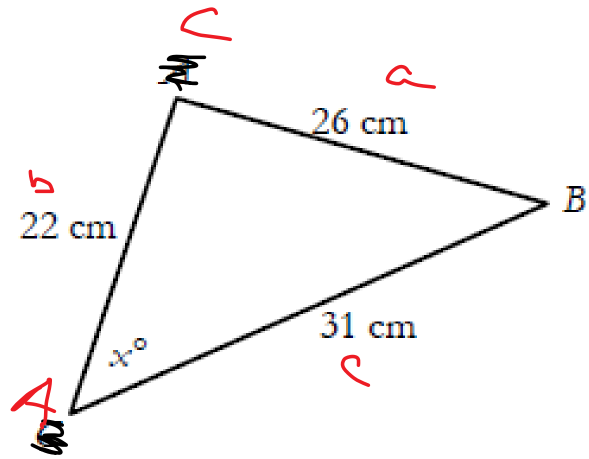
$a^2 = 658.5019$

$\sqrt{a^2} = \sqrt{658.5019}$

$a \approx 26\text{cm}$



(C)



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{22^2 + 31^2 - 26^2}{2(22)(31)}$$

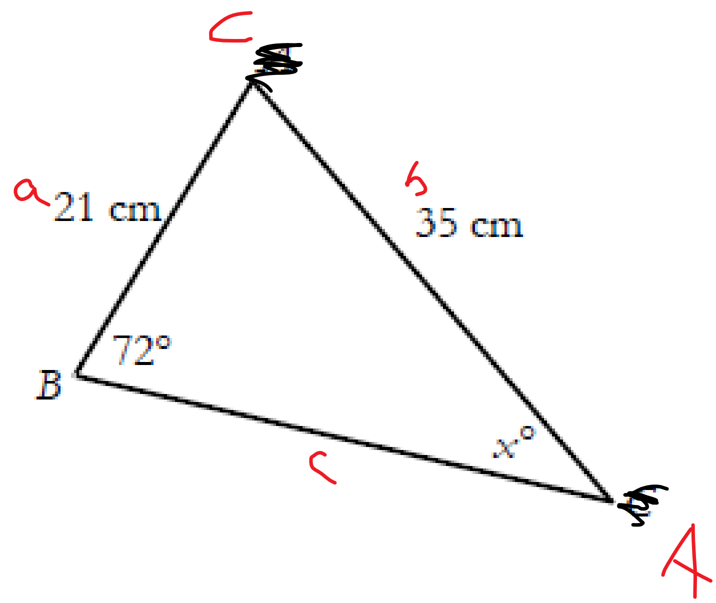
$$\cos A = \frac{769}{1364}$$

$$\cos A = 0.5637$$

$$\theta = \cos^{-1}(0.5637)$$

$$\theta = 56^\circ$$

(D)



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin X}{21} = \frac{\sin 72^\circ}{35}$$

$$\frac{\sin X}{21} = \frac{0.0272}{1}$$

$$\sin X = 0.5712$$

$$X = \sin^{-1}(0.5712)$$

$$X = 35^\circ$$

(E)

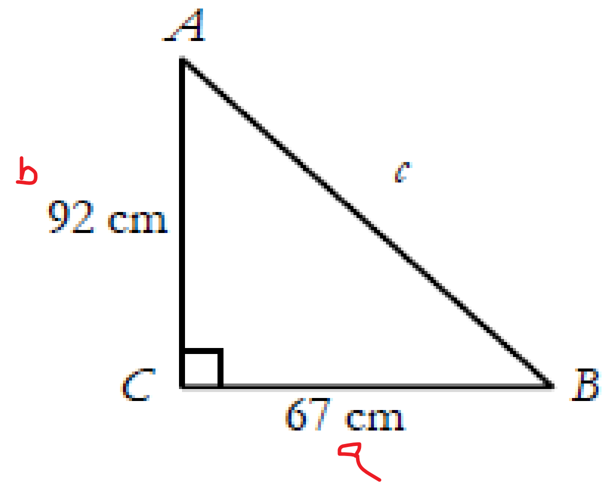
$a^2+b^2=c^2$

$(67)^2+(92)^2=c^2$

$12953=c^2$

$\sqrt{c^2}=\sqrt{12953}$

$c=114$ cm



(F)

~~soh~~ ~~cah~~ $\boxed{\text{toa}}$

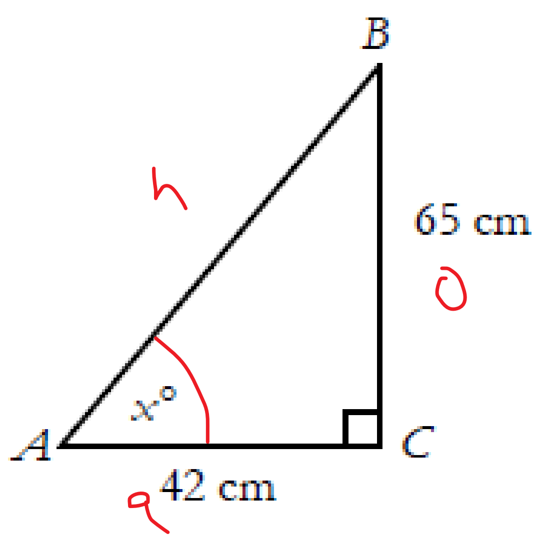
$\tan A=\frac{o}{a}$

$\tan x=\frac{65}{42}$

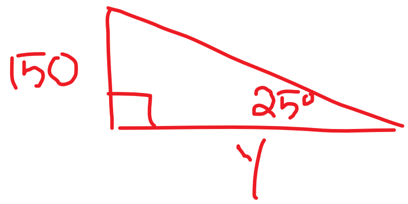
$\tan x=1.5476$

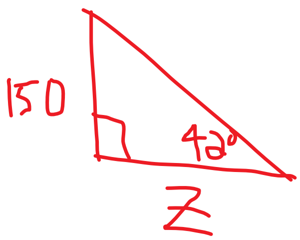
$x=\tan^{-1}(1.5476)$

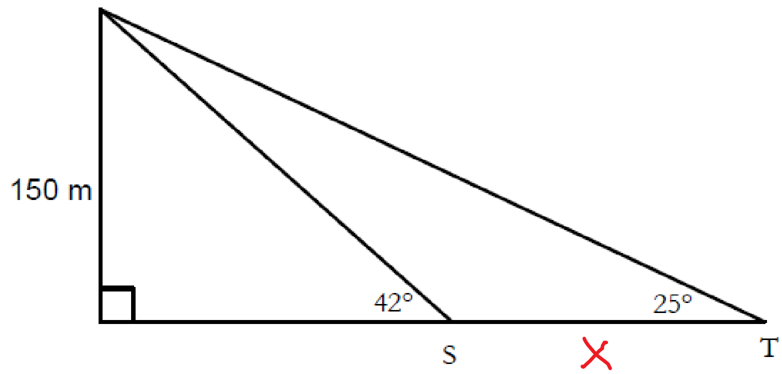
$x=57^\circ$



Example 3
A smokestack is 150 metres high. Two observers, located at positions S and T, look to the top of the smokestack at angles of elevation of 42° and 25° respectively. How far apart are the two people?







$X = Z - Y$

~~SOH~~ ~~CAH~~ TOA

$\tan 25^\circ = \frac{150}{Z}$

$\frac{0.4663}{1} = \frac{150}{Z}$

$\frac{\cancel{0.4663}Z}{\cancel{0.4663}} = \frac{150}{0.4663}$

$Z = 322\text{m}$

$\tan 42^\circ = \frac{150}{Y}$

$\frac{0.9004}{1} = \frac{150}{Y}$

$\frac{\cancel{0.9004}Y}{\cancel{0.9004}} = \frac{150}{0.9004}$

$Y = 167\text{m}$

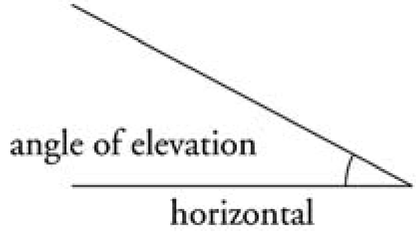
$X = Z - Y$

$X = 322 - 167$

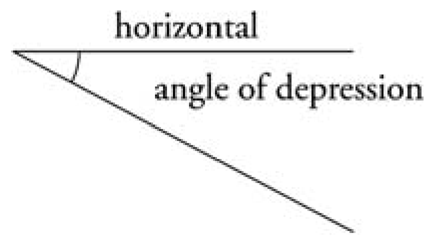
$X = 155\text{m}$

The people are 155m apart.

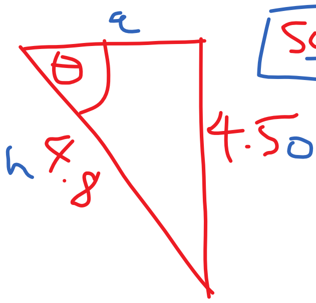
The Angle of Elevation
The angle formed by the horizontal and a line of sight above the horizontal is called the angle of elevation.



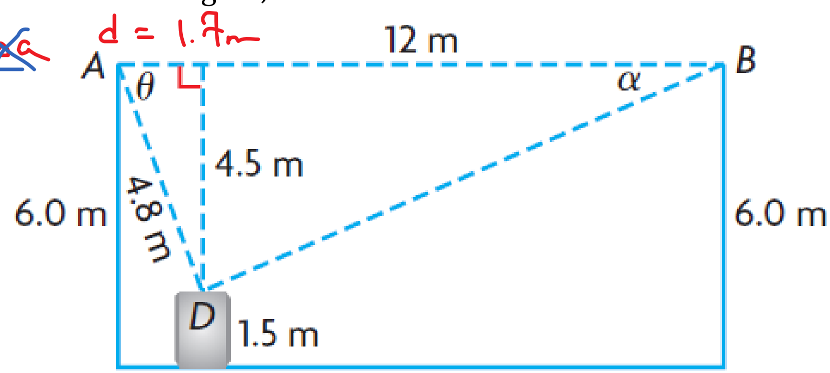
The Angle of Depression
The angle formed by the horizontal and a line of sight below the horizontal is called the angle of depression.



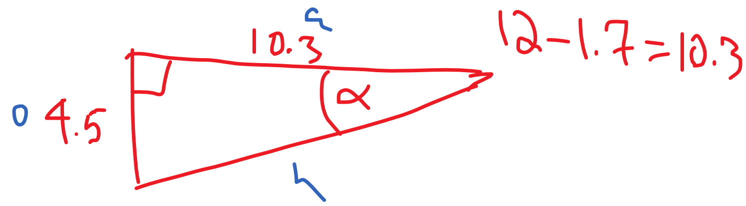
Example 4
Determine the **angles of depression,** to the nearest degree, for each camera.



$\boxed{\text{soh}}$ cah toa



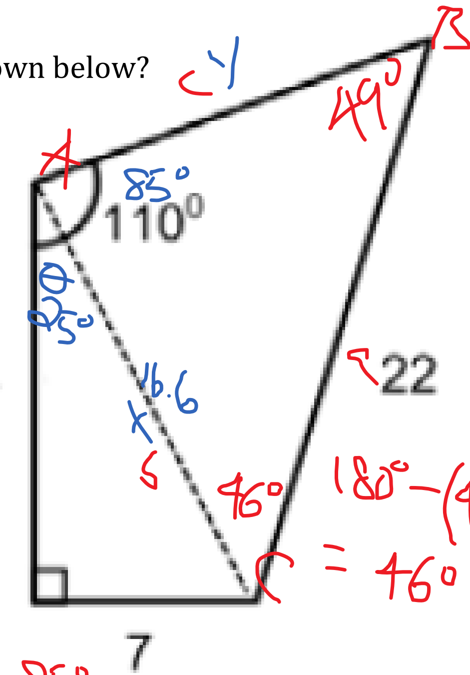
$$\sin\theta = \frac{4.5}{4.8}$$
$$\sin\theta = 0.9375$$
$$\theta = \sin^{-1}(0.9375)$$
$$\theta = 70^\circ$$
$$d^2 = (4.8)^2 - (4.5)^2$$
$$\sqrt{d^2} = \sqrt{2.79}$$
$$d = 1.7\text{ m}$$



$$12 - 1.7 = 10.3$$
$$\tan\alpha = \frac{4.5}{10.3}$$
$$\tan\alpha = 0.4369$$
$$\alpha = \tan^{-1}(0.4369)$$
$$\alpha = 24^\circ$$

Example 5

How much fencing is required to enclose the playground shown below?



$15^2 + 7^2 = x^2$

$225 + 49 = x^2$

$x^2 = 274$

$\sqrt{x^2} = \sqrt{274}$

$x = 16.6$

$110° - 25° = 85°$

$180° - (49° + 85°) = 46°$

$\tan\theta = \frac{7}{15}$

$\tan\theta = 0.4667$

$\theta = \tan^{-1}(0.4667)$

$\theta = 25°$

$\frac{\sin B}{16.6} = \frac{\sin 85°}{22}$

$\frac{\sin B}{16.6} = \frac{0.0453}{1}$

$\sin B = 0.7517$

$B = \sin^{-1}(0.7517)$

$B = 49°$

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{c}{\sin 46°} = \frac{22}{\sin 85°}$

$\frac{c}{0.7193} = \frac{22.0840}{1}$

$c = 15.9$

$15.9 + 15 + 22 + 7$

$= 59.9$

~ 60 units.

Example 6

The world's tallest free-standing totem pole is located in Beacon Hill Park in Victoria, British Columbia. It was carved from a single cedar log by noted carver Chief Mungo Martin of the Kwakiutl (Kwakwaka'wakw), with a team that included his son David and Henry Hunt. It was erected in 1956. While visiting the park, Manuel wanted to determine the height of the totem pole, so he drew a sketch and made some measurements:

- I walked along the shadow of the totem pole and counted 42 paces, estimating each pace was about 1 m.
- I estimated that the **angle of elevation** of the Sun was about 40°.
- I observed that the shadow ran uphill, and I estimated that the angle the hill made with the horizontal was about 5°.

How can Manuel determine the height of the totem pole to the nearest metre?

$\angle D = 180° - (40° + 5°)$

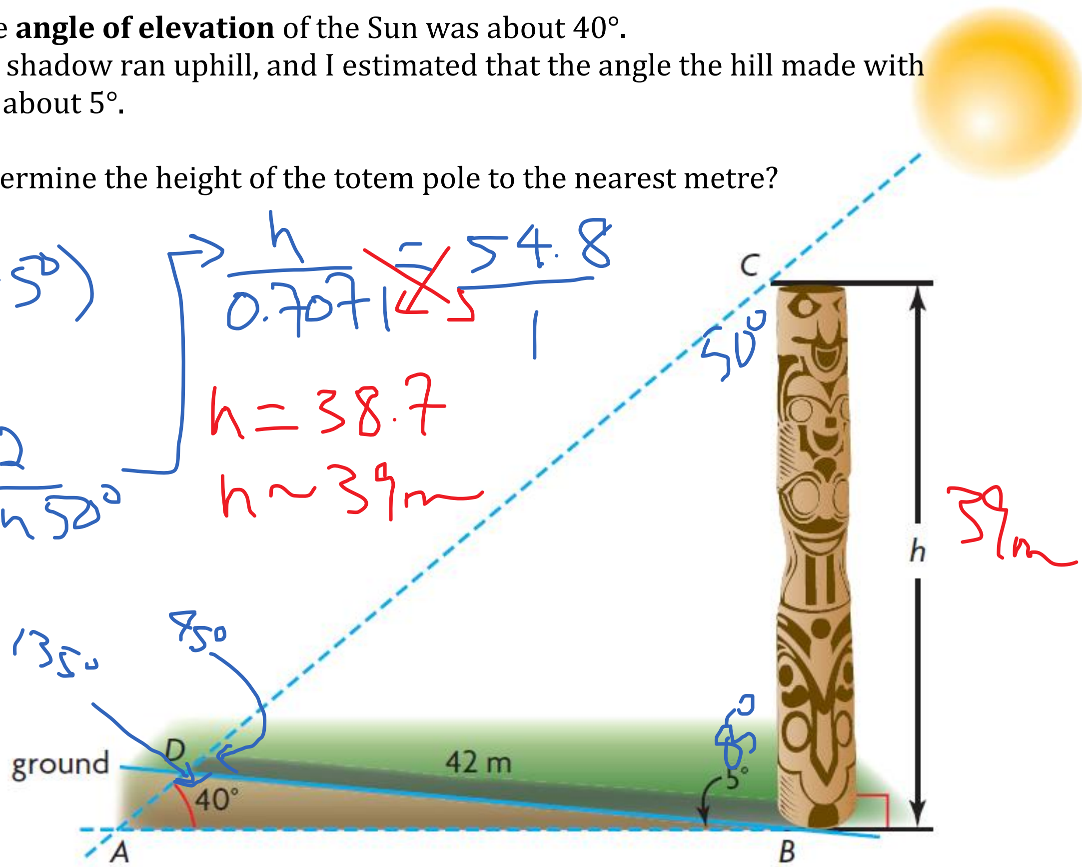
$= 135°$

$\dfrac{h}{\sin 45°} = \dfrac{42}{\sin 50°}$

$\rightarrow \dfrac{h}{0.7071} = \dfrac{54.8}{1}$

$h = 38.7$

$h \sim 39\text{m}$



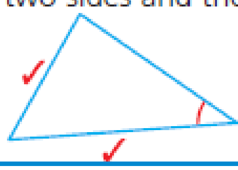
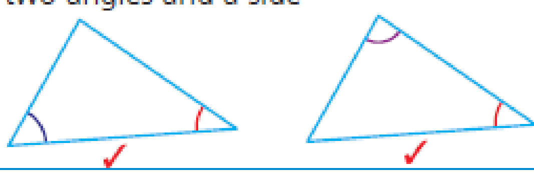
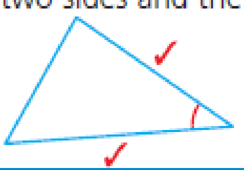
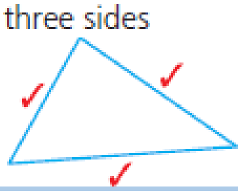
In Summary

Key Idea

- The sine law, the cosine law, the primary trigonometric ratios, and the sum of angles in a triangle may all be useful when solving problems that can be modelled using acute triangles.

Need to Know

- To decide whether you need to use the sine law or the cosine law, consider the information given about the triangle and the measurement to be determined.

Information Given	Measurement to be Determined	Use
two sides and the angle opposite one of the sides 	angle	sine law
two angles and a side 	side	sine law
two sides and the contained angle 	side	cosine law
three sides 	angle	cosine law

- Drawing a clearly labelled diagram makes it easier to select a strategy for solving a problem.

Textbook Questions: page: 161 - 163; # 1, 2, 3, 4, 5, 6, 7, 8