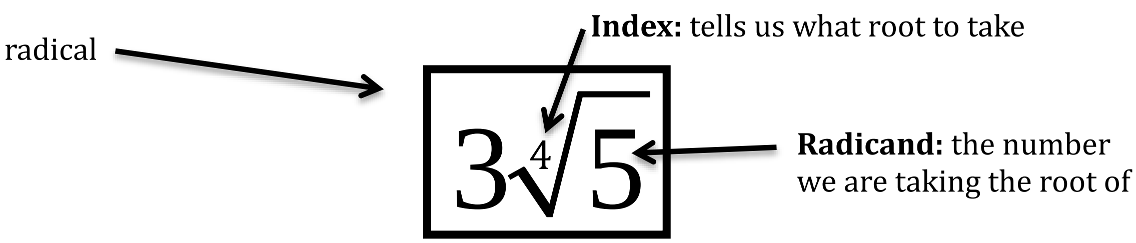
Math 2201 **Date:**____________________

4.1A Review of Radicals

Recall from Level 1, radicals:



Index:
the index tells us what root to take. No index would be the same as the index being 2, or a **square root.** For example:

$\sqrt{9}$ $\qquad\qquad$ $\sqrt[2]{9}$

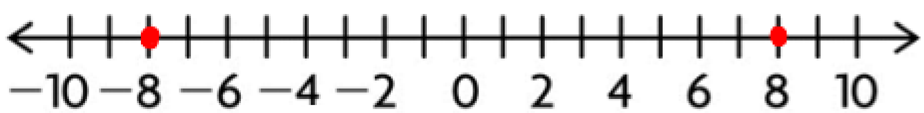
$= 3$ $\qquad\qquad$ $= 3$

When the index is 3 we take the cube root.
When the index is 4 we take the fourth root, etc.

Principal and Secondary Square Roots
Every positive number has two square roots. For example, the square root of 49 is 7 since $7^2 = 49$. Likewise $(-7)^2 = 49$ so -7 is also a square root of 49. The value $\sqrt{49} = 7$ is called the **principal square root** and $\sqrt{49} = -7$ is the **secondary square root**

$+$ $\qquad\qquad$ $-$

Another way to look at it is think of opposite numbers on a number line, for example -8 and 8.



$(-8)^2$ $\qquad\qquad$ 8^2

$= 64$ $\qquad\qquad$ $= 64$

$\therefore \sqrt{64} = -8$ $\qquad\qquad$ $\therefore \sqrt{64} = 8$

Example 1:

(A) Determine the value of x:

$x^2 = 3^2 + 4^2$

$x^2 = 9 + 16$

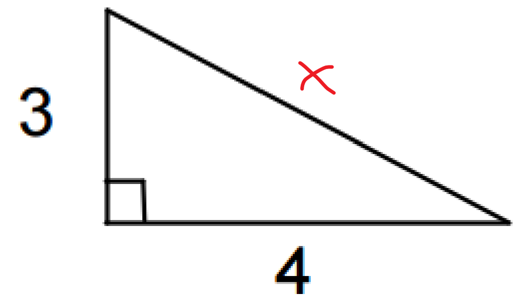
$x^2 = 25$

$\sqrt{x^2} = \sqrt{25}$

$x = 5$

or

$\cancel{x = -5}$



(B) Which would be more appropriate to use, the primary or secondary square root?

Distance is always positive.

Entire and Mixed Radicals

Entire Radical: a radical that is not multiplied by any number other than 1. $\sqrt{x}$

Mixed Radical: a radical that is multiplied by a number other than 1. $y\sqrt{x}$

Example 2:

Classify the following as entire radical or mixed radical.

(A) $3\sqrt{7}$ Mixed

(B) $\sqrt{5}$ Entire

(C) $\sqrt[3]{9}$ Entire

(D) $5\sqrt[5]{5}$ Mixed

Simplifying Radicals

To determine whether a radical can be simplified, use your calculator to take the square/cube/fourth/etc. root (whichever is required). If the answer works out to be a whole number, then you simply need to state that answer.

(A) $\sqrt{625} = 25$

(B) $\sqrt[3]{5832} = 18$

(C) $\sqrt[4]{1296} = 6$

(D) $4\sqrt[4]{196} = 4 \cdot 14 = 56$

(E) $5\sqrt[3]{729} = 5 \cdot 9 = 45$

(F) $12\sqrt[4]{4096} = 12(8) = 96$

Sometimes the radicand may not be a perfect square/cube, etc. In these cases, when we take the root using a calculator, we get a decimal number instead of a whole number. In these cases, we may be able to break the radical down into a mixed radical. There are two methods for doing this:

- the factor tree method
- finding the largest perfect square/cube, etc. factor of the radicand

Factor Tree Method
Here we **prime factorize** the radicand and find groups of prime factors that are equivalent to the index.

Example 3:
Reduce using the factor tree method:

(A) $\sqrt{20}$ index: 2

$\sqrt{20}$

$4 \cdot 5$

$(2 \cdot 2) \; 5$

$2^2 \cdot 5$

$\rightarrow \sqrt{2^2 \cdot 5}$

$= \cancel{\sqrt{}}2^{\cancel{2}}\sqrt{5}$

$= 2\sqrt{5}$

(B) $\sqrt[3]{16}$ index: 3

$2 \cdot 8$

$2 \;\; 2 \cdot 4$

$(2 \; 2 \; 2) \; 2$

$2^3 \cdot 2$

$\sqrt[3]{2^3 \cdot 2}$

$= \cancel{\sqrt[3]{}}2^{\cancel{3}} \cdot \sqrt[3]{2}$

$= 2\sqrt[3]{2}$

Finding the Largest Perfect Square/Cube, etc. Factor of the Radicand
This method involves finding the largest perfect square, cube, etc that divides into the radicand. To be efficient with this method, it's best to have perfect squares and cubes memorized up to 10.

Example 3:

(A) $\sqrt{20}$

number squared	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2
perfect square	1	4	9	16	25	36	49	64	81

$$\sqrt{20}$$
$$= \sqrt{4 \cdot 5}$$
$$= \sqrt{4}\sqrt{5}$$
$$= 2\sqrt{5}$$

(B) $\sqrt[3]{16}$

number cubed	1^3	2^3	3^3	4^3	5^3	6^3	7^3	8^3	9^3
Perfect cube	1	8	27	64	125	216	343	512	729

$$\sqrt[3]{16}$$
$$= \sqrt[3]{8 \cdot 2}$$
$$= \sqrt[3]{8}\sqrt[3]{2}$$
$$= 2\sqrt[3]{2}$$

Example 4:
Simplify using the method of your choice:

(A) $4\sqrt{48}$ index: 2

$\sqrt{48}$

$16 \cdot 3$

$4 \cdot 4 \cdot 3$

$(2 \cdot 2)(2 \cdot 2)\,3$

$2^2 \cdot 2^2 \cdot 3$

$4\sqrt{2^2 2^2 3}$

$= 4\sqrt{2^2}\sqrt{2^2}\sqrt{3}$

$= 4 \cdot 2 \cdot 2 \cdot \sqrt{3}$

$= 16\sqrt{3}$

$4\sqrt{48}$

$= 4\sqrt{16 \cdot 3}$

$= 4\sqrt{16}\sqrt{3}$

$= 4 \cdot 4\sqrt{3}$

$= 16\sqrt{3}$

(B) [illegible] index: 3

$2\sqrt[3]{24}$

$4 \cdot 6$

$(2 \cdot 2\, 2)\, 3$

$2^3 \cdot 3$

$2\sqrt[3]{2^3 \cdot 3}$

$= 2\sqrt[3]{2^3}\sqrt[3]{3}$

$= 2 \cdot 2\sqrt[3]{3}$

$= 4\sqrt[3]{3}$

$2\sqrt[3]{24}$

$= 2\sqrt[3]{8 \cdot 3}$

$= 2\sqrt[3]{8}\sqrt[3]{3}$

$= 2 \cdot 2\sqrt[3]{3}$

$= 4\sqrt[3]{3}$

Your turn:

1. Simplify the following:

(A) $\sqrt{48}$

$= \sqrt{16 \cdot 3}$

$= \sqrt{16}\sqrt{3}$

$= 4\sqrt{3}$

(E) $\sqrt[3]{54}$

$= \sqrt[3]{27}\sqrt[3]{2}$

$= 3\sqrt[3]{2}$

(B) $\sqrt{125}$

$= \sqrt{25 \cdot 5}$

$= \sqrt{25}\sqrt{5}$

$= 5\sqrt{5}$

(F) $\sqrt[3]{375}$

$= \sqrt[3]{125}\sqrt[3]{3}$

$= 5\sqrt[3]{3}$

(C) $4\sqrt{75}$

$= 4\sqrt{25 \cdot 3}$

$= 4\sqrt{25}\sqrt{3}$

$= 4 \cdot 5\sqrt{3}$

$= 20\sqrt{3}$

(G) $2\sqrt[3]{81}$

$= 2\sqrt[3]{27}\sqrt[3]{3}$

$= 2 \cdot 3\sqrt[3]{3}$

$= 6\sqrt[3]{3}$

(D) $-5\sqrt{128}$

$= -5\sqrt{64 \cdot 2}$

$= -5\sqrt{64}\sqrt{2}$

$= -5 \cdot 8\sqrt{2}$

$= -40\sqrt{2}$

(H) $-4\sqrt[3]{343}$

$= -4 \cdot 7$

$= -28$

Textbook Questions: page 182 – 183, #1, 2, 3, 4, 5, 7, 17, 18