Math 2201 Date:____________________

4.1 B Converting Mixed Radicals to Entire Radicals

Expressing Mixed Radicals as Entire Radicals
In Section 4.1A, we learned how to simplify radicals. That often requires us to convert an entire radical to a mixed radical. For example:

$$\sqrt{20}$$

$$= \sqrt{4}\sqrt{5}$$
$$= 2\sqrt{5}$$

Here, we will do the exact opposite. We will start off with mixed radicals and change them over to entire radicals.

In order to do this, we need to take the integer number in front of the root sign, which is called the **index**, and put it under a root sign. In other words, we replace the integer with a radical. Thus, we will end up with two radicals that need to be multiplied. We multiply the two numbers under the roots and place the product under a root sign.

Example 1:
Express each mixed radical as an entire radical:

(A) $4\sqrt{5}$ index: 2

$$= \sqrt{4^2}\sqrt{5}$$
$$= \sqrt{16}\sqrt{5}$$
$$= \sqrt{16 \cdot 5}$$
$$\rightarrow \sqrt{80}$$

(B) $-5\sqrt{2}$ index

$$= -\sqrt{(5)^2}\sqrt{2}$$
$$= -\sqrt{25}\sqrt{2}$$
$$= -\sqrt{25 \cdot 2}$$
$$= -\sqrt{50}$$

(C) $-2\sqrt[3]{5}$ index: 3

$$= \sqrt[3]{(-2)^3}\sqrt[3]{5}$$
$$= \sqrt[3]{-8}\sqrt[3]{5}$$
$$\rightarrow \sqrt[3]{-8 \cdot 5}$$
$$= \sqrt[3]{-40}$$

(D) $3\sqrt[3]{3}$

$$\sqrt[3]{3 \cdot 3 \cdot 3 \cdot (3)}$$
$$\sqrt[3]{81}$$

Writing mixed radicals as entire radicals is useful in that in allows us to more easily to determine which radical in a set is largest. As long as the radicals have the same index, then they can be compared by comparing the radicands.

Example 2:
Arrange the following radicals in order from least to greatest:

$4\sqrt{2}$ $\quad$ $2\sqrt{3}$ $\quad$ $\sqrt{11}$

$= \sqrt{4^2}\sqrt{2}$ $\quad$ $= \sqrt{2^2}\sqrt{3}$

$= \sqrt{16}\sqrt{2}$ $\quad$ $= \sqrt{4}\sqrt{3}$

$= \sqrt{32}$ $\quad$ $= \sqrt{12}$

$\sqrt{11}, \sqrt{12}, \sqrt{32}$

In Summary

Key Ideas

- You can express a radical in simplest form by using prime factors.

  For example:

  $$\sqrt{75} = \sqrt{5^2 \cdot 3} = 5\sqrt{3} \qquad \sqrt[3]{375} = \sqrt[3]{5^3 \cdot 3} = 5\sqrt[3]{3}$$

  When expressing square roots in simplest form, try to combine prime factors to create powers with even exponents. When working with cube roots, try to create powers with exponents that are multiples of 3.
- You can express a mixed radical as an entire radical, by writing the leading number as a radical, then multiplying the radicands.

  For example:

  $$4\sqrt{2} = \sqrt{16} \cdot \sqrt{2} = \sqrt{16 \cdot 2} = \sqrt{32}$$

Need to Know

- A radical is in simplest form when each exponent of the fully factored radicand is less than the index of the radical. For example, $12\sqrt{3^1}$ and $13\sqrt[3]{2^2}$ are in simplest form, while $12\sqrt{2^2}$ is not.
- A square has a principal square root, which is positive, and a secondary square root, which is negative. For example, the principal square root of 16 is $\sqrt{16}$ or 4, and the secondary square root of 16 is $-\sqrt{16}$ or -4. The radical of a square root may be negative, but the radicand of a square root must be positive.
- If you express an answer as a radical, the answer will be exact. If you write a radical in decimal form, the answer will be an approximation, except when the radicand is a perfect square. For example, $\sqrt{12}$ expressed as $2\sqrt{3}$ remains an exact value, while $\sqrt{12}$ expressed as 3.464… is an approximation. Both $\sqrt{9}$ and 3 are exact values.

Textbook Questions: page 182 - 183 #11, 12, 13