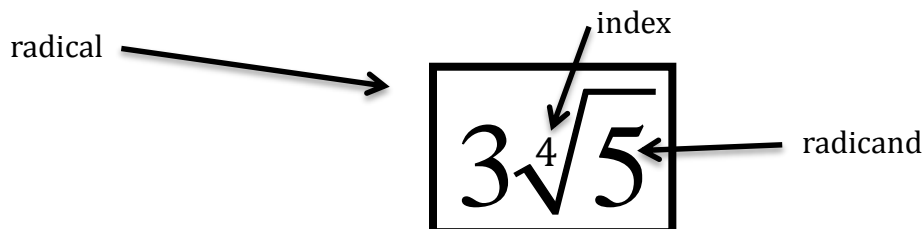


4.2 Adding and Subtracting Radicals

Recall from Section 4.1A, radicals:



Radicals with the same index and radicand are called Like Radicals.

Like Radicals

$$5\sqrt{7} \text{ and } -2\sqrt{7}$$

$$5\sqrt[3]{4} \text{ and } \sqrt[3]{4}$$

Unlike Radicals

$$2\sqrt{5} \text{ and } 5\sqrt{3}$$

$$\sqrt[4]{5} \text{ and } \sqrt[5]{5}$$

When adding and subtracting radicals, only like radicals can be combined. A good way to think about it, is to treat radicals like variables. For example:

$$4x + 3x = 7x$$

$$4\sqrt{5} + 3\sqrt{5} = 7\sqrt{5}$$

Unlike radicals cannot be added or subtracted. Think:

Simplest form \rightarrow $4x + 3y = 4x + 3y$

$$4\sqrt{5} + 3\sqrt{6} = 4\sqrt{5} + 3\sqrt{6}$$

Example 1:

Simplify where possible:

$$\begin{aligned} \text{(A)} \quad 5\sqrt{2} + 3\sqrt{2} &= 8\sqrt{2} \\ &= (5+3)\sqrt{2} \\ &= 8\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{(B)} \quad 6\sqrt{5} - 4\sqrt{5} &= 2\sqrt{5} \\ &= (6-4)\sqrt{5} \\ &= 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{(C)} \quad 5\sqrt{3} + 3\sqrt{2} \\ \text{already in simplest form.} \end{aligned}$$

$$\text{(D)} \quad -4\sqrt{2x} - 2\sqrt{2x} = -6\sqrt{2x}$$

$$\begin{aligned} \text{(E)} \quad 3\sqrt{2} + \sqrt[3]{2} \\ \text{already in simplest forms.} \end{aligned}$$

$$\text{(F)} \quad 4\sqrt{x} + 2\sqrt{x} - 7\sqrt{x} = -\sqrt{x}$$

$$\text{(E)} \quad 2\sqrt{2xy} + 4\sqrt{2xy} - 7\sqrt{2xy} = -\sqrt{2xy}$$

$$\begin{aligned} \text{(F)} \quad 3\sqrt{2} + 4\sqrt{5} - 6\sqrt{2} + 5\sqrt{5} \\ &= 3\sqrt{2} - 6\sqrt{2} + 4\sqrt{5} + 5\sqrt{5} \\ &= -3\sqrt{2} + 9\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{(G)} \quad -5\sqrt{2} + 3\sqrt{7y} + 8\sqrt{2} + 3\sqrt{5} - 2\sqrt{7y} \\ &= 3\sqrt{2} + \sqrt{7y} + 3\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{(H)} \quad 6\sqrt{2} - 3\sqrt[3]{2} + 11\sqrt{2} + 4\sqrt[3]{2} \\ &= 17\sqrt{2} + 3\sqrt[3]{2} \end{aligned}$$

Sometimes like radicals can be hidden. By simplifying each radical to lowest terms, mixed radicals, we can then see which are like and which are unlike.

Example 2:

Simplify each radical expressions:

(A) $2\sqrt{27} - 4\sqrt{3} - \sqrt{12}$

$$= 6\sqrt{3} - 4\sqrt{3} - 2\sqrt{3}$$

$$= 0\sqrt{3}$$

$$= 0$$

① $2\sqrt{27}$

$$= 2\sqrt{9\sqrt{3}}$$

$$= 2 \cdot 3\sqrt{3}$$

$$= 6\sqrt{3}$$

② $4\sqrt{3}$

③ $-\sqrt{12}$

$$= -\sqrt{4\sqrt{3}}$$

$$= -2\sqrt{3}$$

(B) $2\sqrt{24} - 4\sqrt{96} + \sqrt{432}$

$$= 4\sqrt{6} - 16\sqrt{6} + 12\sqrt{3}$$

$$= -12\sqrt{6} + 12\sqrt{3}$$

① $2\sqrt{24}$

$$= 2\sqrt{4\sqrt{6}}$$

$$= 2 \cdot 2\sqrt{6}$$

$$= 4\sqrt{6}$$

② $-4\sqrt{96}$

$$= -4\sqrt{16\sqrt{6}}$$

$$= -4 \cdot 4\sqrt{6}$$

$$= -16\sqrt{6}$$

③ $\sqrt{432}$

$$= \sqrt{144\sqrt{3}}$$

$$= 12\sqrt{3}$$

(C) $2\sqrt{18} + 9\sqrt{7} - \sqrt{63}$

$$= 6\sqrt{2} + 9\sqrt{7} - 3\sqrt{7}$$

$$= 6\sqrt{2} + 6\sqrt{7}$$

① $2\sqrt{18}$

$$= 2\sqrt{9\sqrt{2}}$$

$$= 2 \cdot 3\sqrt{2}$$

$$= 6\sqrt{2}$$

② $9\sqrt{7}$

③ $-\sqrt{63}$

$$= -\sqrt{9\sqrt{7}}$$

$$= -3\sqrt{7}$$

Example 3:

The voltage V required for a circuit is given by $V = \sqrt{PR}$ where P is the power in watts and R is the resistance in ohms. How many more volts are needed to light a 100-W bulb than a 75-W bulb if the resistance for both is 100 ohms? Express your answer in exact value.

$$\textcircled{1} V = \sqrt{PR}$$

$$V = \sqrt{100(100)}$$

$$V = \sqrt{100^2}$$

$$V = 100 \text{ volts}$$

100 - $50\sqrt{3}$ more volts are needed.

$$\textcircled{2} V = \sqrt{PR}$$

$$V = \sqrt{75(100)}$$

$$V = \sqrt{2^2 \cdot 3 \cdot 5^2 \cdot 2 \cdot 5}$$

$$V = 2 \cdot 5 \cdot 5 \sqrt{3}$$

$$V = 50\sqrt{3} \text{ volts}$$

$$\begin{array}{r} 7500 \\ / \\ 75 \quad 100 \\ / \quad / \\ 3 \cdot 25 \quad 10 \cdot 10 \\ / \quad / \quad / \\ 3 \quad 5 \quad 5 \quad 2.5 \quad 2.5 \\ (2 \cdot 2) \cdot 3 \cdot (5 \cdot 5) \cdot (5 \cdot 5) \end{array}$$

Example 4:

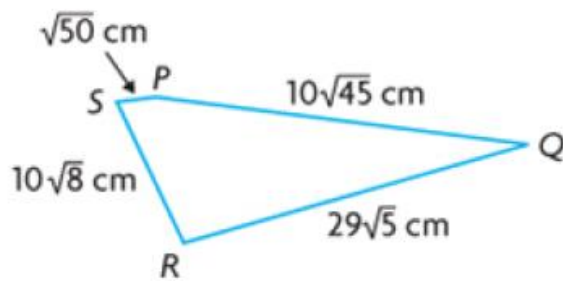
Determine the difference in length between each pair of sides.

(A) PS and SR

$$\begin{aligned} & 10\sqrt{8} - \sqrt{50} \\ &= 10\sqrt{4}\sqrt{2} - \sqrt{25}\sqrt{2} \\ &= 10 \cdot 2\sqrt{2} - 5\sqrt{2} \\ &= 20\sqrt{2} - 5\sqrt{2} \\ &= 15\sqrt{2} \end{aligned}$$

(B) RQ and PQ

$$\begin{aligned} & 10\sqrt{45} - 29\sqrt{5} \\ &= 10\sqrt{9}\sqrt{5} - 29\sqrt{5} \\ &= 10 \cdot 3\sqrt{5} - 29\sqrt{5} \\ &= 30\sqrt{5} - 29\sqrt{5} \\ &= \sqrt{5} \end{aligned}$$



Example 5:

Identify and correct the error in the following solution:

$$\begin{aligned} & 25\sqrt{5} + 13\sqrt{5} \\ & = 38\sqrt{10} \leftarrow \text{mistake} \\ & \rightarrow = 38\sqrt{5} \end{aligned}$$