Math 2201 **Date:**____________________

4.3A Multiplying Radicals

When multiplying radicals, multiply the coefficients and multiply the radicands. You can only multiply radicals if they have the same index. However, unlike adding and subtracting, you can multiply and divide radicals with a different radicand.

Multiplying Radicals

In general, $(c\sqrt{a})(d\sqrt{b}) = cd\sqrt{ab}$, where $a \geq 0$ and $b \geq 0$.

Example 1:

Multiply:

(A) $\sqrt{3} \cdot \sqrt{2}$

$= \sqrt{3 \cdot 2}$

$= \sqrt{6}$

(B) $\sqrt{5} \cdot \sqrt{7}$

$= \sqrt{5 \cdot 7}$

$= \sqrt{35}$

(C) $-\sqrt{6} \cdot \sqrt{3}$

$= -\sqrt{6 \cdot 3}$

$= -\sqrt{18}$

$= -\sqrt{9}\sqrt{2}$

$\rightarrow = -3\sqrt{2}$

(D) $(-\sqrt{2})(-\sqrt{7})$

$= (-1)(-1)\sqrt{2 \cdot 7}$

$= \sqrt{14}$

(E) $2\sqrt{3} \cdot 4\sqrt{5}$

$= (2 \cdot 4)\sqrt{3 \cdot 5}$

$= 8\sqrt{15}$

(F) $-5\sqrt{5} \cdot 6\sqrt{2}$

$= (-5 \cdot 6)\sqrt{5 \cdot 2}$

$= -30\sqrt{10}$

(G) $4\sqrt{3} \cdot (-5\sqrt{9})$

$= 4\sqrt{3}(-5 \cdot 3)$

$= 4\sqrt{3}(-15)$

$= 4 \cdot (-15)\sqrt{3}$

$= -60\sqrt{3}$

(H) $(-10\sqrt{6})(-2\sqrt{5})$

$= (-10)(-2)\sqrt{6 \cdot 5}$

$= 20\sqrt{30}$

You are less likely to make simplification errors if you simplify radicals before multiplying. For example:

$$\sqrt{80} \times \sqrt{12}$$

Multiply First:

$= \sqrt{80 \cdot 12}$

$= \sqrt{960}$

$= \sqrt{64}\sqrt{15}$

$= 8\sqrt{15}$

Simplify first:

$= \sqrt{16}\sqrt{5} \times \sqrt{4}\sqrt{3}$

$= 4\sqrt{5} \times 2\sqrt{3}$

$= (4 \cdot 2)\sqrt{5 \cdot 3}$

$= 8\sqrt{15}$

Which method do you prefer? Simplify first.

What happens when the radicand is the same in each expression. Consider the example:

*A radical multiplied by itself is the radicand.

$$7\sqrt{2} \times 3\sqrt{2}$$

$= (7 \cdot 3)\sqrt{2 \cdot 2}$

$= 21\sqrt{4}$

$= 21 \cdot 2$

$= 42$

Were you surprised that the answer was 42? Why is the product of two identical radical expressions an integer even though each factor is an irrational number?

Example 2:

Multiply and simplify:

(A) $\sqrt{3} \cdot \sqrt{12}$

$\sqrt{3} \cdot \sqrt{4}\sqrt{3}$

$= \sqrt{3} \cdot 2\sqrt{3}$

$= 2\sqrt{3 \cdot 3}$

$= 2\sqrt{9}$

$= 2 \cdot 3$

$= 6$

(B) $\sqrt{48} \cdot \sqrt{16}$

$= \sqrt{16}\sqrt{3} \cdot \sqrt{16}$

$= 16\sqrt{3}$

or $\sqrt{16}\sqrt{3}\sqrt{16}$

$= 4\sqrt{3} \cdot 4$

$= 16\sqrt{3}$

(C) $\sqrt{9} \cdot \sqrt{8}$

$= 3\sqrt{4}\sqrt{2}$

$= 3 \cdot 2\sqrt{2}$

$= 6\sqrt{2}$

(D) $2\sqrt{9} \cdot 3\sqrt{9}$

$= (2 \cdot 3)(9)$

$= 54$

The Distributive Property

Recall Math 1201 and multiplying polynomials. For example:

i. $3x(x+2)$

$= 3x^2 + 6x$

ii. $(3x+4)(x-7)$

① $3x(x-7) + 4(x-7)$

$= 3x^2 - 21x + 4x - 28$

$= 3x^2 - 17x - 28$

First
Outside
Inside
Last

② $(3x+4)(x-7)$

$= 3x^2 - 21x + 4x - 28$

$= 3x^2 - 17x - 28$

We can use the same techniques when multiplying radicals as well.

Example 3:
Express in simplest form

(A) $4\sqrt{2}(7\sqrt{5}+\sqrt{3})$

$= (4\sqrt{2})(7\sqrt{5}) + (4\sqrt{2})(\sqrt{3})$

$= (4\cdot 7)\sqrt{2\cdot 5} + 4\sqrt{2\cdot 3}$

$= 28\sqrt{10} + 4\sqrt{6}$

(B) $2\sqrt{3}(\sqrt{12}-\sqrt{7})$

$\sqrt{12}$
$= \sqrt{4}\sqrt{3}$
$= 2\sqrt{3}$

$= 2\sqrt{3}(2\sqrt{3}-\sqrt{7})$

$= (2\cdot 2)(3) - 2\sqrt{3\cdot 7}$

$= 4\cdot 3 - 2\sqrt{21}$

$= 12 - 2\sqrt{21}$

(D) $(8\sqrt{2}-5)(9\sqrt{5}+6\sqrt{10})$

$=(8\sqrt{2})(9\sqrt{5})+(8\sqrt{2})(6\sqrt{10})-5(9\sqrt{5})-5(6\sqrt{10})$

$=(8\cdot 9)\sqrt{2\cdot 5}+(8\cdot 6)\sqrt{2\cdot 10}-45\sqrt{5}-30\sqrt{10}$

$=72\sqrt{10}+48\sqrt{20}-45\sqrt{5}-30\sqrt{10}$

$=42\sqrt{10}-45\sqrt{5}+48\sqrt{20}$

$=42\sqrt{10}-45\sqrt{5}+48(2\sqrt{5})$

$=42\sqrt{10}-45\sqrt{5}+96\sqrt{5}$

$=42\sqrt{10}+51\sqrt{5}$

$\sqrt{20}$
$=\sqrt{4}\sqrt{5}$
$=2\sqrt{5}$

(E) $(\sqrt{2}+\sqrt{6})^2$

$=(\sqrt{2}+\sqrt{6})(\sqrt{2}+\sqrt{6})$

$=(\sqrt{2})(\sqrt{2})+(\sqrt{2}\sqrt{6})+(\sqrt{6})(\sqrt{2})+\sqrt{6}\sqrt{6}$

$=2+\sqrt{12}+\sqrt{12}+6$

$=2+2\sqrt{12}+6$

$=2+2(2\sqrt{3})+6$

$=8+4\sqrt{3}$

$\sqrt{12}$
$=\sqrt{4}\sqrt{3}$
$=2\sqrt{3}$

(F) $(3\sqrt{8}-4)(2+7\sqrt{3})$

$\sqrt{8}$
$=\sqrt{4}\sqrt{2}$
$=2\sqrt{2}$

$(6\sqrt{2}-4)(2+7\sqrt{3})$

$=12\sqrt{2}+42\sqrt{6}-8-28\sqrt{3}$

(G) $(\sqrt{20}+\sqrt{24})(3\sqrt{12}-5\sqrt{32})$

$=(2\sqrt{5}+2\sqrt{6})(6\sqrt{3}-20\sqrt{2})$

$=12\sqrt{15}-40\sqrt{10}+12\sqrt{18}-40\sqrt{12}$

$=12\sqrt{15}-40\sqrt{10}+36\sqrt{2}-80\sqrt{3}$

$\sqrt{20}$
$=\sqrt{4}\sqrt{5}$
$=2\sqrt{5}$

$\sqrt{24}$
$=\sqrt{4}\sqrt{6}$
$=2\sqrt{6}$

$\sqrt{12}$
$=\sqrt{4}\sqrt{3}$
$=2\sqrt{3}$

$\sqrt{32}$
$=\sqrt{16}\sqrt{2}$
$=4\sqrt{2}$

$\sqrt{18}$
$=\sqrt{9}\sqrt{2}$
$=3\sqrt{2}$

Example 4:

Express $(\sqrt{3}-\sqrt{2})^2$ in simplest form.

(A) 1
(B) $5-2\sqrt{6}$
(C) $1-2\sqrt{3}$
(D) 5

$(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})$
$= 3-\sqrt{6}-\sqrt{6}+2$
$= 5-2\sqrt{6}$

Explain your reasoning:

A radical multiplied by itself is the radicand.

Example 5:

Identify any errors and provide the correct solutions:

(i) $(2\sqrt{3})(4\sqrt{5}) = 8 + 2\sqrt{5} + \cancel{4\sqrt{3}} + \sqrt{15}$

$= 8\sqrt{15}$

Student applied distributive property.

(ii) $(2\sqrt{3}+5\sqrt{2})^2 = \cancel{4\sqrt{9} + 25\sqrt{4}}$

Did not use distributative property.

$(2\sqrt{3}+5\sqrt{2})(2\sqrt{3}+5\sqrt{2})$
$= 4(3)+10\sqrt{6}+10\sqrt{6}+25(2)$
$= 12 + 20\sqrt{6} + 50$
$= 62 + 20\sqrt{6}$

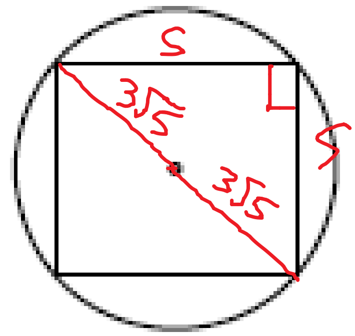
(iii) $2-5\sqrt{7}(3+4\sqrt{7}) = \cancel{(2-5\sqrt{7})}(3+4\sqrt{7})$

Put brackets around first "binomial".

$= 2-15\sqrt{7}-20(7)$
$= 2-15\sqrt{7}-140$
$= -138-15\sqrt{7}$

Example 6:

The radius of a circle is $3\sqrt{5}$ m. If a square is inscribed in a circle:



(A) Determine the exact length of the diagonal of the square.

① $d = 3\sqrt{5} + 3\sqrt{5} = 6\sqrt{5}$ m

or

② $d = 2(3\sqrt{5}) = 6\sqrt{5}$ m

(B) Determine the exact perimeter of the square.

$$a^2 + b^2 = c^2$$

$$s^2 + s^2 = (6\sqrt{5})^2$$

$$2s^2 = 36(5)$$

$$\frac{\cancel{2}s^2}{\cancel{2}} = \frac{180}{2}$$

$$\sqrt{s^2} = \sqrt{90}$$

$$s = \sqrt{9}\sqrt{10}$$

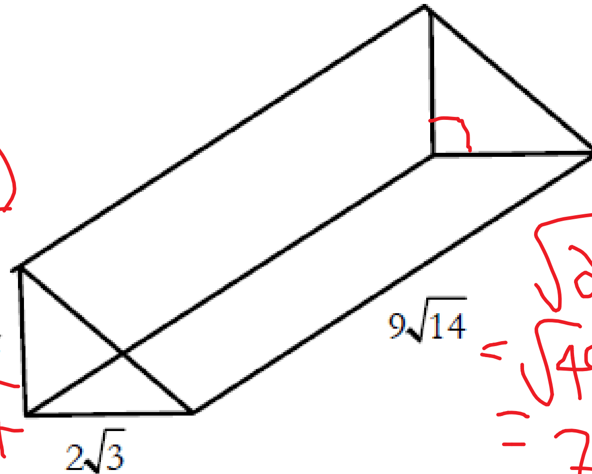
$$s = 3\sqrt{10} \text{ m}$$

Perimeter: $4(3\sqrt{10}\text{ m})$

$= 12\sqrt{10}$ m

Example 7:

Given the right triangular prism:



(A) Determine the volume.

$A_B = \frac{1}{2} b \cdot h = \frac{(\cancel{2}\sqrt{3})(6\sqrt{7})}{\cancel{2}}$

$= 6\sqrt{21}$

$V = (6\sqrt{21})(9\sqrt{14}) = 54\sqrt{294}$

$= 54(7\sqrt{6})$

$= 378\sqrt{6}$

$\sqrt{294} = \sqrt{49}\sqrt{6} = 7\sqrt{6}$

(B) Determine the surface area.

$6\sqrt{7}$, $2\sqrt{3}$, $c = 2\sqrt{66}$

$a^2 + b^2 = c^2$

$(6\sqrt{7})^2 + (2\sqrt{3})^2 = c^2$

$36(7) + 4(3) = c^2$

$\sqrt{264} = \sqrt{c^2}$

$c = \sqrt{4}\sqrt{66}$

$c = 2\sqrt{66}$

$6\sqrt{7}$, $9\sqrt{14}$

$A = l \cdot w = (6\sqrt{7})(9\sqrt{14})$

$= 54\sqrt{98}$

$= 54\sqrt{49}\sqrt{2}$

$= 378\sqrt{2}$

$2\sqrt{3}$, $9\sqrt{14}$

$A = l \cdot w = (2\sqrt{3})(9\sqrt{14})$

$= 18\sqrt{42}$

$2\sqrt{66}$, $9\sqrt{14}$

$A = l \cdot w = (2\sqrt{66})(9\sqrt{14})$

$= 18\sqrt{924}$

$= 18\sqrt{4}\sqrt{231}$

$= 36\sqrt{231}$

$SA = 36\sqrt{231} + 18\sqrt{42} + 378\sqrt{2} + 6\sqrt{21}$

Textbook Questions: page 198 - 199 #1, 4, 5, 6, 7, 8, 9, 10, 11, 20a