

4.3B Dividing Radicals

Dividing Radicals

When dividing radicals, divide the coefficients and then divide the radicands. You can only divide radicals that have the same index.

In general, $\frac{c\sqrt{a}}{d\sqrt{b}} = \frac{c}{d} \cdot \sqrt{\frac{a}{b}}$, a , b , c , and d are real numbers. $n \neq 0$ and $b \neq 0$. If the index is even, then $a \geq 0$ and $b > 0$.

Example 1:

$$\begin{aligned} \text{(A)} \quad & \sqrt{\frac{25}{4}} \\ &= \frac{\sqrt{25}}{\sqrt{4}} \\ &= \frac{5}{2} \end{aligned}$$

$$\begin{aligned} \text{(B)} \quad & \frac{\sqrt{12}}{\sqrt{6}} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{(C)} \quad & \frac{4\sqrt{6}}{2\sqrt{3}} \\ &= 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{(D)} \quad & \frac{6\sqrt{48}}{3\sqrt{6}} \\ &= 2\sqrt{8} \\ &= 2\sqrt{4}\sqrt{2} \\ &= 2 \cdot 2\sqrt{2} \\ &= 4\sqrt{2} \end{aligned}$$

Rationalizing the Denominator

It's considered bad form to leave a radical in the denominator of a fraction. There are two methods we use to remedy this, depending on the type of expression that is in the denominator.

A monomial denominator can simply be multiplied by 1 in the form of that denominator over itself:

Recall:

$$\frac{5}{\sqrt{3}} = 1 \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

$5 \cdot 1 = 5$, $3 \cdot 1 = 3$

Example 3:

(A) $\frac{2\sqrt{3} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$

$$= \frac{2\sqrt{15}}{5}$$

(B) $\frac{\sqrt{12}}{5}$

$$= \frac{\sqrt{12} \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{60}}{5} = \frac{\sqrt{4 \cdot 15}}{5} = \frac{2\sqrt{15}}{5}$$

(C) $\frac{2\sqrt{3} \cdot \sqrt{5}}{7\sqrt{5} \cdot \sqrt{5}}$

$$= \frac{2\sqrt{15}}{7 \cdot 5} = \frac{2\sqrt{15}}{35}$$

(D) $\frac{6\sqrt{48}}{3\sqrt{6}}$

$$= 2\sqrt{8} = 2\sqrt{4 \cdot 2} = 4\sqrt{2}$$

Expressions With Multiple Operations

Some expressions may have addition or subtraction in the numerator or denominator. When possible work these operations before you divide.

Example 3:

Simplify:

$$(A) \quad \frac{3\sqrt{6}+5\sqrt{6}}{4\sqrt{2}}$$

$$= \frac{8\sqrt{6}}{4\sqrt{2}}$$

$$= 2\sqrt{3}$$

$$(B) \quad \frac{4\sqrt{12}-10\sqrt{6}}{2\sqrt{3}}$$

$$= \frac{4\sqrt{12}}{2\sqrt{3}} - \frac{10\sqrt{6}}{2\sqrt{3}}$$

$$= 2\sqrt{4} - 5\sqrt{2}$$

$$= 2 \cdot 2 - 5\sqrt{2}$$

$$= 4 - 5\sqrt{2}$$