Math 2201 **Date:**_____________________

4.4A Restrictions on Radicals Containing Variables

Radical expressions contain variables under the root sign. For example: $\sqrt{x-3}$

As long as we are working within the real number system, we **cannot take the square root of a negative number**. Thus, we must be careful to ensure that we do not choose values for a variable that would result in taking the square root of a negative number. To avoid this situation, we state **restrictions** on the variable.

Restrictions on Algebraic Expressions Involving Radicals

Restrictions are the values of the variable in an expression that ensure the expression is defined. Consider the example:

$$\sqrt{x}$$

Give three examples of numbers that we could use for x in the expression.

$x = 9, 16, 7, 0$

Give three examples of numbers that we could **not** use for x in the expression.

$x \neq -8, -9, -2, -1000283796$

State the restrictions on the variable in the expression.

$x \geq 0$

Radical Expressions that Contain a Single Term Radicand (Monomials)

Example 1: $\in$: belongs to , R: Real numbers

State any restrictions on the variable.

(A) $\sqrt{x^2}$, $x \in R$

$-$: $x=-2$, $\sqrt{(-2)^2} = \sqrt{4} = 2$

0: $x=0$, $\sqrt{0^2} = \sqrt{0} = 0$

$+$: $x=2$, $\sqrt{2^2} = \sqrt{4} = 2$

(B) $\sqrt{x^3}$, $x \geq 0$

$-$: $x=-2$, $\sqrt{(-2)^3} = \sqrt{-8}$ Not possible

0: $x=0$, $\sqrt{0^3} = \sqrt{0} = 0$

$+$: $x=2$, $\sqrt{2^3} = \sqrt{8} = 2\sqrt{2}$

(C) $\sqrt{x^4}$, $x \in R$

$-$: $x=-2$, $\sqrt{(-2)^4} = \sqrt{16} = 4$

0: $x=0$, $\sqrt{0^4} = \sqrt{0} = 0$

$+$: $x=2$, $\sqrt{2^4} = \sqrt{16} = 4$

(D) $\sqrt{15x}$, $x \geq 0$

$-$: $x=-2$, $\sqrt{15(-2)} = \sqrt{-30}$ Not possible

0: $x=0$, $\sqrt{15(0)} = \sqrt{0} = 0$

$+$: $x=2$, $\sqrt{15(2)} = \sqrt{30} = 5.47\ldots$

Summary: whenever you have a single term (monomial) that contains a variable under a root sign:

- if the exponent on the variable is even, there will be **no** restrictions on the variable. $x \in R$
- if the exponent on the variable is odd, there will be a restriction on the variable, usually, $x \geq 0$ where $x \in R$.

Radical Expressions that Contain a Multiple Term Radicand (+ or − signs)

The entire expression under the root sign **cannot** be equal to a negative number. That is, it must equal zero or any positive value. To determine the restrictions on the variable:

- Write an inequality in which the expression ≥ 0.
- Solve the inequality for the variable, just as you would with an equation.

Example 2:

State restrictions on the variable.

(A) $\sqrt{x-3}$

Think: $x-3=0$

$x=3$

$x-3\geq 0$

$x\geq 3$

(B) $\sqrt{4+x}$

$4+x\geq 0$

$x\geq -4$

(C) $\sqrt{2x-8}$

$2x-8\geq 0$

$2x\geq 8$

$\frac{2x}{2}\geq\frac{8}{2}$

$x\geq 4$

(D) $\sqrt{5-4x}$

* When you divide by negatives, you must reverse the inequality.

$5-4x\geq 0$

$-4x\geq -5$

$\frac{-4x}{-4}\geq\frac{-5}{-4}$

$x\leq 5/4$

Radical Expressions that Contain Fractions

Recall that the denominator of a fraction **cannot** equal 0.

$\frac{0}{3} = 0$

$\frac{3}{0} =$ undefined

For the expression $\frac{1}{x}$, $x \neq 0$ where $x \in R$

If we see radicals in the denominator of a fraction, we must keep in mind that in addition to any restrictions that might apply to the radical itself, we must also ensure that the radicand cannot equal 0. This often results in dropping the "equal to" part in the inequality sign.

Example 3:
State any restriction for the following.

(A) $\frac{1}{\sqrt{x}}$, $X > 0$

Recall: $\sqrt{X}$, $X \geq 0$

Consider: $\frac{1}{\sqrt{X}}$

$\frac{0}{1}$

$\frac{1}{\sqrt{0}} = \frac{1}{0}$ undefined

(B) $\frac{1}{\sqrt{x^2}}$, $X > 0$, $X < 0$

$X \neq 0$

Recall: $\sqrt{X^2}$, $X \in R$

Consider: $\frac{1}{\sqrt{X^2}}$

$\frac{1}{\sqrt{0^2}}$

$= \frac{1}{\sqrt{0}} = \frac{1}{0}$ undefined

(C) $\frac{1}{\sqrt{2x-3}}$, $X > \frac{3}{2}$

Recall: $\sqrt{2X-3}$

$2X - 3 \geq 0$

$2X \geq 3$

$\frac{2X}{2} \geq \frac{3}{2}$

$X \geq 3/2$

Consider:

$\frac{1}{\sqrt{2X-3}}$

$\frac{1}{\sqrt{\cancel{2}\left(\frac{3}{\cancel{2}}\right)-3}}$

$\frac{1}{\sqrt{3-3}} = \frac{1}{\sqrt{0}} = \frac{1}{0}$ undefined

Textbook Questions: page 211 - 213 #1, 2, 3 restrictions only, 11