Math 2201 **Date:**_____

4.4B Radicals Containing Variables

Some radicals can contain variables. In Mathematics 1201, students expressed a radical as a power with rational exponents. The focus here will be exclusive to the square root of a radical with variable radicands, for example, $\sqrt{x^n} = x^{\frac{n}{2}}$.

Simplifying Algebraic Expressions Involving Radicals

$\sqrt{x}$ cannot be simplified, since x is the smallest possible variable radicand. However, other powers of x under a root sign can be simplified.

Example 1:
Simplify the following.

(A) $\sqrt{x^2}$

$= x$

(B) $\sqrt{x^3}$

$= \sqrt{(x \cdot x)x}$

$= \sqrt{x^2 \cdot x}$

$= \sqrt{x^2}\sqrt{x}$

$= x\sqrt{x}$

(C) $\sqrt{x^4}$

$= \sqrt{(x \cdot x)(x \cdot x)}$

$= \sqrt{x^2 \cdot x^2}$

$= \sqrt{x^2}\sqrt{x^2}$

$= x \cdot x$

$= x^2$

Example 2:
Complete the table:

	Greatest Perfect Square	Prime Factorization
$\sqrt{125}$	$\sqrt{25 \times 5} = 5\sqrt{5}$	$\sqrt{5 \times 5 \times 5} = 5\sqrt{5}$
$\sqrt{x^3}$	$\sqrt{x^2 \times x} = x\sqrt{x}$	$\sqrt{(x \times x) \times x} = x\sqrt{x}$
$\sqrt{x^4}$	$\sqrt{x^2 \times x^2} = x \times x = x^2$	$\sqrt{x \times x \times x \times x} = x \times x = x^2$
$\sqrt{x^5}$	$\sqrt{x^2 \cdot x^2 \cdot x} = x \cdot x\sqrt{x} = x^2\sqrt{x}$	$\sqrt{x \cdot x \cdot x \cdot x \cdot x} = x \cdot x\sqrt{x} = x^2\sqrt{x}$
$\sqrt{x^6}$	$\sqrt{x^2 \cdot x^2 \cdot x^2} = x \cdot x \cdot x = x^3$	$\sqrt{x \cdot x \cdot x \cdot x \cdot x \cdot x} = x \cdot x \cdot x = x^3$

When the radical consists of a radicand that contains a variable and a numerical coefficient, simply split the radical into two separate radicals. For example:

$\sqrt{18x^2}$

$= \sqrt{18}\sqrt{x^2}$

$= \sqrt{9}\sqrt{2}$

$= 3\sqrt{2}x$

$= 3x\sqrt{2}$

Example 3:
Simplify the following and state any restrictions:

(A) $\sqrt{4x^2}$

$= \sqrt{4}\sqrt{x^2}$

$= 2x, x \in R$

(B) $4\sqrt{18x^3}$

$= 4\sqrt{18}\sqrt{x^3}$

$= 4\sqrt{9}\sqrt{2}\sqrt{x \cdot x \cdot x}$

$= 4 \cdot 3\sqrt{2}x\sqrt{x}$

$= 12x\sqrt{2x}, x \geq 0$

(C) $\sqrt{54x^5}$

$= \sqrt{54}\sqrt{x^5}$

$= \sqrt{9}\sqrt{6}\sqrt{x \cdot x \cdot x \cdot x \cdot x}$

$= 3\sqrt{6}x \cdot x\sqrt{x}$

$= 3x^2\sqrt{6x}, x \geq 0$

(D) $-7y^2\sqrt{8y^5}$

$= -7y^2\sqrt{8}\sqrt{y^5}$

$= -7y^2\sqrt{4}\sqrt{2}\sqrt{y \cdot y \cdot y \cdot y \cdot y}$

$= -7y^2 2\sqrt{2}y \cdot y\sqrt{y}$

$= -14y^4\sqrt{2y}, y \geq 0$

(E) $4\sqrt{8x^3y^4}$

$= 4\sqrt{8}\sqrt{x^3}\sqrt{y^4}$

$= 4\sqrt{4}\sqrt{2}\sqrt{x \cdot x}x\sqrt{y \cdot y \cdot y \cdot y}$

$= 4 \cdot 2\sqrt{2} \cdot x\sqrt{x} \cdot y \cdot y$

$= 8xy^2\sqrt{2x}, x \geq 0, y \in R$

(F) $\sqrt{y-5}$

simplest form

$y - 5 \geq 0$

$y \geq 5$

Textbook Questions: page 211 #2