Math 2201 **Date:**____________________

4.4C Adding, Subtracting, Multiplying and Dividing Radical Expressions

Adding and Subtracting Radicals with Variables
As with radicals that contain numerical radicands only, to add and subtract, radicals must have the same index and radicand.

Compare adding radicals with variables and without:

$6\sqrt{3} + 2\sqrt{3}$

$= 8\sqrt{3}$

$6\sqrt{x} + 2\sqrt{x}$

$= 8\sqrt{x}$

Example 1:
Solve each expression and state any restrictions:

(A) $\sqrt{x} + 5\sqrt{x}$

$= 6\sqrt{x}, x \geq 0$

(B) $3\sqrt{z} - 5\sqrt{z}$

$= -2\sqrt{z}, z \geq 0$

As with strictly numerical radicands, sometimes you might have to simplify the radicals before you can add or subtract.

Example 2:
Simplify and state any restrictions:

$$2\sqrt{4x^2} - \sqrt{8x^4}$$

$= 2\sqrt{4}\sqrt{x^2} - \sqrt{8}\sqrt{x^4}$

$= 2 \cdot 2\sqrt{\cancel{x \cdot x}} - \sqrt{4}\sqrt{2}\sqrt{\cancel{x \cdot x} \cdot \cancel{x \cdot x}}$

$= 4x - 2\sqrt{2} \cdot x \cdot x$

$= 4x - 2x^2\sqrt{2},\ x \in R$

Example 3:
Identify and correct the error in the following solution:

$\sqrt{18x^3} + 2\sqrt{8x^3}$

$= 3\sqrt{2x^3} + 4\sqrt{2x^3}$

$= 7\sqrt{4x^6}$

$= 14x^3$

$= \sqrt{18}\sqrt{x^3} + 2\sqrt{8}\sqrt{x^3}$

$= \sqrt{9}\sqrt{2}\sqrt{\cancel{x \cdot x} \cdot x} + 2\sqrt{4}\sqrt{2}\sqrt{\cancel{x \cdot x} \cdot x}$

$= 3\sqrt{2}x\sqrt{x} + 2 \cdot 2\sqrt{2}x\sqrt{x}$

$= 3x\sqrt{2x} + 4x\sqrt{2x}$

$= 7x\sqrt{2x},\ x \geq 0$

Multiply the terms outside the roots, and then multiply the terms under the root signs. Let's compare multiplying radicals with variables and without.

$2\sqrt{3} \times 4\sqrt{6}$

$= (2 \cdot 4)\sqrt{3 \cdot 6}$

$= \boxed{8\sqrt{18}}$ ← compare →

$= 8\sqrt{9}\sqrt{2}$

$= 8 \cdot 3\sqrt{2}$

$= 24\sqrt{2}$

$2\sqrt{x} \times 4\sqrt{x^2}$

$= (2 \cdot 4)\sqrt{x \cdot x^2}$

$= \boxed{8\sqrt{x^3}}$

$= 8\cancel{\sqrt{\ }}\cancel{x \cdot x} \cdot x$

$= 8x\sqrt{x}$

Example 4:
Simplify and state any restrictions.

(A) $(-3\sqrt{x})(6\sqrt{x^3})$

$= (-3 \cdot 6)\sqrt{x \cdot x^3}$

$= -18\sqrt{x^4}$

$= -18\cancel{\sqrt{\ }}\cancel{x \cdot x} \cdot \cancel{x \cdot x}$

$= -18 \cdot x \cdot x$

$= -18x^2, x \geq 0$

(B) $(5\sqrt{6x^2})(-2x\sqrt{2x})$

$= [5 \cdot (-2x)]\sqrt{(6x^2)(2x)}$

$= -10x\sqrt{12x^3}$

$= -10x\sqrt{12}\sqrt{x^3}$

$= -10x\sqrt{4}\sqrt{3}\cancel{\sqrt{\ }}\cancel{x \cdot x} \cdot x$

$= -10x\,2\sqrt{3}\,x\sqrt{x}$

$= -20x^2\sqrt{3x}, x \geq 0$

Example 5:
Simplify and state any restrictions.

(A) $-3\sqrt{x}(2\sqrt{2}-3x)$

$= -6\sqrt{2x} + 9x\sqrt{x}, \; x \geq 0$

FOIL

(B) $(3\sqrt{x}+2)(3-5\sqrt{x})$

$= 9x - 15(x) + 6 - 10\sqrt{x}$

$= -6x + 6 - 10\sqrt{x}, \; x \geq 0$

(C) $(2\sqrt{x}+3)(5-3\sqrt{x})$

$= 10\sqrt{x} - 6(x) + 15 - 9\sqrt{x}$

$= \sqrt{x} - 6x + 15, \; x \geq 0$

Dividing

Try to divide the terms outside the roots, and the terms under the root signs. If you are left with a radical in the denominator, you must rationalize it. Let's compare dividing radicals with variables and without.

$$\sqrt{\frac{1}{8}}$$

$$= \frac{\sqrt{1}}{\sqrt{8}}$$

$$= \frac{1}{\sqrt{4}\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2(2)}$$

$$= \frac{\sqrt{2}}{4}$$

$$\sqrt{\frac{1}{x^3}}, x > 0$$

$$= \frac{\sqrt{1}}{\sqrt{x^3}}$$

$$= \frac{1}{\sqrt{\cancel{x} \cdot \cancel{x} \cdot x}}$$

$$= \frac{1}{x\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$

$$= \frac{\sqrt{x}}{x(x)}$$

$$= \frac{\sqrt{x}}{x^2}$$

Example 6:

State any restrictions on the variable, then simplify each expression.

(A) $\frac{5x}{\sqrt{5x}}$

$$= \frac{5x}{\sqrt{5x}} \cdot \frac{\sqrt{5x}}{\sqrt{5x}}$$

$$= \frac{5\cancel{x}\sqrt{5x}}{5\cancel{x}}$$

$$= \sqrt{5x}, x > 0$$

(B) $\frac{\sqrt{x^5}}{\sqrt{x}}$

$$= \sqrt{\frac{x^{\cancel{5}\,4}}{\cancel{x}}}$$

$$= \sqrt{x^4}$$

$$= \sqrt{\cancel{x \cdot x} \cdot \cancel{x \cdot x}}$$

$$= x \cdot x$$

$$= x^2, x > 0$$

(C) $\frac{\sqrt{27x^3}}{\sqrt{3x}}$

$= \sqrt{\frac{27x^{\cancel{3}\,2}}{3\cancel{x}}}$

$= \sqrt{9x^2}$

$= \sqrt{9}\,\cancel{\sqrt{x^2}}$

$= 3x, x > 0$

(D) $\frac{15\sqrt{x^3}}{-3\sqrt{x^2}}$

$= -5\sqrt{\frac{x^{\cancel{3}}}{\cancel{x^2}}}$

$= -5\sqrt{x}, x > 0$

(E) $\frac{8\sqrt{32x^4}}{2x^2}$

$= \frac{4\sqrt{16}\sqrt{2}\,\cancel{\sqrt{x \cdot x \cdot x \cdot x}}}{x^2}$

$= \frac{4 \cdot 4\sqrt{2} \cdot \cancel{x \cdot x}}{\cancel{x^2}}$

$= 16\sqrt{2}, x \neq 0$

(F) $\frac{\sqrt{27x}+\sqrt{3x}}{\sqrt{3x}}$

$= \frac{\sqrt{9}\sqrt{3x} + \sqrt{3x}}{\sqrt{3x}}$

$= \frac{3\sqrt{3x} + \sqrt{3x}}{\sqrt{3x}}$

$= \frac{4\cancel{\sqrt{3x}}}{\cancel{\sqrt{3x}}}$

$= 4, x > 0$

(G) $\frac{6\sqrt{5}-2\sqrt{24x^3}}{2\sqrt{x}}$

$$= \frac{6\sqrt{5}-2\sqrt{4}\sqrt{6}\sqrt{x\cdot x\cdot x}}{2\sqrt{x}}$$

$$= \frac{6\sqrt{5}-2(2)\sqrt{6}\,x\sqrt{x}}{2\sqrt{x}}$$

$$= \frac{6\sqrt{5}-4x\sqrt{6x}}{2\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}$$

$$= \frac{\cancel{6}^{3}\sqrt{5x}-\cancel{4}^{2}x\sqrt{6}x}{\cancel{2}x}$$

$$= \frac{3\sqrt{5x}-2x^2\sqrt{6}}{x},\ x>0$$

Example 7:
Einstein's famous formula $E = mc^2$ relates energy E, mass m and the speed of light c. Express c in terms of E and m and rationalize the denominator.

$$E = mc^2$$

$$\frac{mc^2}{m} = \frac{E}{m}$$

$$c^2 = \frac{E}{m}$$

$$\sqrt{c^2} = \sqrt{\frac{E}{M}}$$

$$c = \frac{\sqrt{E}}{\sqrt{m}}\cdot\frac{\sqrt{m}}{\sqrt{m}}$$

$$c = \frac{\sqrt{EM}}{M}$$

Textbook Questions: page 211 - 213 #3, 4, 5, 6, 8, 9, 10, 12, 13, 15, 17