Math 2201 **Date:**____________________

4.5 Radical Equations

Exploring and Solving Radical Equations

Radical Equation: an equation which contains a radical. Sometimes, we may be asked to solve for a variable that is found under a radical sign.

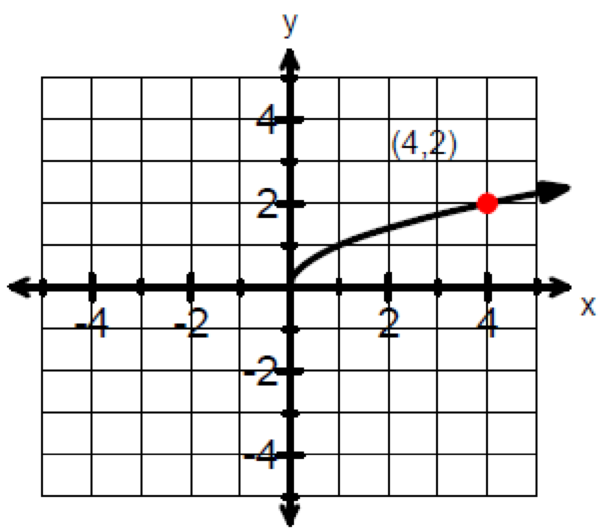
Inverse Operations

- Squaring a number and taking a square root are inverse operations. $\sqrt{4^{\cancel{2}}} = 4$
- Cubing a number and taking a cube root are inverse operations. $\sqrt[\cancel{3}]{5^{\cancel{3}}} = 5$

Restrictions

As you work with radical equations containing square roots, it would be beneficial to first compare the radical equation to its graph to develop an understanding of restrictions for the variable and the points that satisfy the equation. For example, let's look at the graph of $y = \sqrt{x}$:

x	y
1	1
2	1.4
3	1.7
4	2



Focusing on the point (4, 2) how would you algebraically solve the equation given only the y-coordinate 2.

$$y = \sqrt{x}$$

$$2 = \sqrt{x}$$

$$(2)^2 = (\cancel{\sqrt{}}x)^{\cancel{2}}$$

$$4 = x$$

$$x = 4$$

Now consider $\sqrt{2x-1} = -3$.

$(\sqrt{2x-1})^2 \neq (-3)^2$

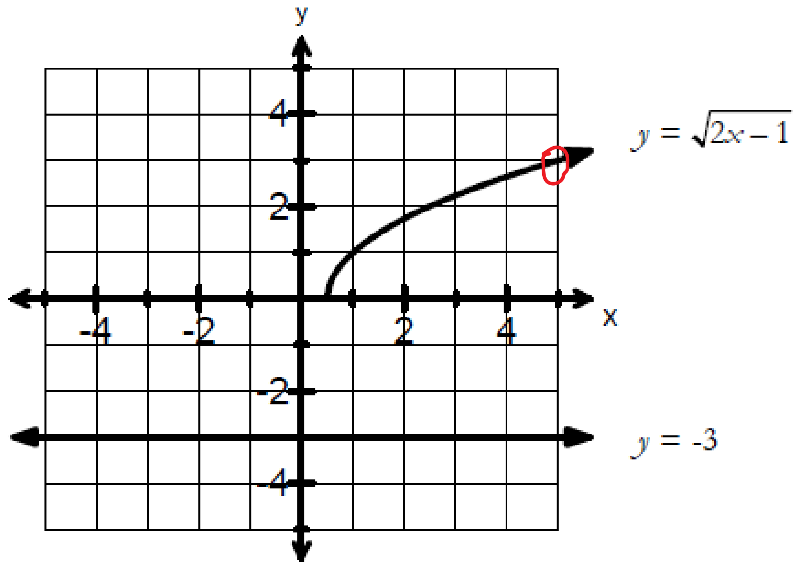
$2x-1 = 9$

$2x = 9+1$

$2x = 10$

$\frac{2x}{2} = \frac{10}{2}$

$x \neq 5$



The left-hand side of the equation calls for a positive square root, but the right-hand side of the equation is negative. Intuitively, there can be no solution. Therefore:

$$\sqrt{2x-1} \neq -3$$ extraneous root.

Solving Radical Equations for a Variable that is Under a Radical Sign

Steps:

- Get the radical by itself on one side of the equation.
- Get rid of the radical by carrying out the inverse operation on BOTH sides of the equation.
- Solve for the variable.

Extraneous Roots

Extraneous roots occur because squaring both sides and solving the quadratic equation may result in roots that do not satisfy the original equation.

Example 1:
Solve each equation for x and verify the answers.

(A) $\sqrt{3x} = 6$

$(\sqrt{3x})^2 = (6)^2$

$3x = 36$

$\frac{3x}{3} = \frac{36}{3}$

$x = 12,\ x \geq 0$

Check:

$\sqrt{3(12)} = 6$

$\sqrt{36} = 6$

$6 = 6$ ✓

(B) $\sqrt{4x} = 8$

$(\sqrt{4x})^2 = 8^2$

$4x = 64$

$\frac{4x}{4} = \frac{64}{4}$

$x = 16,\ x \geq 0$

Check:

$\sqrt{4(16)} = 8$

$\sqrt{64} = 8$

$8 = 8$ ✓

(C) $1+\sqrt{2x} = 3$

$\sqrt{2x} = 3-1$

$\sqrt{2x} = 2$

$(\sqrt{2x})^2 = 2^2$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2,\ x \geq 0$

Check:

$1+\sqrt{2(2)} = 3$

$1+\sqrt{4} = 3$

$1+2 = 3$

$3 = 3$ ✓

(D) $\sqrt{x+4} = 5$

$(\sqrt{x+4})^2 = 5^2$

$x+4 = 25$

$x = 25-4$

$x = 21,\ x \geq -4$

$x+4 \geq 0$

$x \geq -4$

Check:

$\sqrt{21+4} = 5$

$\sqrt{25} = 5$

$5 = 5$ ✓

(E) $\sqrt{2x-3} = -2$

$(\sqrt{2x-3})^2 = (-2)^2$

$2x-3 = 4$

$2x = 4+3$

$2x = 7$

$\frac{2x}{2} = \frac{7}{2}$

$\cancel{x = 7/2},\ x \geq \frac{3}{2}$

$2x-3 \geq 0$

$2x \geq 3$

$x \geq \frac{3}{2}$

Check:

$\sqrt{\cancel{2}\left(\frac{7}{\cancel{2}}\right)-3} = -2$

$\sqrt{7-3} = -2$

$\sqrt{4} = -2$

$2 \neq -2$

no solution.

(F) $\sqrt[3]{x-2} = -3$

$(\cancel{\sqrt[3]{x-2}})^{\cancel{3}} = (-3)^3$

$x-2 = -27$

$x = -27+2$

$x = -25,\ x \in \mathbb{R}$

Check:

$\sqrt[3]{-25-2} = -3$

$\sqrt[3]{-27} = -3$

$-3 = -3$ ✓

Working With Inequalities

- The same number can be added to both sides of an inequality.
- The same number can be subtracted from both sides of an inequality.
- Both sides of an inequality can be multiplied or divided by the same positive number.
- If an inequality is multiplied or divided by a negative number, then: $>$ becomes $<$

$\geq$ becomes $\leq$

$<$ becomes $>$

$\leq$ becomes $\geq$

Example 2:
Solve each inequality for x:

(A) $-7x \geq 98$

$$\frac{-7x}{-7} \geq \frac{98}{-7}$$

$$x \leq -14$$

(B) $-12x + 4 \leq 0$

$$\frac{-12x}{-12} \leq \frac{-4}{-12}$$

$$x \geq \frac{1}{3}$$

Example 3:

What is the value of x in the equation $\sqrt{-3x+6} = 6$?

(A) -10
(B) -2
(C) 0
(D) 10

Explain your reasoning:

$$\left(\sqrt{-3x+6}\right)^2 = 6^2$$

$$-3x + 6 = 36$$

$$-3x = 36 - 6$$

$$\frac{-3x}{-3} = \frac{30}{-3}$$

$$x = -10$$

Rather than squaring both sides of a radical equation, students sometimes mistakenly square the individual terms. When solving $3+\sqrt{2x+1}=7$, they may not isolate the radical. Squaring each term results in the incorrect equation, for example:

$$3+\sqrt{2x+1}=7$$

$$(3)^2+\left(\sqrt{2x+1}\right)^2=(7)^2$$

~~$9+2x+1=49$~~ wrong!

$3+\sqrt{2x+1}=7$

$\sqrt{2x+1}=7-3$

$\sqrt{2x+1}=4$

$\left(\sqrt{2x+1}\right)^2=4^2$

$2x+1=16$

$2x=16-1$

$\dfrac{2x}{2}=\dfrac{15}{2}$

$x=15/2$, $x\geq\dfrac{1}{2}$

Check:

$3+\sqrt{\cancel{2}\left(\frac{15}{\cancel{2}}\right)+1}=7$

$3+\sqrt{15+1}=7$

$3+\sqrt{16}=7$

$3+4=7$

$7=7$ ✓

Example 4:
State any restrictions and solve for x.

(A) $3+\sqrt{2x+1}=7$

(B) $\sqrt{3n-5}-2=3$

$\sqrt{3n-5}=3+2$

$\sqrt{3n-5}=5$

$\left(\sqrt{3n-5}\right)^2=5^2$

$3n-5=25$

$3n=25+5$

$\dfrac{3n}{3}=\dfrac{30}{3}$

$n=10$, $n\geq 5/3$

$3n-5\geq 0$

$\dfrac{3n}{3}\geq\dfrac{5}{3}$

$n\geq 5/3$

$\sqrt{3(10)-5}-2=3$

$\sqrt{25}-2=3$

$5-2=3$

$3=3$ ✓

$2x - 1 \geq 0 \rightarrow x \geq \frac{1}{2}$

$2x \geq 1$

Check:

(C) $5 + \sqrt{2x-1} = 12$

$\sqrt{2x-1} = 12 - 5$

$\sqrt{2x-1} = 7$

$(\sqrt{2x-1})^2 = (7)^2$

$\rightarrow 2x - 1 = 49$

$2x = 49 + 1$

$\frac{2x}{2} = \frac{50}{2}$

$x = 25, x \geq \frac{1}{2}$

$5 + \sqrt{2(25)-1} = 12$

$5 + \sqrt{49} = 12$

$5 + 7 = 12$

$12 = 12$ ✓

Example 5:
Is the solution to the following radical equation correct? Justify your answer.

$3 + 2\sqrt{n+4} = 5$

$5\sqrt{n+4} = 5$

$\sqrt{n+4} = 1$

$\left(\sqrt{n+4}\right)^2 = 1^2$

$n + 4 = 1$

$n = -3$

$2\sqrt{n+4} = 5 - 3$

$\frac{2\sqrt{n+4}}{2} = \frac{2}{2}$

$(\sqrt{n+4}) = (1)^2$

$n + 4 \geq 0 \quad n + 4 = 1$

$n \geq -4$

$\rightarrow n = 1 - 4$

$n = -3,$

$n \geq -4$

Check:

$3 + 2\sqrt{(-3)+4} = 5$

$3 + 2\sqrt{1} = 5$

$3 + 2 = 5$

$5 = 5$ ✓

The Area Model
The area model can also be used to solve radical equations. Recall that in Grade 8, you viewed the area of the square as the perfect square number, and the side length of the square as the square root. Recall that if a square has an area of 4, then its side has a length of 2. Similarly, if a square has an area of 3, then its side has a length of $\sqrt{3}$.

Consider the following example: Solve $\sqrt{x-7} = 3$.

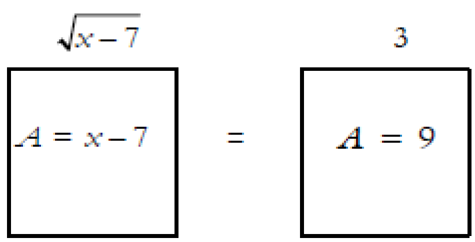
$A = s^2$

$A = (\cancel{\sqrt{}}x - 7)^2$

$A = x - 7$

$x - 7 \geq 0$

$x \geq 7$



$A = s^2$

$A = (3)^2$

$A = 9$

$x - 7 = 9$

$x = 9 + 7$

$x = 16, x \geq 7$

Check: $\sqrt{16-7} = 3$

$\sqrt{9} = 3$

$3 = 3$ ✓

Example 6:

The period T (in seconds) is the time it takes a pendulum to make one complete swing back and forth. This is modelled by $T = 2\pi\sqrt{\frac{L}{32}}$, where L is the length of the pendulum in feet. Determine the period of the pendulum if its length is 2 ft.

$$T = 2\pi\sqrt{\frac{2}{32}}$$

$$T = 2\pi\sqrt{\frac{1}{16}}$$

$$T = 2\pi\frac{\sqrt{1}}{\sqrt{16}}$$

$$T = 2\pi\frac{1}{4}$$

$$\rightarrow T = \frac{2\pi}{4}$$

$T = \frac{\pi}{2}$ feet.

($T = 1.57$ ft.)

Example 7:

The radius of a cylinder can be found using the equation $r = \sqrt{\frac{V}{\pi h}}$ where r is the radius, V is the volume, and h is the height. A cylindrical tank can hold 105.62 m^3 of water. If the height of the tank is 2 m, what is the radius of its base?

$$r = \sqrt{\frac{105.62}{\pi(2)}}$$

$$r = \sqrt{16.8099}$$

$\rightarrow r = 4.10$ m

Example 8:

The surface area, S, of a sphere with radius r can be found using the equation $S = 4\pi r^2$.

(A) Using the given equation, how could you find the radius of a sphere given its surface area? Write the equation.

$$\frac{S}{4\pi} = \frac{\cancel{4\pi} r^2}{\cancel{4\pi}}$$

$$\sqrt{\frac{S}{4\pi}} = \sqrt{r^2}$$

$$\rightarrow r = \sqrt{\frac{S}{4\pi}}$$

(B) The surface area of a ball is 426.2 cm^2. What is its radius?

$$r = \sqrt{\frac{426.2}{4\pi}} = \sqrt{33.9} = 5.8 \text{ cm}$$

Textbook Questions: page 222 - 223 #1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 15