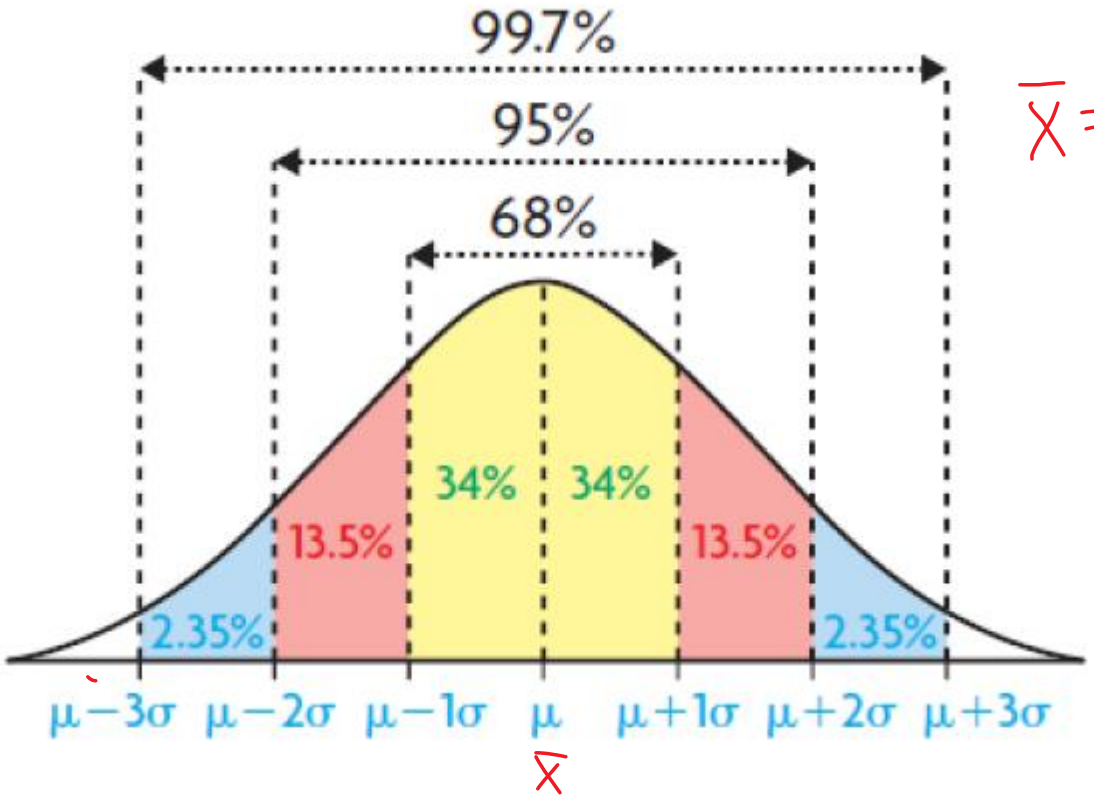
Math 2201 **Date:**_____

5.4 Normal Distribution

Normal Distribution and Normal Curves
In many situations, if a controlled experiment is repeated and the data is collected to create a histogram, the histogram becomes more and more bell-shaped as the number of data in the distribution increases. Such repeated measurements will produce bell-shaped distributions which exhibit the characteristics of a **normal curve.**

Normal Curve: a symettrical curve that represents the normal distribution; also called a **bell curve.**



$\bar{x} = \mu$

Normal Distribution: Data that, when graphed as a histogram or a frequency polygon, results in a unimodal symmetric distribution about the mean.

uni-model

The general shape of the histogram will begin to approach a normal curve as more trials are added. The characteristics are:

- Normal distributions are symmetrical with a single peak at the mean of the data.
- The curve is bell shaped with the graph falling off evenly on either side of the mean.

Graphing a normal distribution by hand can be tedious and time consuming. Using technology, such as graphing calculators and Desmos Graphing, students can investigate the properties of a normal distribution and work with data that is specifically one, two and three standard deviations from the mean.

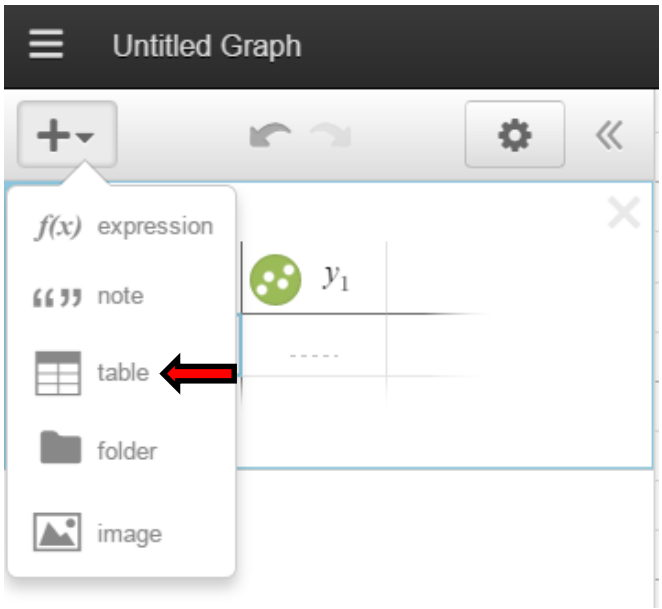
Example 1:

Number of hours playing video games in a week (15year old students)									
21.1	21.8	17.7	23.4	14.9	20.8	19.1	18.6	25.6	23.3
18.5	19.1	18.8	18.3	20.2	15.5	21.4	16.7	14.7	20.5
17.9	20.9	24.5	22.2	17.1	24.5	21.1	18.1	21.0	16.2
23.7	18.9	21.3	16.0	20.2	27.3	17.6	20.7	19.7	14.9

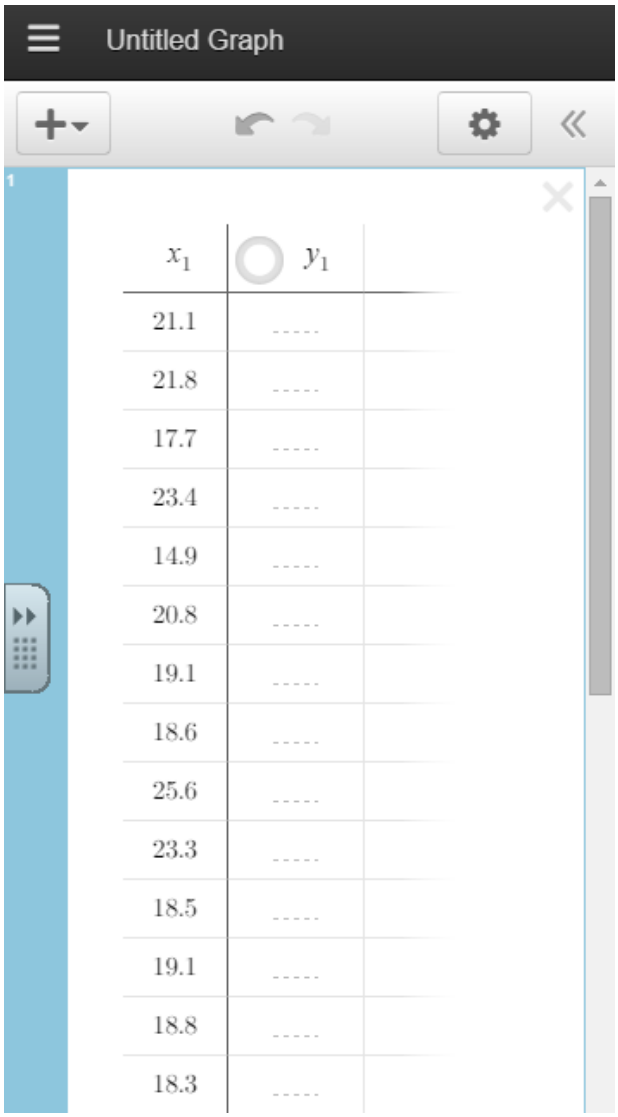
(A) Determine the mean, median and standard deviation. What do you notice about the values?

Using Desmos Graphing:

Click the "+" button and then choose the table option.



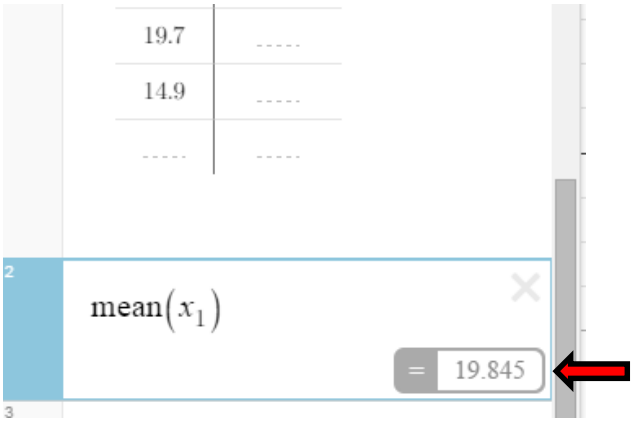
Enter the data points into column x_1:



Scroll down and choose window 2:

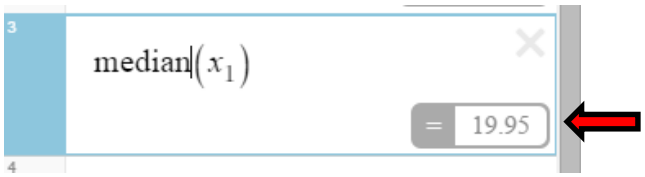
Type the command: mean($x1$)

The mean will appear in the bottom right corner.



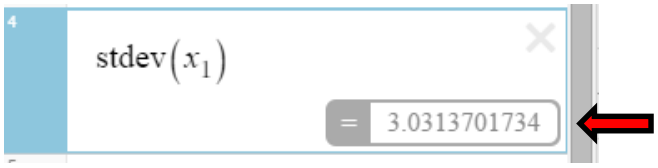
In window 3 type the command: median($x1$)

The median will appear in the bottom right corner.



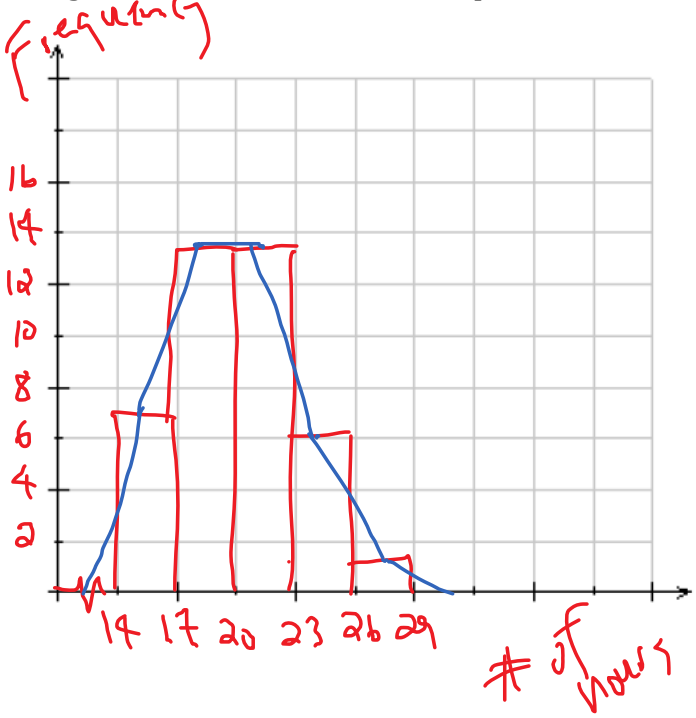
In window 4 type the command: stdev($x1$)

The standard deviation will appear in the bottom right corner.



(B) Construct a frequency table and generate a histogram. Use an interval width equal the standard deviation.

# of hours	tally	frequency
14-17	𝍸 II	7
17-20	𝍸 𝍸 III	13
20-23	𝍸 𝍸 III	13
23-26	𝍸 I	6
26-29	I	1



(C) Discuss the symmetry of the histogram.

Data is close to being symmetrical on each side of the middle.
mean & median, (mode) are in the middle of the graph.

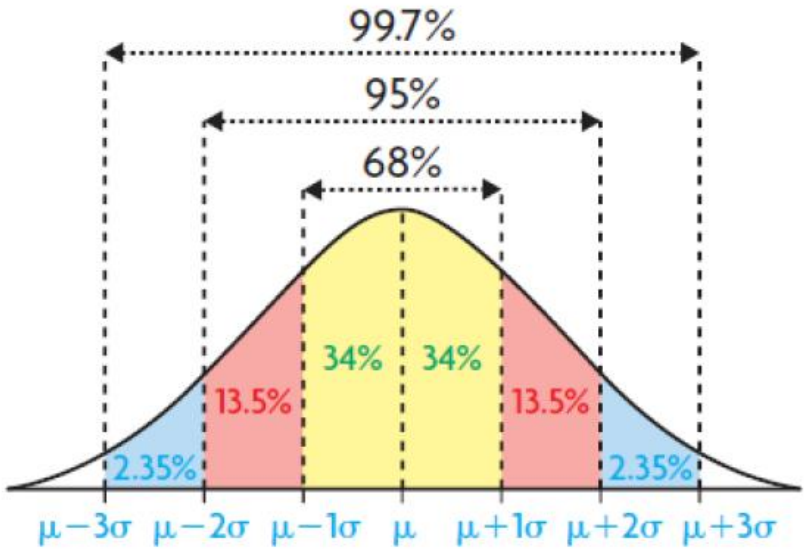
(D) Draw a frequency polygon and explain its shape.

Peak in the middle.
Similar on each side.
"Bell shaped"

(E) Where do the mean, median and (mode) lie?

middle

In a normal distribution the data is symmetrical about the mean. Since the mean lies in the middle of the data it is also the median. As the mean is located where the curve is at its highest point, it is also the mode. Therefore the mean, median and mode are equal for a normal distribution.

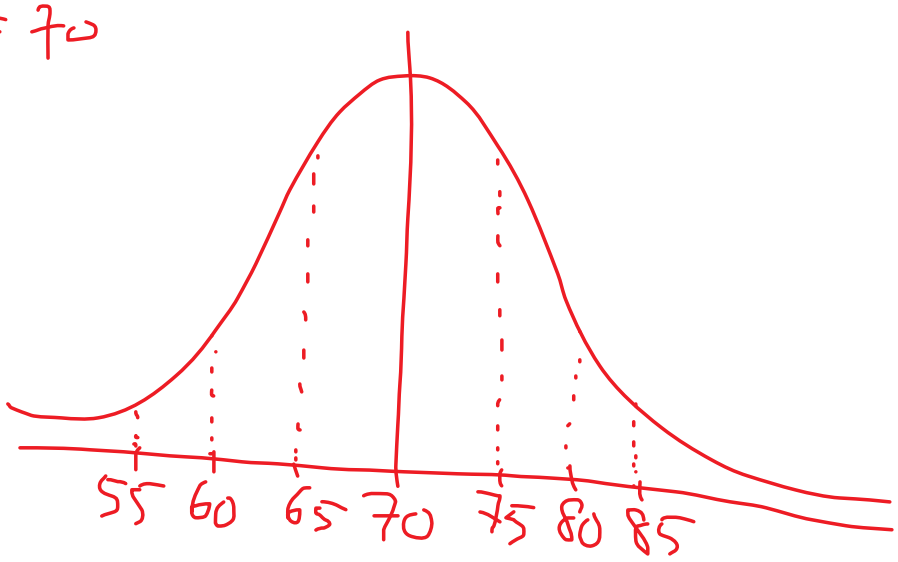


- 68% of all data points lie within one standard deviation of the mean. More specifically, on each side of the mean.
- 95% of all data points lie within two standard deviations of the mean.
- 99.7% of all data points lie within three standard deviations of the mean.

Example 2:
A data set has a mean, $\bar{x} = 70$ and a standard deviation, $\sigma = 5$. Construct a normal distribution curve for this data set.

$\bar{x} = \mu = 70$

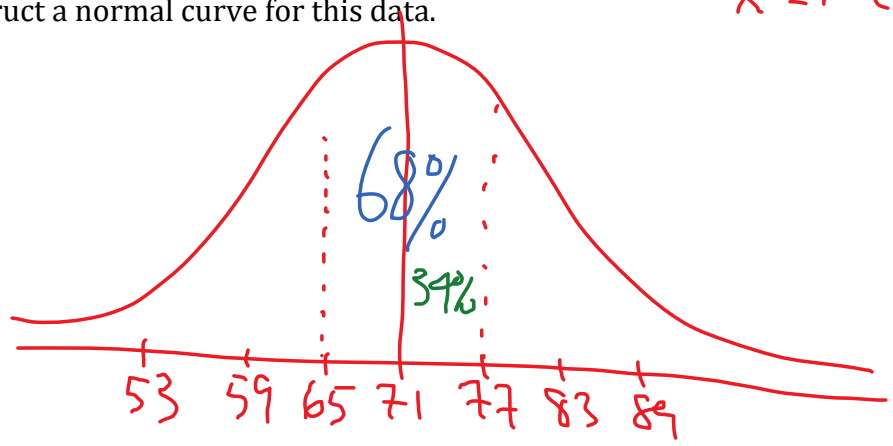
$\sigma = 5$



Example 3:
Mr. Payne's Math 2201 class writes a test for Chapter 5. The class average is 71% with a standard deviation of 6%. He observes the test scores to be normally distributed.

(A) Construct a normal curve for this data.

$\bar{x} = m = 71\%$



(B) Between which values would you find 68% of the test scores.

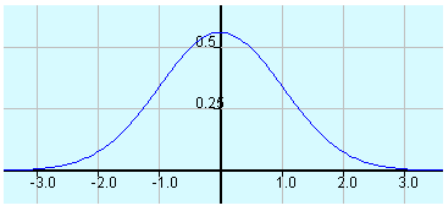
Between 65% and 77%

(C) What percentage of the data lies between 71% and 77%?

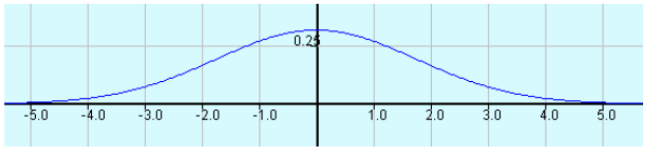
34%

Example 4:
Which normal distribution curve has the largest standard deviation? Explain your reasoning.

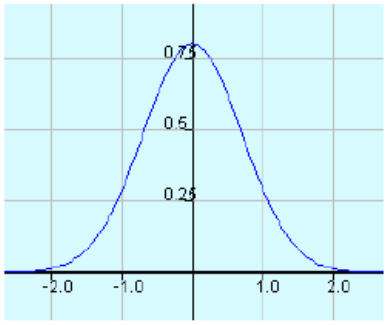
(A)



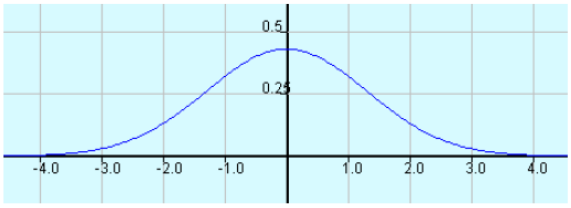
(C)



(B)



(D)



Example 5:
A data set of 50 items is given below with a standard deviation of 1.8. Ask students to answer the following questions:

6	9	6	7	6	7	6	7	6	6
5	8	5	8	6	7	7	4	8	6
5	4	6	8	6	6	7	6	6	8
8	7	6	6	0	4	7	8	5	3
6	7	6	7	1	7	7	4	4	3

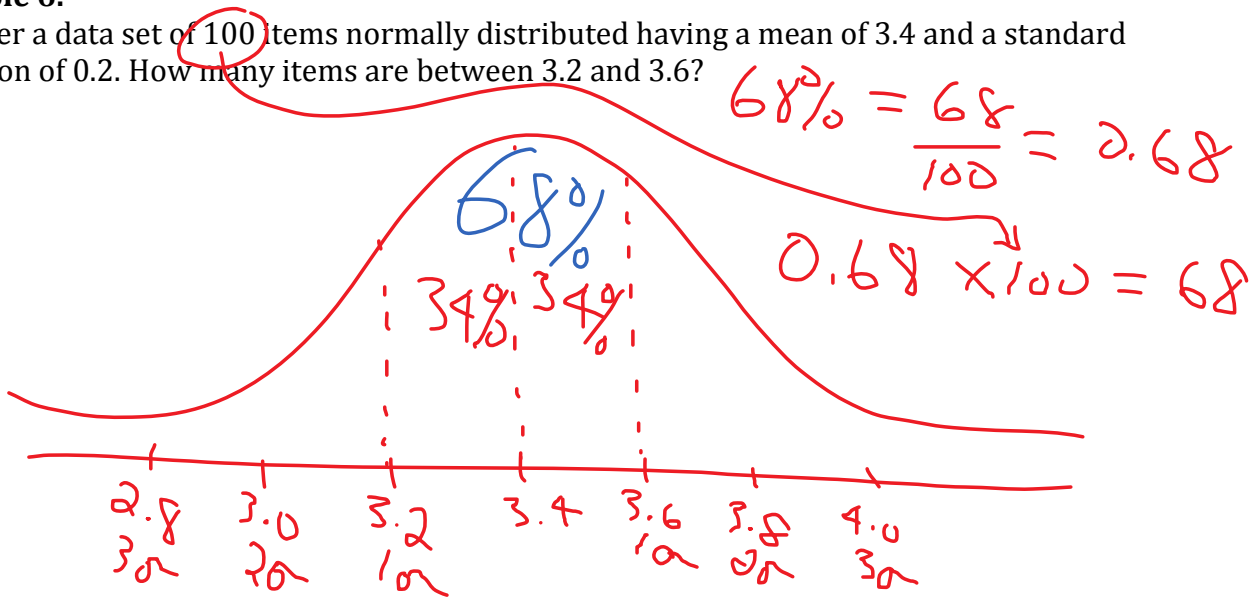
(A) What is the value of the mean, median and mode?

mean = 5.96, median = 6, mode = 6

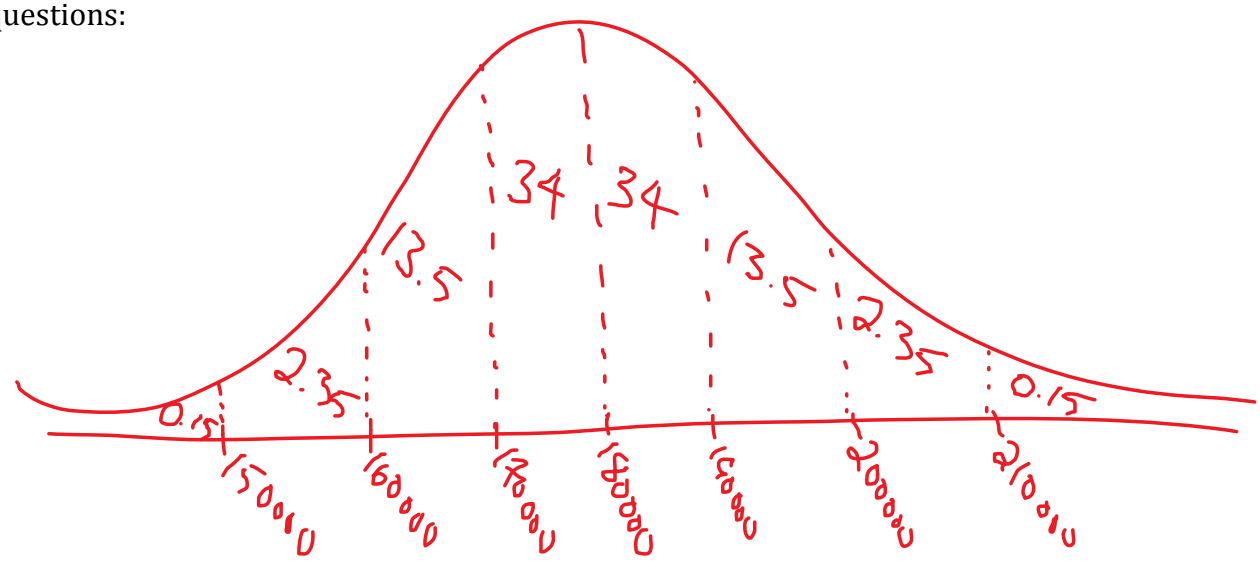
(B) Is the data normally distributed? Explain your reasoning.

Yes. Data is normally distributed when the mean, median & mode are the same.

Example 6:
Consider a data set of 100 items normally distributed having a mean of 3.4 and a standard deviation of 0.2. How many items are between 3.2 and 3.6?



Example 7:
The assessed value of housing in Rocky Harbour is normally distributed. There are 1200 houses with a mean assessed value of $180 000. The standard deviation is $10 000. Construct a normal curve to represent the data and use it to answer the following questions:



(A) What percentage of houses have an assessed value between $170 000 and $200 000?

$34\% + 34\% + 13.5\% = 81.5\%$

(B) What percentage of houses have an assessed value of less than $200 000?

$0.15\% + 2.35\% + 13.5\% + 13.5\% + 34\% + 34\% = 97.5\%$

(C) About how many houses have an assessed value between $170 000 and $200 000?

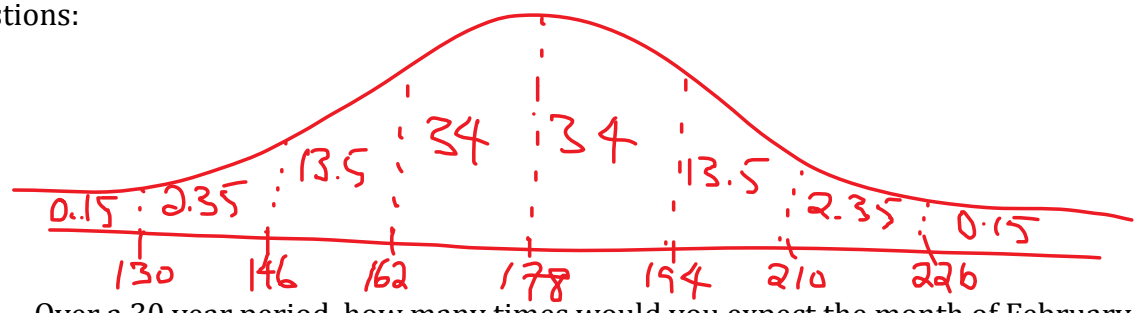
$\frac{81.5\%}{100} = 0.815$ $\quad 0.815 \times 1200 = 978$ houses.

(D) About how many houses have an assessed value greater than $190 000?

$13.5\% + 2.35\% + 0.15\% = 16\%$

$\frac{16\%}{100} = 0.16$ $\quad 0.16 \times 1200 = 192$ houses.

Example 8:
The average snowfall in St. John's for February is 178 cm with a standard deviation of 16 cm. Draw a normal curve to represent the data and use it to answer the following questions:



(A) Over a 30 year period, how many times would you expect the month of February to have between 162 cm and 194 cm?

$34\% + 34\% = 68\%$ $0.68 \times 30 = 20.4$

$\frac{68\%}{100} = 0.68$ or 20 times.

(B) Over a 30 year period, how many times would you expect the month of February to have between 146 cm and 210 cm?

$13.5\% + 34\% + 34\% + 13.5\% = 95\%$

$\frac{95\%}{100} = 0.95$ $0.95 \times 30 = 28.5$ or 29 times.

(C) Over a 30 year period, how many times would you expect the month of February to have less than 146 cm or more than 210 cm?

$0.15\% + 2.35\% + 2.35\% + 0.15\% = 5\%$

$0.05 \times 30 = 1.5$ or 2 times.

Textbook Questions: page 279 - 281 #1, 2, 3, 6, 10, 11, 13