Math 2201 **Date:**____________________

5.5B Z-Scores

Z-Scores
Z-Scores can be used to compare two or more sets of data.

Defining ZScores
For any data point (x) in a set that is normally distributed, we can determine how many standard deviations (σ) the data point is from the mean value (μ):

$$\mu + z\sigma = x$$

The number of standard deviations that a data point is located away from the mean is called the **z-score** (z).

We can rearrange the previous equation to solve for the z-score (z):

$$\mu + z\sigma = x$$

$$\frac{z\sigma}{\sigma} = \frac{x-\mu}{\sigma}$$

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

Given on evalutions.

Example 1:
Hailey runs a 200 m sprint at two locations: in Vancouver and at Lake Louise. The data is given in the table below:

Location	Club Mean Time: μ for 200 m (s)	Club Standard Deviation: σ (s)	Hailey's Run Time (s)
Vancouver	25.75	0.62	24.95
Lake Louise	25.57	0.60	24.77

Determine the location at which her run time was better, when compared to the club results.

Vancouver

$\mu = 25.75$
$\sigma = 0.62$
$x = 24.95$
$z = ?$

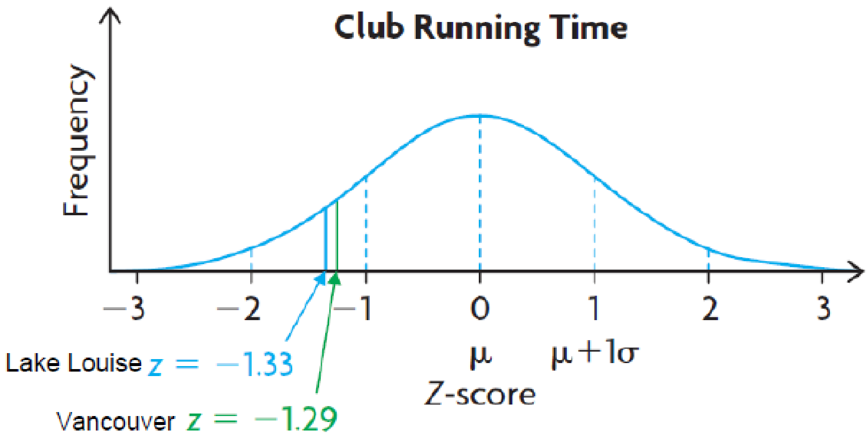
$$z = \frac{x-\mu}{\sigma}$$
$$= \frac{24.95-25.75}{0.62}$$
$$= \frac{-0.8}{0.62}$$
$$z = -1.29$$

Lake Louise

$\mu = 25.57$
$\sigma = 0.60$
$x = 24.77$
$z = ?$

$$z = \frac{24.77-25.57}{0.60}$$
$$= \frac{-0.8}{0.6}$$
$$z = -1.33$$

What do the z-scores actually mean?



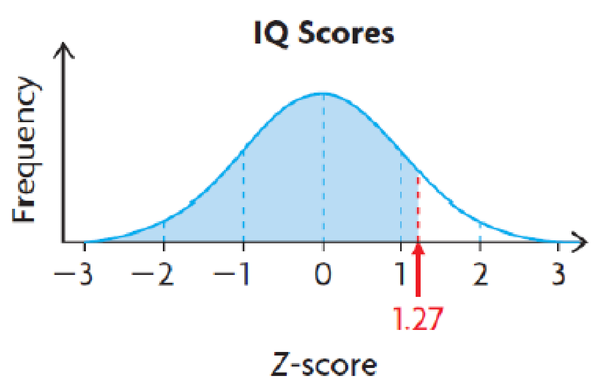
The z-scores show us how many standard deviations each running time is away from the mean value.

Important Note Recall that the data we are looking at is run time for a race. In a race, lower run times are considered the best, since they indicate that a race was completed in a shorter period of time the runner was faster. Therefore, the best run time will be the lower time.

Which run time was best? Lake Louise

Using Z-Scores to Determine the Percent of Data Less Than a Given Value.
Once you calculate a z-score, you can use it to determine the percentage of data that would fall below the given data point, for example, to the left of the data point, on a standard normal curve.

We can determine the percentage of data that would lie in the shaded region on the graph shown.



We do this by looking up our z-score value in the z-score table on pages 580 – 581 in our textbook.

0.8980 × 100 = 89.8%. If your z-score on a test is 1.27, then you scored higher than 89.8% of people in your class.

Example 2:
IQ tests are sometimes used to measure a person's intellectual capacity at a particular time. IQ scores are normally distributed, with a mean of 100 and a standard deviation of 15. If a person scores 119 on an IQ test, how does this score compare with the scores of the general population?

Calculate the z-score:

$\mu = 100$

$\sigma = 15$

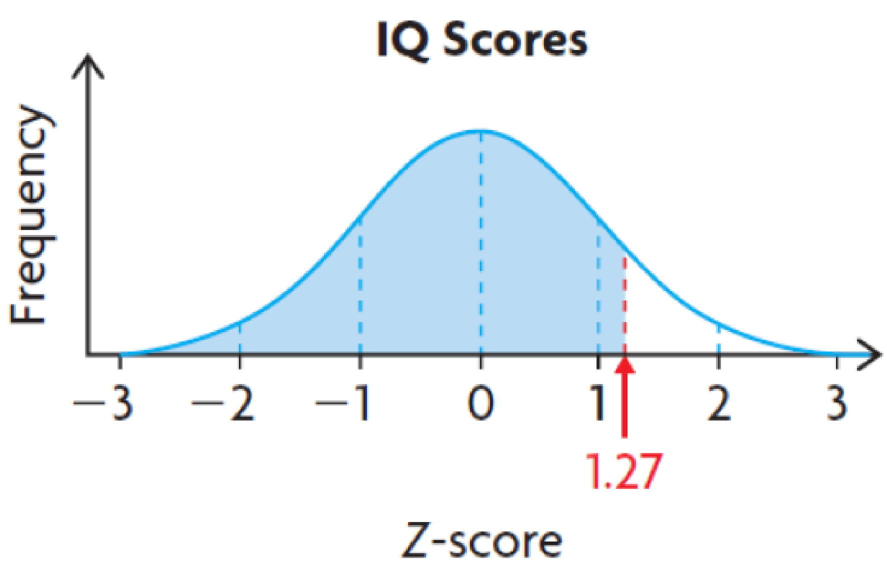
$x = 119$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{119 - 100}{15}$$

$$z = 1.27$$

A z-score of 1.27 means that an IQ of 119 is 1.27 standard deviations above the mean, as shown below:



How do we find the percentage of data below the given data point, IQ = 119? Use a **z-score table** in this case, we are looking up a z-score of 1.27

z-score table

A table that displays the fraction of data with a *z*-score that is less than any given data value in a standard normal distribution.
(There is a *z*-score table on pages 580 to 581.)

last digit in our number

z	0.0	0.01	0.06	0.07
0.0	0.5000	0.5040	0.5239	0.5279
0.1	0.5398	0.5438	0.5636	0.5675
1.1	0.8643	0.8665	0.8770	0.8790
1.2	0.8849	0.8869	0.8962	**0.8980**
1.3	0.9032	0.9049	0.9131	0.9147

1st two digits in our number

119 IQ is 0.8980 or higher than 89.8% of the population.

Example 3:
Sally has a height of 1.75 m and lives in a city where the average height is 1.60 m and the standard deviation is 0.20 m. Leah is 1.80 m and lives in a city where the average height is 1.70 m and the standard deviation is 0.15 m. Identify which of the two is considered to be taller compared to their fellow citizens. Explain your reasoning.

Sally

$x = 1.75$
$\mu = 1.6$
$\sigma = 0.2$

$z = \frac{x - \mu}{\sigma}$

$z = \frac{1.75 - 1.6}{0.2}$

$z = 0.75$

From z table: 0.773
or 77.3%

Leah

$x = 1.8$
$\mu = 1.7$
$\sigma = 0.15$

$z = \frac{1.8 - 1.7}{0.15}$

$= 0.67$

From z-table: 0.7486
or 74.86%

Sally is taller compared to her fellow citizens.

Example 4:
On her first math test, Susan scored 70%. The mean class score was 65% with a standard deviation of 4%. On her second test she received 76%. The mean class score was 73% with a standard deviation of 10%. On which test did Susan perform better with respect to the rest of her class?

Test 1

$x = 0.7$

$\mu = 0.65$

$\sigma = 0.04$

$z = \frac{x - \mu}{\sigma}$

$= \frac{0.7 - 0.65}{0.04}$

$= 1.25$

Test 2

$x = 0.76$

$\mu = 0.73$

$\sigma = 0.1$

$z = \frac{0.76 - 0.73}{0.1}$

$= 0.3$

Example 5:
It is known that the mean value of repairs, due to an accident, for both cars is $3500. The standard deviation for the Corvette is $1200, while the standard deviation for the Civic is only $800. If the cost of repairs is normally distributed, determine the probability that the repairs costs will be over $5000 for both cars.

Corvette

$x = 5000$

$\mu = 3500$

$\sigma = 1200$

$z = \frac{x - \mu}{\sigma}$

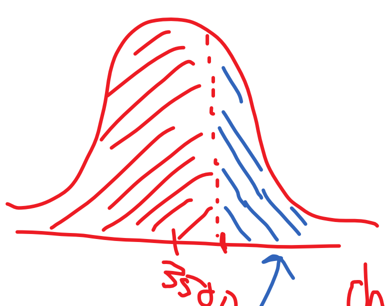
$z = \frac{5000 - 3500}{1200}$

$z = 1.25$

From table: 0.8944

$1 - 0.8944 = 0.1056$

or 10.56%



There's a 10.56% probability that the repairs cost over $5000.

Civic

$x = 5000$

$\mu = 3500$

$\sigma = 800$

$z = \frac{5000 - 3500}{800}$

$z = 1.88$

From z-table: 0.9699

$1 - 0.9699 = 0.031$

or 3.1%

There's a 3.1% chance the repairs cost over $5000.

Example 6:

A manufacturuer produces tires that have an average thickness of 179 mm, with a standard deviation of 0.9 mm. To be classified as “supreme quality”, tires must have a thickness between 177.8 mm and 180.7 mm. What percent, to the nearest whole number, of the total production can be rated as “supreme quality” tires?

* two x-values

$x = 177.8$
$\mu = 179$
$\sigma = 0.9$

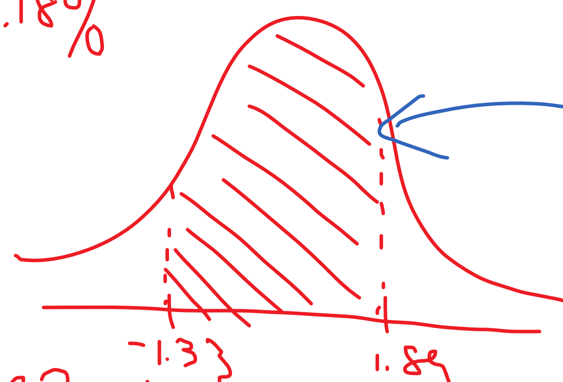
$$z = \frac{x-\mu}{\sigma} = \frac{177.8-179}{0.9} = -1.33$$

From table: 0.0918 or 9.18%

$x = 180.7$

$$z = \frac{180.7-179}{0.9} = 1.89$$

From table: 0.9706 or 97.06%



$97.06\% - 9.18\% = 87.88\%$

Example 7:
A manufacture of personal music players has determined that the mean life of the players is 32.4 months, with a standard deviation of 6.3 months. What length of warranty should be offered if the manufacturuer wants to restrict repairs to less than 1.5% of all players sold?

$\mu = 32.4$

Percentage: 1.5% or 0.015

$\sigma = 6.3$

$z = -2.17$

$x = ?$

$z = ?$

$$z = \frac{x-\mu}{\sigma}$$

$$\frac{-2.17}{1} = \frac{x-32.4}{6.3}$$

$$x - 32.4 = (-2.17)(6.3)$$

$$x - 32.4 = -13.671$$

$$x = -13.671 + 32.4$$

$$x = 18.729$$

Warranty should be 18-19 months.

SUMMARY

Need to Know

- A *z*-score indicates the number of standard deviations that a data value lies from the mean. It is calculated using this formula:

$$z = \frac{x - \mu}{\sigma}$$

- A positive *z*-score indicates that the data value lies above the mean. A negative *z*-score indicates that the data value lies below the mean.
- The area under the standard normal curve, to the left of a particular *z*-score, can be found in a *z*-score table or determined using a graphing calculator.

Textbook Questions: page 292, 293 #1, 5, 6, 7, 8, 10, 11