

5.6 Confidence Intervals

Consider the following scenario:

"A telephone survey of 600 randomly selected people was conducted in an urban area. The survey determined that 76% of people, from 18 to 34 years of age, have a social networking account. The results are accurate within plus or minus 4 percent points, 19 times out of 20".

This means that $76 \pm 4\%$ of people within the given age group have a social networking site. We can determine the range of percentages that this includes:

$$\begin{aligned} 76 - 4\% &= 72\% \\ 76 + 4\% &= 80\% \end{aligned}$$

Thus, 72% to 80% of people within the given age group have social networking sites.

Margin of Error: this is the plus or minus percent that defines the range. In our example, it was $\pm 4\%$.

Confidence Interval: this is the interval or range of values defined by the mean and margin of error. In our example it was 72% to 80%.

Confidence Level: this is the likelihood that the result lies within the given confidence interval. In our example, it was "19 times out of 20", or we could change it to a percent: 95%.

Example 1:

A Rent-A Car-company surveys customers and finds that 50 percent of the respondents say its customer service is "very good." The confidence level is cited as 95 percent and the margin of error is ± 3 percent.

(A) What is the margin of error?

$$\pm 3\%$$

(B) What is the confidence interval?

$$50 \pm 3\%$$

$$50 - 3\% = 47\%$$

$$50 + 3\% = 53\%$$

Range of values we are confident.

$$47\% \text{ to } 53\%$$

(C) What is the confidence level? What does this mean?

95%. If the survey was repeated, the results would be accurate 95% of the time.
(95 times out of 100)

Example 2:

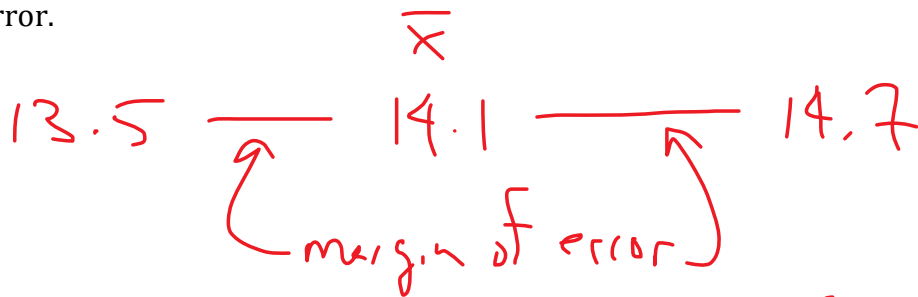
A brand of battery has a mean life expectancy of 12.6 hours with a margin of error of 0.7 hours. How long would we expect the battery to last?

$$12.6 \pm 0.7 \quad 12.6 - 0.7 = 11.9$$
$$12.6 + 0.7 = 13.3$$

You would expect the battery to last between 11.9 and 13.3 hours.

Example 3:

The expiration time of milk is said to occur between 13.5 and 14.7 days. The mean expiration time would be the midpoint of the interval, $\frac{13.5+14.7}{2} = 14.1$. Calculate the margin of error.



$$14.7 - 14.1 = 0.6$$

or

$$14.1 - 13.5 = 0.6$$

\therefore margin of error = ± 0.6

Example 5:

A report claims that the average family income in a large city is \$32 000. It states the results are accurate 19 times out of 20 and have a margin of error of ± 2500 .

(A) What is the confidence level in this situation? Explain what it means.

(B) Explain the meaning of a margin of error of ± 2500 .

Example 6:

Explain what is meant by a 90%, a 95%, and a 99% confidence interval. How are these intervals similar? How are they different?

90% : accurate 90 times out of 100.

95% : " 95 " " " " "

99% : " 99 " " " " "

All out of 100. All relatively high. Interval gets smaller as confidence level gets higher.

Example 7:

The town of Conception Bay South is trying to determine the location of a new recreation centre. Make a prediction about the confidence interval and margin of error if 100 people were surveyed, if 1000 people were surveyed or if all the people in the town were surveyed.

Example 8:

In a national survey of 400 Canadians from the ages of 20 to 35, 37.5% of those interviewed claimed they exercise for at least four hours a week. The results were considered accurate within 4%, 9 times out of 10. Ask students to answer the following questions:

(A) Are you dealing with a 90%, 95%, or 99% confidence interval? How do you know?

(B) How many people in the survey claimed to exercise at least four hours a week?

(C) What is the margin of error?

(D) What is the confidence interval? Explain its meaning.

Example 10:

The results of a survey show that 72% of residents in Mount Pearl own cell phones. The margin of error for the survey was 2.3%. If there are 24 600 people in Mount Pearl, determine the range of the number of people that own cell phones.

$$72 \pm 2.3\%$$
$$72 - 2.3\% = 69.7\%$$
$$72 + 2.3\% = 74.3\%$$
$$69.7\% \text{ to } 74.3\%$$
$$0.697 \quad 0.743$$
$$24600 \times 0.697 = 17146.2$$
$$24600 \times 0.743 = 18277.8$$

Between 17146 and 18278 people in Mount Pearl own cell phones.

Need to Know

- A confidence interval is expressed as the survey or poll result, plus or minus the margin of error.
- The margin of error increases as the confidence level increases (with a constant sample size). The sample size that is needed also increases as the confidence level increases (with a constant margin of error).
- The sample size affects the margin of error. A larger sample results in a smaller margin of error, assuming that the same confidence level is required.

For example:

- A sample of 1000 is considered to be accurate to within $\pm 3.1\%$, 19 times out of 20.
- A sample of 2000 is considered to be accurate to within $\pm 2.2\%$, 19 times out of 20.
- A sample of 3000 is considered to be accurate to within $\pm 1.8\%$, 19 times out of 20.

Textbook Questions: page 302, 303 #1, 2, 3, 4, 5