Math 2201 **Date:**_____

6.1 Exploring Quadratic Functions

Degree of a Function - refers to the highest exponent on the variable in an expression or equation.

In Math 1201, you learned about linear functions. These have a degree of 1. When the exponent on the variable is 1, we don't include it in the equation.

Examples of Linear functions:

$y = 2x + 5$ $y = -7x + 2$ $10x - 3y = 12$

Notice that in each of these equations, the exponent on the variable x is 1.

Quadratic Functions
These have a degree of 2. That is, highest power of any x is 2.

Examples of Quadratic functions:

$y = 2x^2 + 5x - 4$ $y = -3x^2 - 7x + 1$ $y = (x - 3)(x + 4)$

$y = x(x + 4)$ $y = -3(x + 4)^2 + 2$ $y = (3x - 2)^2$

Example 1:
Describe the reasoning used to decide whether each statement is true or false.

Polynomial Function	Classification	True or False	Explain/Justify
$y = 5(x + 3)$	Linear	T	1 x
$y = 5(x^2 + 3)$	Quadratic	T	2 xs
$y = 5^2(x + 3)$	Quadratic	F	1 x
$y = 5x(x + 3)$	Linear	F	2 xs
$y = (5x + 1)(x + 3)$	Quadratic	T	2 xs
$y = 5(x + 3)^2 + 2$	Quadratic	T	2 xs

The Parabola

Parabola: the shape of the graph of any quadratic relation. It is a u-shaped graph, that can open upward or downward.

Watch "Projectile Motion – Science of NFL Football"

https://www.youtube.com/watch?v=HB4ws7RoA3M

What are some other natural events that involve parabolas?

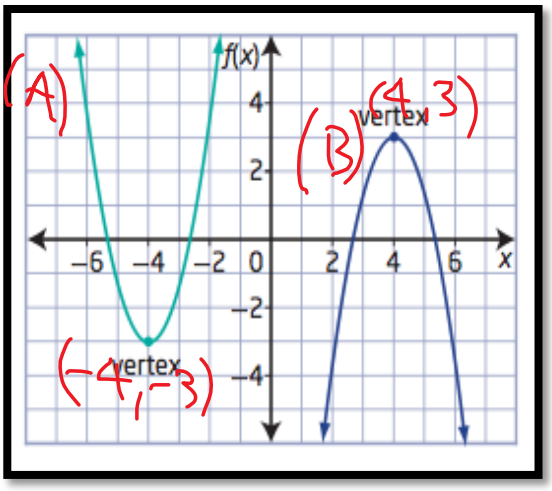
Baseball hit. Cannon ball. Arch ways.

The **vertex** of a parabola is the lowest point of the graph if the graph opens upward, or the highest point of the graph if the graph opens downward.

Example 1:
What is the vertex of the following graphs?

(A) (-4, -3)

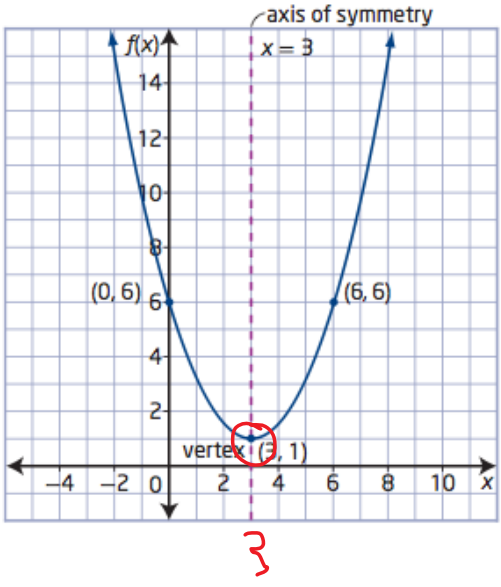
(B) (4, 3)



The **axis of symmetry** is a line through the vertex that divides the graph of a quadratic function into two congruent halves. It is defined by the x-coordinate of the vertex.

Example 2:
What is the axis of symmetry of the following graph?

x = 3



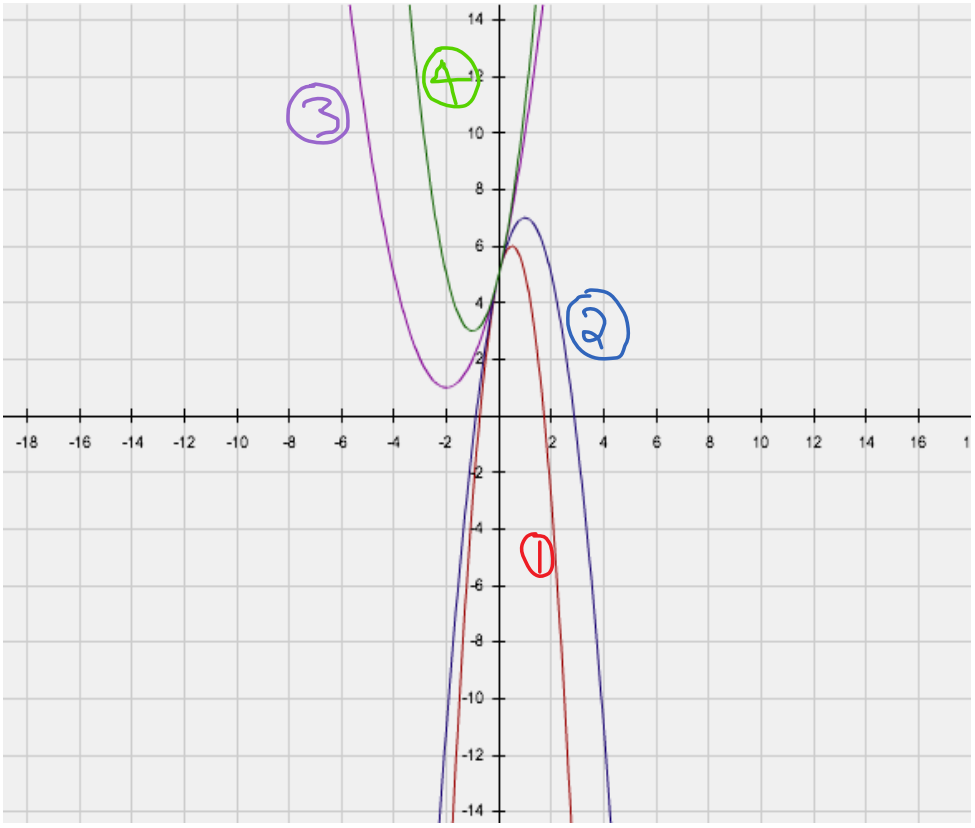
Standard Form

The **standard form** of a quadratic is:

$$f(x) = ax^2 + bx + c \qquad \text{or} \qquad y = ax^2 + bx + c$$

where a, b, and c are real numbers and $a \neq 0$.

The Effect of Parameter a

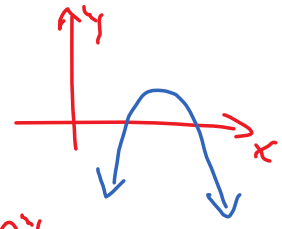


1. $f(x) = -4x^2 + 4x + 5$
2. $f(x) = -2x^2 + 4x + 5$
3. $f(x) = x^2 + 4x + 5$
4. $f(x) = 2x^2 + 4x + 5$

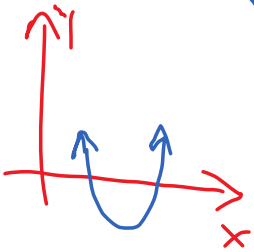
Questions:

1. What happens to the direction of the opening of the quadratic if $a < 0$ and $a > 0$?

$a < 0$ opens down (negative)



$a > 0$ opens up (positive)



2. Is the shape of the parabola affected by the parameter a? Are some graphs wider or narrower compared to the graph of $y = x^2$?

Excluding the negative or positive, the larger a is, the more narrow the graph.

3. What happens to the x-intercepts as the value of a is manipulated?

The larger the a value, the closer the x-intercepts get to each other.

4. What is the impact on the graph if $a = 0$?

$y = ax^2 + bx + c$, $a = 0$

$y = 0x^2 + bx + c$

$y = bx + c$ ← not a quadratic. (Linear)

5. If the quadratic opens upward, is the vertex a maximum or minimum point? Explain your reasoning. What if the quadratic opens downward?

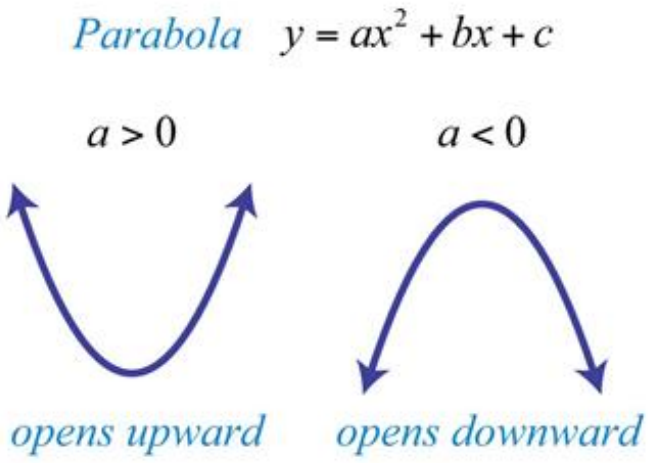
$a > 0$, opens up → minimum point

$a < 0$, opens down → maximum point

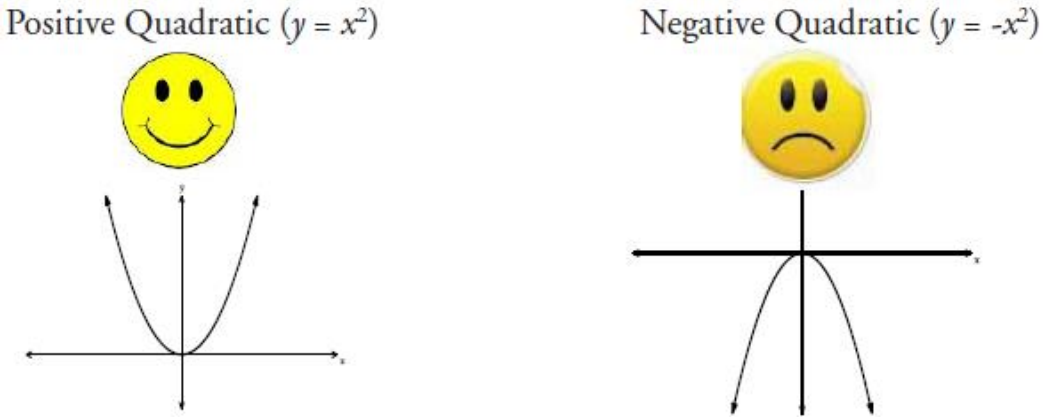
6. Explain how changing the parameter a affects the graph of the function $f(x) = ax^2 + bx + c$.

- a affects the direction of opening and the shape of the graph.
- $a > 0$ → opens up
- $a < 0$ → opens down
- excluding the sign (positive or negative), the larger a is, the more narrow the graph.

Summary:



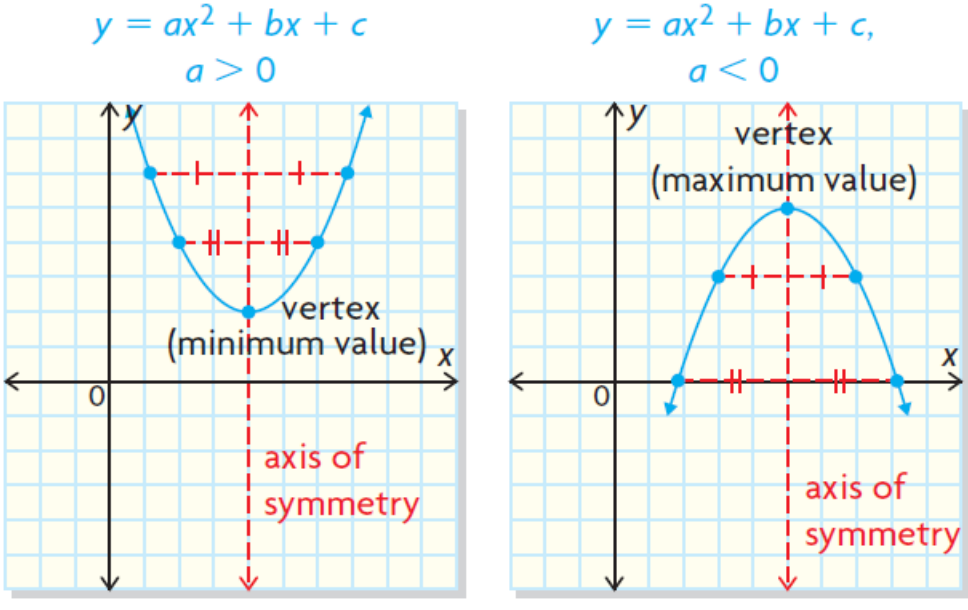
One way to remember how to determine direction of opening is shown here.



Maximum and Minimum Values

The **minimum value** of a function is the least $y-$ value in the range of a function.

The **maximum value** of a function is the greatest $y-$ value in the range of a function.



The Effect of Parameter *b*

1. $f(x) = -x^2 + 2x + 5$
2. $f(x) = -2x^2 + 5$
3. $f(x) = -x^2 - 2x + 5$
4. $f(x) = -x^2 - 4x + 5$

Questions:

1. What is the effect of parameter b in $y = ax^2 + bx + c$?

b affects the position of the vertex.

2. Is the parabola's line of symmetry changing?

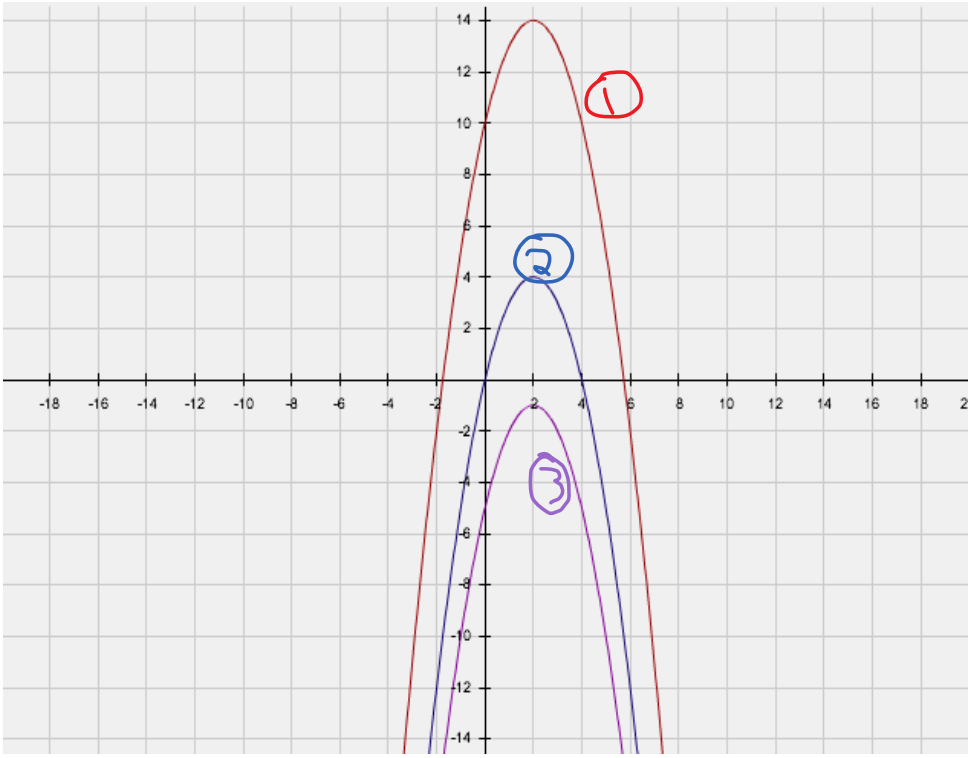
Yes. It moves as b changes.

The Effect of Parameter c

(1) $f(x) = -x^2 + 4x + 10$

(2) $f(x) = -x^2 + 4x$

(3) $f(x) = -x^2 + 4x - 5$



Questions:

1. What is the effect of parameter c?

c is the y-intercept.

2. How can you identify the y-intercept from the equation in standard form?

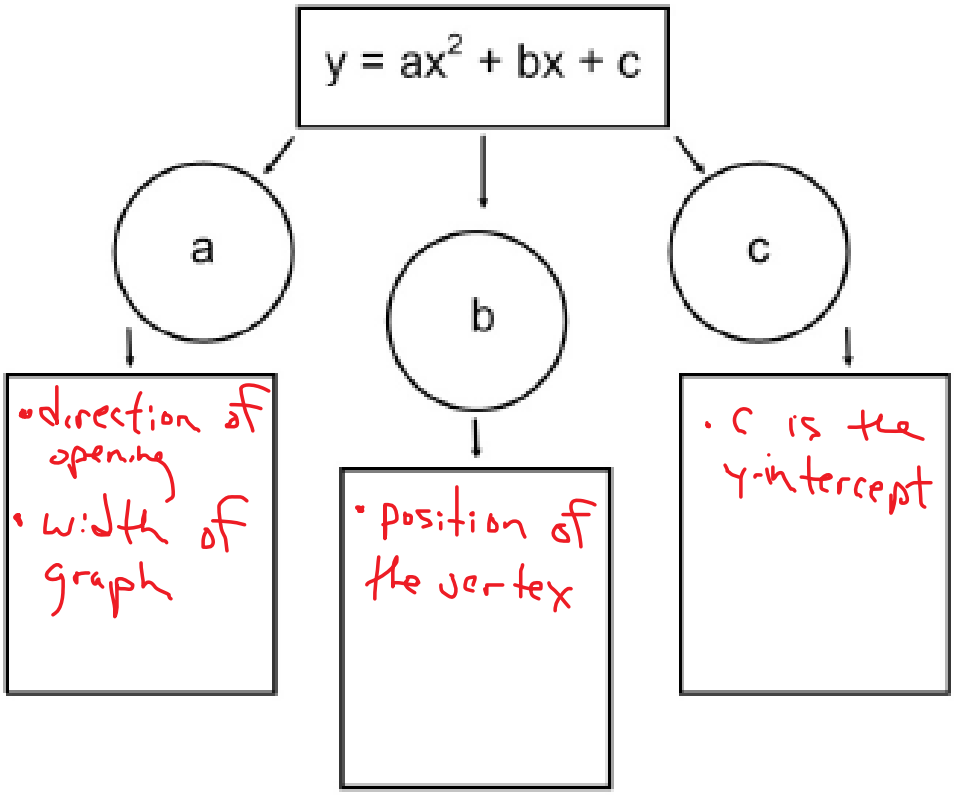
It is the c-value.

3. Is the line of symmetry affected by the parameter c?

No. Only the y-coordinate of the vertex is affected.

Your turn:

1. Complete the webbing to describe the effects of the parameters a, b and c on the quadratic function $y = ax^2 + bx + c$.



2. Identify whether the following equations represent quadratic functions.

(A) $y = 3x^2 + 2x + 6$ yes

(B) $y = (x - 7)(x + 5)$ yes

(C) $y = 2x(x - 1)^2$ No

3. For each quadratic function, state whether the parabola will open up or down, whether the vertex will be a maximum or minimum, and state the y-intercept.

(A) $y = 3x^2 + 5x - 7$

$a > 0$ → opens up

vertex is a minimum

y-intercept: -7

(B) $y = -2x^2 - 4x + 5$

$a < 0$ → opens down

vertex is a maximum

y-intercept: 5

Textbook Questions: page 324, #1, 2, 3, 4, 5, 6