Math 2201 **Date:**____________________

6.3 Factored Form of a Quadratic Function

Example 1:
Using technology, graph the following functions and determine the x-intercepts. Then answer the following questions.

9
1, 9
3, 3

i. $y = x^2 - 6x + 9$ $y=(x-3)(x-3)$

ii. $y = 2x^2 - 4x - 6$

iii. $y = (x - 2)(x + 3)$

iv. $y = (x + 1)(x - 4)$

(A) Can you determine the x-intercepts by looking at a quadratic function? Explain.
Yes, if the function is in factored form.

(B) Which form of the quadratic function did you find the easiest to use when determining the x-intercepts?

Factored form.

(C) What is the connection between the factors and the x-intercepts?
The factors have the opposite sign of the x-intercepts.

(D) What is the value of the y-coordinate at the point where the graph crosses the x-axis? The y-coordinate is 0 where the graph crosses the x-axis.

(E) What would happen if the factors of the quadratic function were identical?

We get a single root at the opposite number of the factor. This single root is also the vertex.

(F) How can you find the x-intercepts of a quadratic function without the graph or a table of values?

By factoring.

Factored Form

The factored form of a quadratic is:

$$f(x) = a(x - r)(x - s) \qquad \text{or} \qquad y = a(x - r)(x - s)$$

where a, r, and s are real numbers and $a \neq 0$.

Factored form can be obtained by factoring standard form. For example:

$$f(x) = 2x^2 + 14x + 12$$

$y = 2(x^2 + 7x + 6)$ $\quad \frac{6}{1,6 \quad 2,3}$

$y = 2(x+1)(x+6)$

There are a number of properties that can be read from factored form.

Direction of Opening: as with standard form, the a value gives the direction of opening and indicates whether the parabola will have a maximum or a minimum point.

Zeros: these are the value of the variable x that make the function $f(x)$ equal to zero. These correspond to the x-intercepts on a graph.

Zeros = x-intercepts = roots

y-intercept: obtained by setting x equal to zero and solving for $f(x)$.

Axis of Symmetry: obtained by taking the average of the x-intercepts/zeros.

Vertex: x-value is the same as the axis of symmetry. y-value of obtained by substituting the x-value into the equation and solving for y.

Domain and Range: can be determined once we know the vertex and direction of opening.

Example 1:
For the function given below in factored form, find:

$\therefore$ therefor

$$f(x) = 2(x+1)(x+6)$$

(A) Direction of opening:

$a > 0$, $\therefore$ opens up

(B) Zeros

$(-1, 0), (-6, 0)$

(C) y-intercept

$y = 2(0+1)(0+6) = 2(1)(6) = 12$ $(0, 12)$

(D) Axis of symmetry

$\frac{-1+(-6)}{2} = -\frac{7}{2}$ or -3.5 $x = -3.5$

(E) Vertex

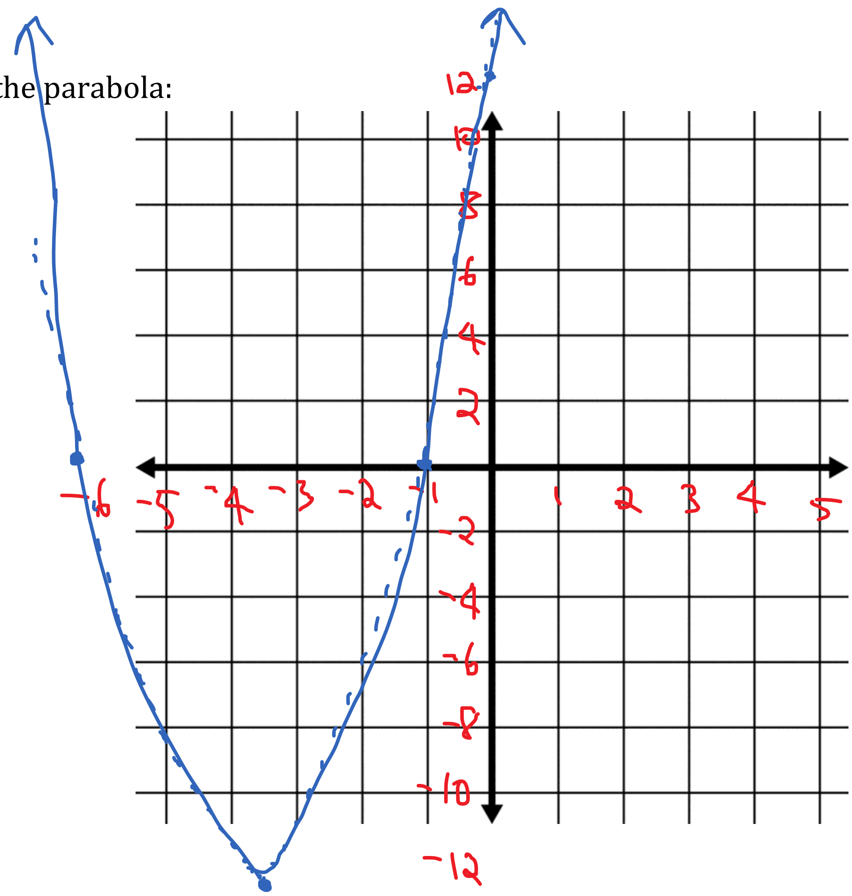
$y = 2(-3.5+1)(-3.5+6) = 2(-2.5)(2.5) = -12.5$

Vertex: $(-3.5, -12.5)$

(F) Domain and Range

Domain: $\{x \mid x \in R\}$, Range: $\{y \mid y \geq -12.5, y \in R\}$

(G) Sketch the parabola:



Example 2:
For the function given below ~~in factored form~~, find:

$$f(x) = x^2 + 2x - 8$$

$f(x) = (x-2)(x+4)$

8
1, 8
2, 4

(A) Direction of opening:

a > 1, opens up

(B) Zeros

(2, 0), (-4, 0)

(C) y-intercept

(0, -8)

(D) axis of symmetry

$\frac{2+(-4)}{2} = \frac{-2}{2} = -1 \quad x = -1$

(E) vertex

$y=(-1-2)(-1+4)=(-3)(3)=-9$

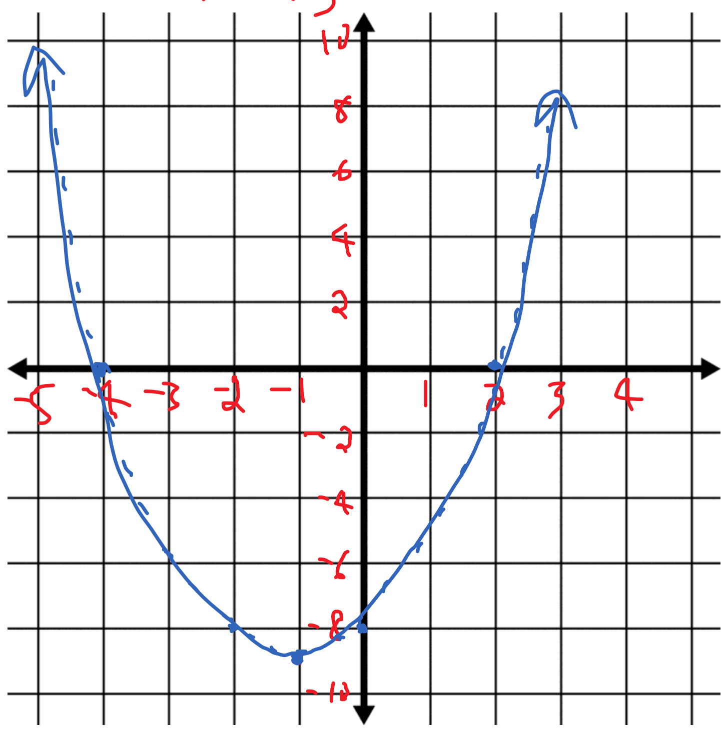
vertex: $(-1,-9)$

(F) Domain and Range

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq -9, y \in R\}$

(G) Sketch the parabola:



Example 3:
Determine the quadratic function with factors $(x+3)$ and $(x-5)$ and a y-intercept of -5.

$y=a(x-r)(x-s)$ * Find a. $(0,-5)$ x y

$y=a(x+3)(x-5)$ → $a=\frac{1}{3}$

$-5=a(0+3)(0-5)$

$-5=a(3)(-5)$

$\frac{-5}{-15}=\frac{-15a}{-15}$

Function:

$y=\frac{1}{3}(x+3)(x-5)$

Example 4:
Ask students to answer the following questions:

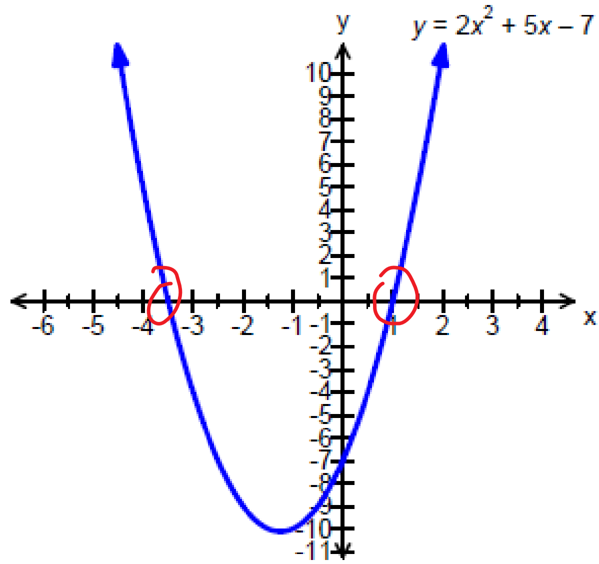
(A) Using technology, what are the zeros of $f(x) = 2x^2 + 5x - 7$?

$(-3.5, 0), (1, 0)$

(B) What are the x-intercepts of the graph?

x-int: -3.5

x-int: 1



(C) What do you notice about the answers to the above questions?

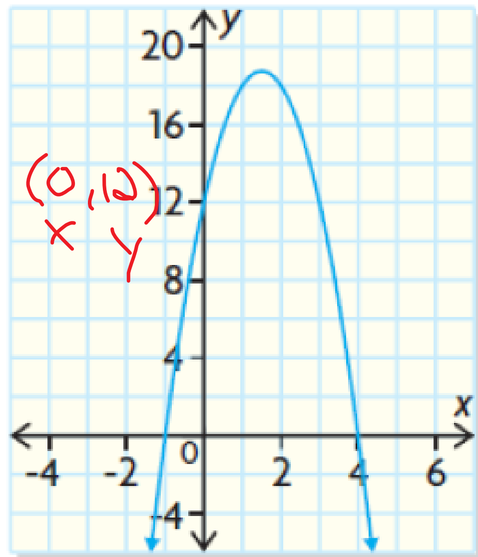
The numbers are the same.

Example 5:

(A) Determine the function that defines this parabola in factored form.

$y = a(x-r)(x-s)$

$y = a(x+1)(x-4)$



$12 = a(0+1)(0-4)$

$12 = a(1)(-4)$

$\frac{12}{-4} = \frac{-4a}{-4}$

$a = -3$ Function: $y = -3(x+1)(x-4)$

(B) Write the function in standard form. FOIL

$y = -3(x+1)(x-4)$

$y = -3(x^2 - 4x + 1x - 4)$

$y = -3(x^2 - 3x - 4)$

$y = -3x^2 + 9x + 12$

Example 6:
A quadratic function has x-intercepts at $x = 2$ and $x = 3$, and it passes through the point $(1, 8)$.

$x \; y$ $\qquad x-2 \quad x-3$

(A) Write the equation of the quadratic function in factored form.

$y = a(x-r)(x-s)$

$y = a(x-2)(x-3)$ Function:

$8 = a(1-2)(1-3)$ $\quad y = 4(x-2)(x-3)$

$8 = a(-1)(-2)$

$\frac{8}{2} = \frac{2a}{2}$

$a = 4$

(B) Write the equation of the quadratic function in standard form.

$y = 4(x-2)(x-3)$

$y = 4(x^2 - 3x - 2x + 6)$

$y = 4(x^2 - 5x + 6)$

$y = 4x^2 - 20x + 24$

Example 7:
Answer the following questions about quadratic functions:

(A) Are two x-intercepts enough information to sketch a unique graph? Explain.

No. You need the vertex as well.

(B) Is the vertex enough information to sketch a unique graph? Explain.

No. You need to have the x-intercepts as well. The y-intercept helps, but is not necessary.

(C) What are the minimum requirements to sketch a unique quadratic graph? Explain.

x-intercepts and the vertex.

Textbook Questions: page 346 - 348, #1, 2, 3, 4 a,d,e, 11, 12