6.4 Vertex Form of a Quadratic Function

Recall from 6.1 and 6.2:
Standard Form

The **standard form** of a quadratic is:

$$f(x) = ax^2 + bx + c \qquad \text{or} \qquad y = ax^2 + bx + c$$

where a, b, and c are real numbers and $a \neq 0$.

Benefits of Standard Form:

- Direction of opening can be quickly determined by looking at the a value.
- y-intercept is equal to the c value.

Disadvantages of Standard Form:

- Finding the equation of axis of symmetry and the coordinates of the vertex requires us to carry out calculations.

Factored Form

Quadratics written in factored form:

$$f(x) = a(x - r)(x - s) \qquad \text{or} \qquad y = a(x - r)(x - s)$$

The factored form tells you the x-intercepts, $x = r$, $x = s$, and the direction of opening.

Benefits of Factored Form:

- Direction of opening can be quickly determined by looking at the a value.
- The x-intercepts can be quickly determined using the two factors in the equation and zero product property.

Disadvantages of Factored Form:

- Finding the equation of axis of symmetry and the coordinates of the vertex requires us to carry out calculations.
- Finding the y-intercept requires us to carry out calculations.

Using Desmos Graphing, compare graphs such as:

$$y = x^2 \text{ and } y = (x+2)^2$$

$$y = x^2 \text{ and } y = x^2 + 3$$

$$y = x^2 \text{ and } y = (x-1)^2 - 5$$

Observe the vertical translation, k, and the horizontal translation, h, and make a prediction about the vertex (h, k) of each equation.

$y = a(x-h)^2 + k$

- changing h, moves (translates) the graph horizontally (left or right), opposite the sign of h.
- Changing k, moves (translates) the graph vertically (up or down) the same direction as the sign of k.

Vertex: (h, k)

Rules about Vertex Form

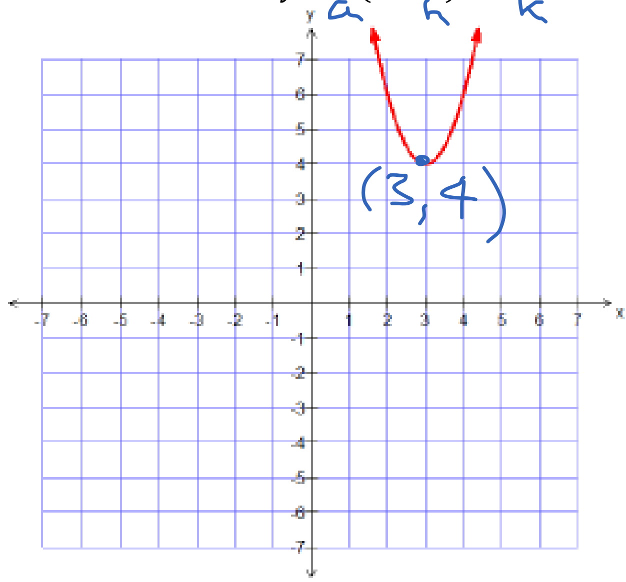
For a quadratic written in the form $y = a(x-h)^2 + k$

- Direction of opening is determined by a. If $a > 0$, the parabola opens upward. If $a < 0$, it opens downward.
- The vertex is given by (h, k), where h is called the horizontal translation. It tells us how far the parabola has been shifted horizontally. k is the vertical translation. It tells us how far the parabola has been shifted vertically.
- The equation of the axis of symmetry is $x = h$.

Example 1:

The following graph shows the function $y = 2(x-3)^2 + 4$.

a h k



(A) What is the direction of opening of the graph. How could you tell this from looking at the equation?

Opens up. $a > 0$

(B) What is the vertex of the graph. How does this relate to the equation?

(3, 4) h is opposite sign

k is same sign

(C) What is the equation of the axis of symmetry of the graph? How does this relate to the equation?

$x = 3$. Axis of symmetry = h

Example 2:
A quadratic function is given by the equation $y = -3(x+4)^2 - 1$ Determine:

(A) the direction of opening.

opens down. $a < 0$

(B) the vertex.

$(-4, -1)$

(C) the equation of the axis of symmetry.

$x = -4$

(D) state the Domain and Range

Domain: $\{x \mid x \in R\}$, Range: $\{y \mid y \leq -1, y \in R\}$

Sketching a Graph Given Vertex Form

- Read off the vertex.
- Calculate the y-intercept by setting $x = 0$.
- Plot the vertex and y-intercept, then use these points to sketch the parabola.

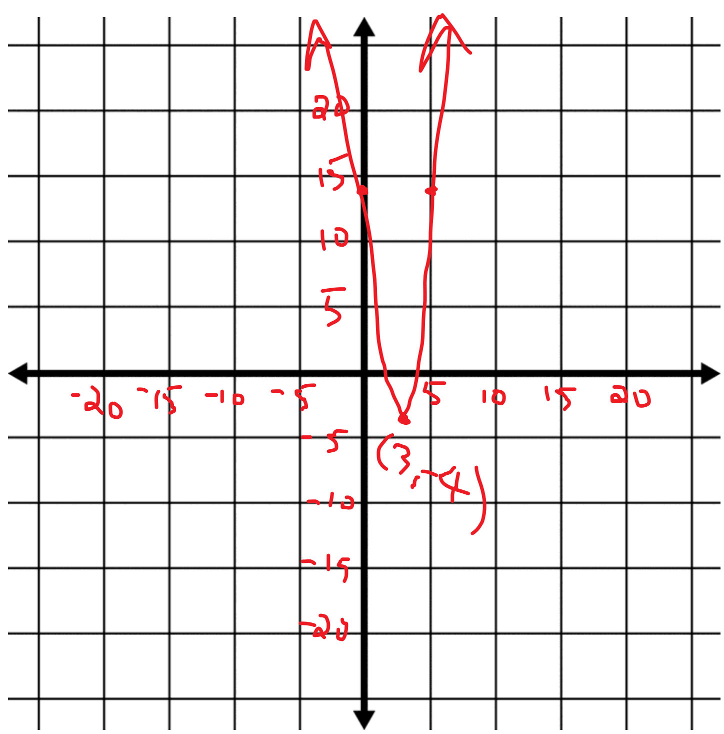
Example 3:
Sketch the graph of the quadratic function given by the equation $y = 2(x-3)^2 - 4$

Vertex: $(3, -4)$

y-int: $x = 0$

$y = 2(0-3)^2 - 4$

$y = 14$



Finding Equations in Vertex Form

You can find equations in vertex form once you have the vertex and one other point.

Example 4:

(A) Find the equation of the quadratic in vertex form given the vertex of the parabola is $(5, -2)$ and passes through the point $(-1, -4)$.

h k x y Find a!!!

$y = a(x-h)^2 + k$

BEDMAS

$-4 = a(-1-5)^2 - 2$

$-4 = a \cdot 36 - 2$

$-4 + 2 = 36a$

$\frac{-2}{36} = \frac{36a}{36}$

$a = -\frac{2}{36}$

$a = -\frac{1}{18}$

$y = -\frac{1}{18}(x-5)^2 - 2$

(B) State the Domain and Range.

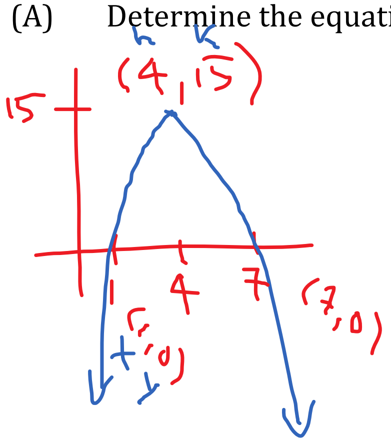
Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \leq -2, y \in R\}$

Example 5:

A parabola has x-intercepts $x = 7$ and $x = 1$. The parabola has a maximum height of 15.

(A) Determine the equation of the parabola in vertex form. Find a!!



$y = a(x-h)^2 + k$

$0 = a(1-4)^2 + 15$

$0 = 9a + 15$

$\frac{-15}{9} = \frac{9a}{9}$

$a = -\frac{5}{3}$

$y = -\frac{5}{3}(x-4)^2 + 15$

Example 6:
A parabola has vertex $(1, 7)$ and the point $(2, 9)$ also lies on the parabola. Find $\underline{\underline{a}}$.

h k x y

(A) Determine the equation in vertex from.

$y = a(x-h)^2 + k$

$9 = a(2-1)^2 + 7$

$9 = a + 7$

$9 - 7 = a$

$a = 2$

$y = 2(x-1)^2 + 7$

(B) Change the quadratic function to standard form.

$y = 2(x-1)^2 + 7$

$y = 2[(x-1)(x-1)] + 7$

$y = 2(x^2 - x - x + 1) + 7$

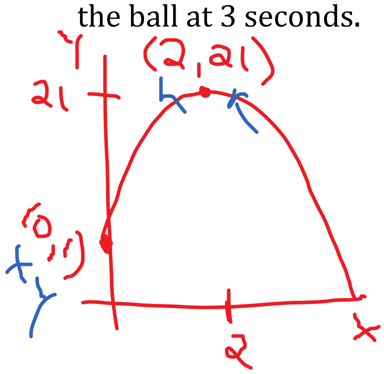
$y = 2(x^2 - 2x + 1) + 7$

$y = 2x^2 - 4x + 2 + 7$

$y = 2x^2 - 4x + 9$

Example 7:

A ball is thrown from an initial height of 1m and follows a parabolic path. After 2 seconds, the ball reaches a maximum height of 21m. Algebraically determine the quadratic function that models the path followed by the ball, and use it to determine the approximate height of the ball at 3 seconds.



$y = a(x-h)^2 + k$ Find $\underline{\underline{a}}$.

$1 = a(0-2)^2 + 21$

$1 = 4a + 21$

$1 - 21 = 4a$

$\frac{-20}{4} = \frac{4a}{4}$

$a = -5$

$y = -5(x-2)^2 + 21$

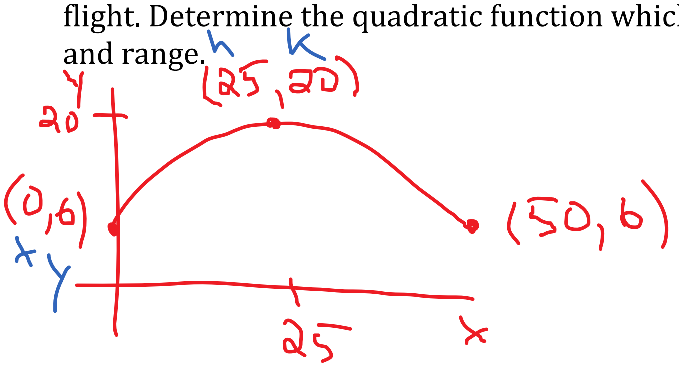
$x = 3\text{ sec}$

$y = -5(3-2)^2 + 21$

$y = 16\text{ m}$

The ball is 16m high at 3s.

Example 8:
A quarterback throws the ball from an initial height of 6 feet. It is caught by the receiver 50 feet away, at a height of 6 feet. The ball reaches a maximum height of 20 feet during its flight. Determine the quadratic function which models this situation and state the domain and range.



$$y = a(x-h)^2 + k$$
$$6 = a(0-25)^2 + 20$$
$$6 = 625a + 20$$
$$6 - 20 = 625a$$
$$\frac{-14}{625} = \frac{625a}{625}$$
$$a = -\frac{14}{625}$$
$$y = -\frac{14}{625}(x-25)^2 + 20$$

Applied math problems have real-world restrictions on the domain and range.

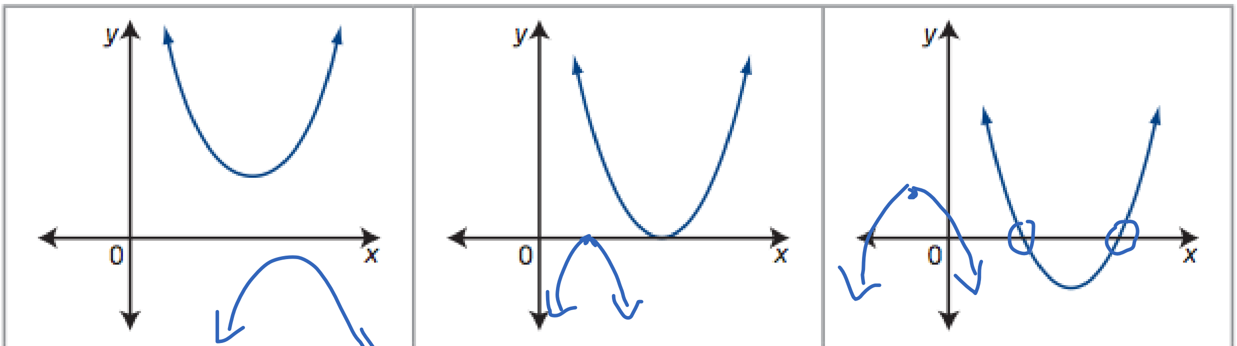
Domain: $\{x \mid 0 \le x \le 50, x \in R\}$

Range: $\{y \mid 6 \le y \le 20, y \in R\}$

The Number of Zeros

What will affect the number of x-intercepts?

- The direction of the opening of the parabola
- The location of the vertex



No real x-intercepts No real zeros No real root	One real x-intercept One real zero One real root	Two real x-intercepts Two real zeros Two distinct real roots

To determine how many zeros a quadratic function has, make a rough sketch of the graph using the vertex and direction of opening.

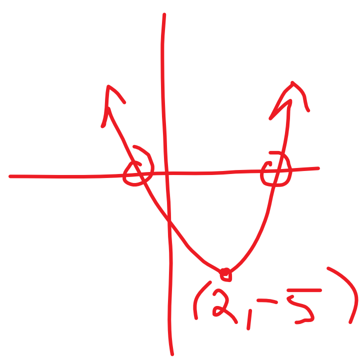
Example 9:

Determine the number of zeros for each quadratic function:

(A) $y = 2(x-2)^2 - 5$

a>0, opens up.

vertex: (2, -5)

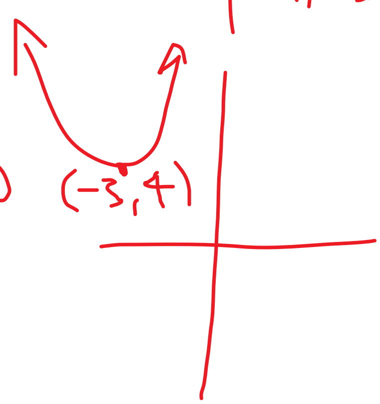


2 zeros

(B) $y = 2(x+3)^2 + 4$

a>0, opens up

vertex: (-3, 4)



no zeros.

Textbook Questions: page 363, #1, 2, 3, 4, 5, 8, 11, 12, 14, 15

page 377 #3, 4, 6, 7