Math 2201 Date:____________________

6.5 Solving Problems Using Quadratic Function Models

Maximization/Minimization (Optimization) Problems

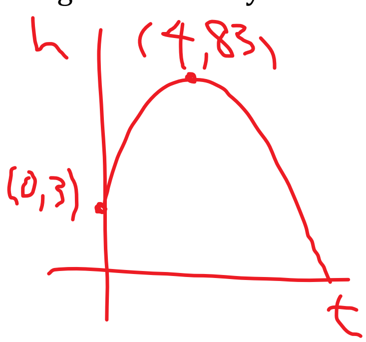
There are two main types of "Max/Min" problems.

1. The equation is given. You have to find the maximum or minimum value of the function, depending on if the parabola opens upwards or downwards. This of course is the vertex, (h, k).

2. You have to make the equation yourself using information given and then use that equation to find the maximum or minimum value. Typically you are asked to find the maximum area of some square or rectangle such as a garden or floor space of a house.

Type 1: Equation Given

Example 1:
An arrow is fired from a bow and its height, h, in metres above the ground, t seconds after being fired, is given by $h(t) = \underset{a}{-5}t^2 + \underset{b}{40}t + \underset{c}{3}$. Algebraically determine the maximum height attained by the arrow and the time taken to reach this height.

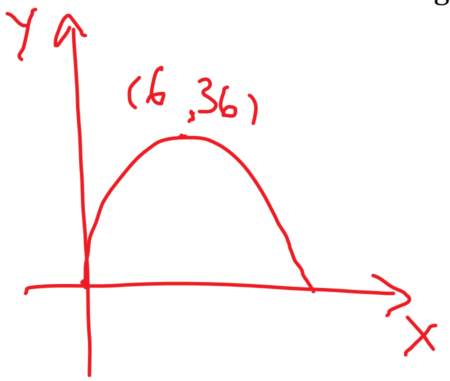


$$h = \frac{-b}{2a} = \frac{-40}{2(-5)} = \frac{-40}{-10} = 4\text{ s}$$

$$k = -5(4)^2 + 40(4) + 3 = 83\text{ m}$$

The arrow reaches a max. height of 83 m at 4 s.

Example 2:
The path of a model rocket can be described by the quadratic function $y = -x^2 + 12x$, where y represents the height of the rocket, in metres, at time x seconds after takeoff. Identify the maximum height reached by the rocket and determine the time at which the rocket reached its maximum height.



$y = \underset{a}{-}x^2 + \underset{b}{12}x + \underset{c}{0}$

$h = \frac{-b}{2a} = \frac{-12}{2(-1)} = 6s$

$k = -(6)^2 + 12(6) = 36m$

The rocket reaches a max. height of 36m at 6s.

Example 3:
A baseball is hit and follows a parabolic path described by the function $h(t) = \underset{a}{-3}t^2 + \underset{b}{12}t + \underset{c}{1}$, where t is time in seconds after the ball is hit and $h(t)$ is the height of the ball above ground in metres. Algebraically determine the maximum height reached by the ball and the time it takes the ball to reach its maximum height.

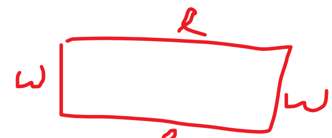
$h = \frac{-b}{2a} = \frac{-12}{2(-3)} = \frac{-12}{-6} = 2s$

$k = -3(2)^2 + 12(2) + 1 = 13m$

The baseball reaches a max. height of 13m at 2s.

Type 2: Equation Not Given

Example 4:
A rancher has 100 m of fencing available to build a rectangular corral.



(A) Write a quadratic function in standard form to represent the area of the corral.

Step 1: Draw pic.

Step 2: Perimeter Equation

① $2l + 2w = 100$

Step 3: Area Equation

② $A = l \cdot w$

Step 4: Solve ① for l or w.

$\frac{2l}{2} = \frac{-2w + 100}{2}$

$l = -w + 50$

Step 5: Sub ① into ②

$A = (-w + 50)w$

$\to A = -w^2 + 50w$

Think:

$y = -x^2 + 50x$

(B) What are the coordinates of the vertex? What does the vertex represent in this situation?

$h = \frac{-b}{2a} = \frac{-50}{2(-1)} = 25$ width: 25m

$l = -(25) + 50 = 25$ length: 25m

$A = -(25)^2 + 50(25) = 625$ Area: 625 m^2

(C) Sketch the graph for the function you determined in part (A).

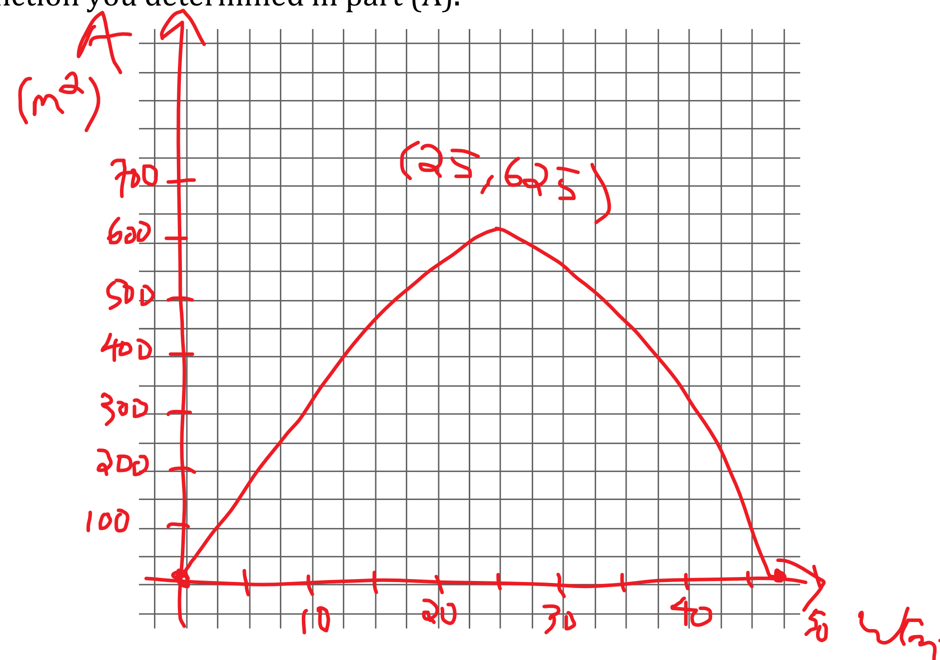
2 extremes:

$w = 0$

$l = 50$

$w = 50$

$l = 0$

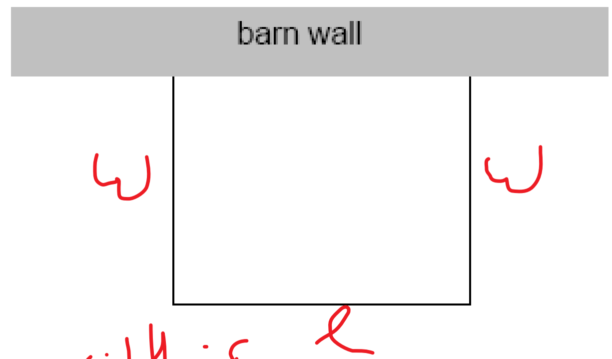


(D) Determine the domain and range.

Domain: $\{w \mid 0 \le w \le 50, w \in R\}$

Range: $\{A \mid 0 \le A \le 625, A \in R\}$

Example 5:
A farmer is constructing a pig pen and is using his barn wall as one side of the pen. If he has 32 m of fencing and wants to use it all, write the quadratic function that models the area of the pig pen, and use it to determine the maximum area of the pen.



① $l + 2w = 32$

② $A = l \cdot w$

$l = -2w + 32$

$A = (-2w + 32)w$

$A = -2w^2 + 32w$

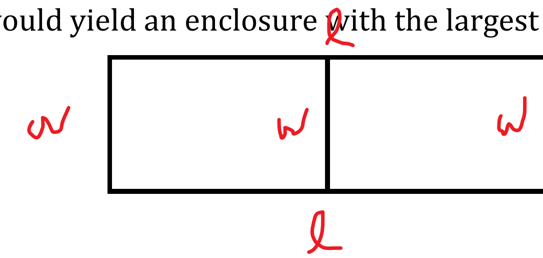
$h = -\frac{b}{2a} = \frac{-32}{2(-2)} = \frac{-32}{-4} = 8m$

width: 8m

$A = -2(8)^2 + 32(8) = 128m^2$

Max. area is $128m^2$.

Example 6:
You have 600 meters of fencing and a large field. You want to make a rectangular enclosure split into two equal lots. What dimensions would yield an enclosure with the largest area?



① $2l + 3w = 600$

② $A = l \cdot w$

$\frac{2l}{2} = \frac{-3w + 600}{2}$

$l = -1.5w + 300$

$A = (-1.5w + 300)w$

$A = -1.5w^2 + 300w$

$h = -\frac{b}{2a} = \frac{-300}{2(-1.5)} = 100m$

$w = 100m$

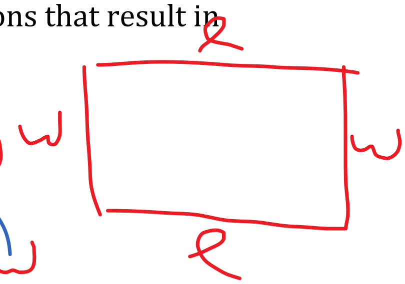
$l = -1.5(100) + 300$

$l = 150m$

$A = -1.5(100)^2 + 300(100)$

$A = 15000m^2$

Example 7:
A farmer is constructing a rectangular fenced in pen to contain rabbits. There is 120m of fencing. Determine the maximum area of the pen and the dimensions that result in maximum area.



① $2l + 2w = 120$

② $A = l \cdot w$

Solve ① for l:

$$\frac{2l}{2} = \frac{-2w + 120}{2}$$

$$l = -w + 60$$

Sub ① into ②

$$A = (-w + 60)w$$

$$A = -w^2 + 60w$$

$$h = \frac{-b}{2a} = \frac{-60}{2(-1)} = 30$$

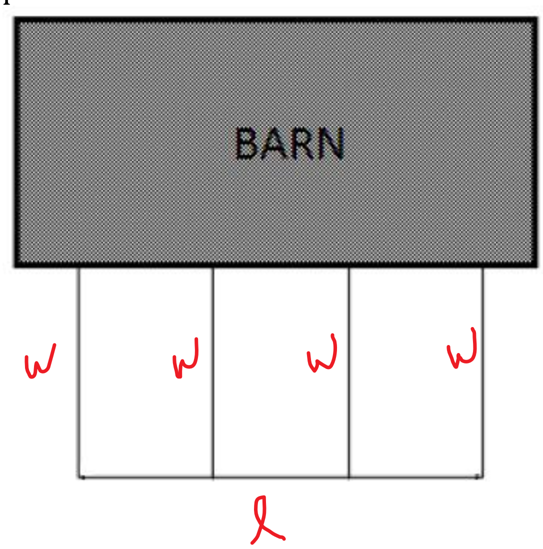
$\therefore$ $w = 30m$

$l = -(30) + 60 = 30m$

$$A = -(30)^2 + 60(30)$$

$$A = 900m^2$$

Example 8:
A barn which contains different livestock will use 240 m of fencing to construct three equal rectangular regions. There is no fencing along the side of the barn so livestock can move in and out of the barn. Determine the maximum area of the pen.



① $l + 4w = 240$

② $A = l \cdot w$

$$l = -4w + 240$$

$$A = (-4w + 240)w$$

$$A = -4w^2 + 240w$$

$$h = \frac{-b}{2a} = \frac{-240}{2(-4)} = 30$$

$w = 30m$

$l = -4(30) + 240 = 120m$

$$A = -4(30)^2 + 240(30)$$

$$A = 3600m^2$$