**Math 2201** Date:__________________

# 7.1 Solving Quadratic Equations by Graphing

In Mathematics 1201, students factored difference of squares, perfect square trinomials and polynomials of the form $x^2 + bx + c$ and $ax^2 + bx + c$.

In the previous unit, students were introduced to quadratic functions expressed in standard form, factored form and vertex form. Graphs were sketched using characteristics such as $x$ and $y$-intercepts, vertex, axis of symmetry and domain and range. Students also expressed quadratic functions in factored form relating the $x$-intercepts of the parabola to the factors of the function.

In this unit, we will solve quadratic equations by graphing, factoring and using the quadratic formula. Choosing the most efficient method is important when solving the quadratic equation.

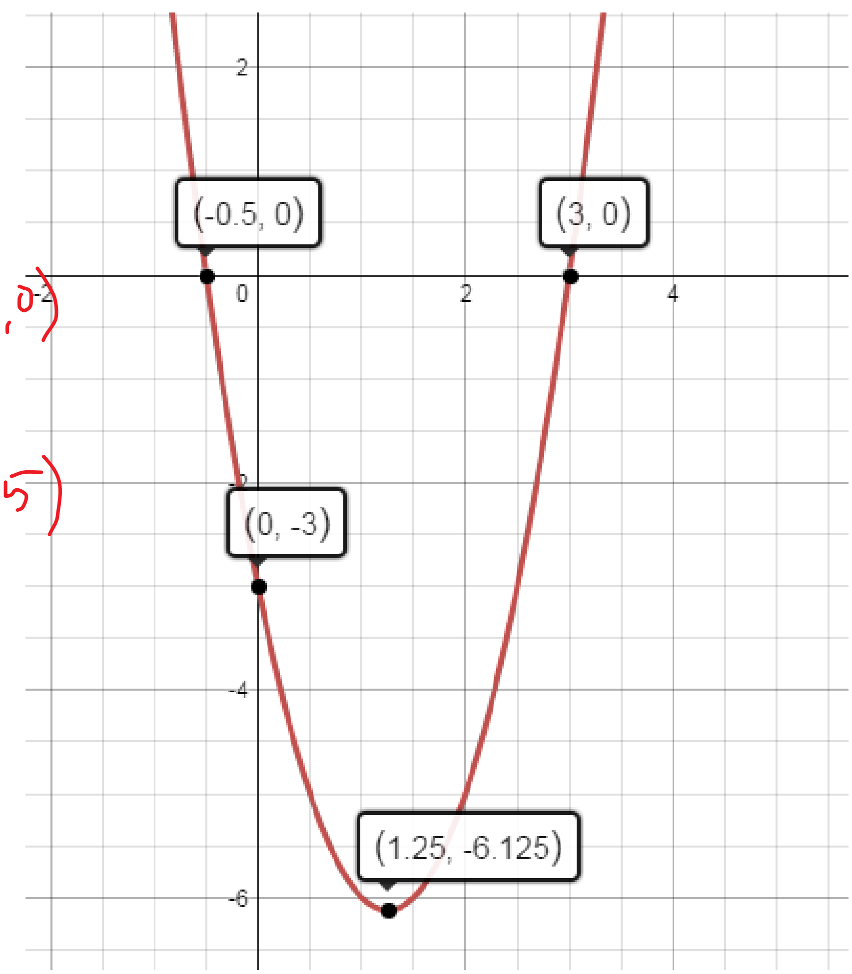
**Solving Quadratic Equations Using Technology**
Using https://www.desmos.com/calculator, or a TI-80 series graphing calculator we can graph any quadratic function and solve the corresponding equation by simply identifying the zeros or $x$-intercepts.

**Example 1:**
Using technology, find the:

- $x$-intercepts
- $y$-intercept
- vertex

for $y = 2x^2 - 5x - 3$.

x-ints: (-0.5, 0), (3, 0)
y-int: (0, -3)
vertex: (1.25, -6.125)



**Example 2:**

(A) Using technology, find the:

- $x$-intercepts
- $y$-intercept
- vertex

for $y = 2x^2 + 4x - 3$.

x-ints: (-2.581, 0), (0.581, 0)
y-int: (0, -3)
vertex: (-1, -5)

(B) Using technology, find the:

- $x$-intercepts
- $y$-intercept
- vertex

for $y = -3x^2 - 6x + 7$.

x-ints: (-2.826, 0), (0.826, 0)
y-int: (0, 7)
vertex: (-1, 10)

It is important to realize that not all $x$-intercepts are integers. Whether you draw a graph by hand or use technology, you may have to approximate the $x$-intercepts using the graph as a visual representation.

**Example 3:**

The flight time for a long distance water ski jumper depends on the initial velocity of the jump and the angle of the ramp. For one particular jump, the ramp has a vertical height of 5.0 m above water level. The height of the ski jumper in flight, $h(t)$, in meters, over time, $t$, in seconds, can be modelled by the following function:

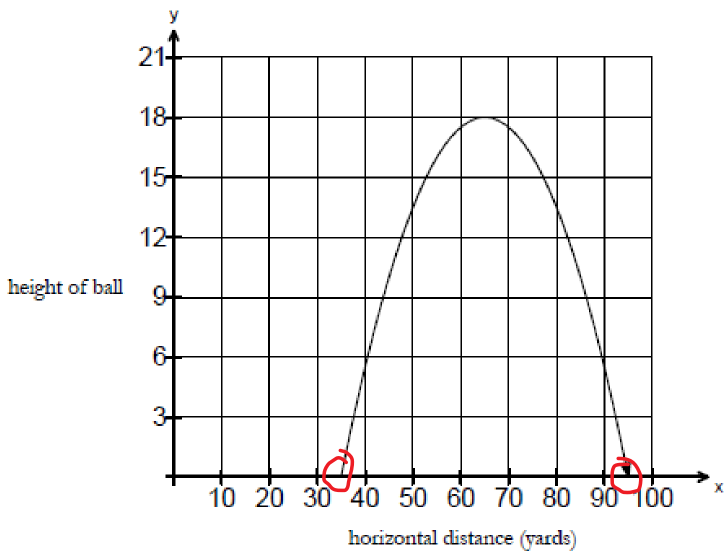
$$h(t) = 5.0 + 24.46t - 4.9t^2 \rightarrow y = 5.0 + 24.46x - 4.9x^2$$

How long does this water ski jumper hold his flight pose?

Using Desmos, water skier is in the air for 5.189 s.

**Example 3:**
The path of a football at one particular kick-off can be modelled using the function $h(d) = -0.02d^2 + 2.6d - 66.5$ where $h$ is the height of the ball above the ground, in yards, and $d$ is the horizontal distance from the kicking team's goal line, in yards.

The graph that models this quadratic function is shown below:




(A) What are the $x$-intercepts? What do they mean in the context of the problem? Why are there two $x$-intercepts?

- (35,0) and (95,0)
- The ball left the ground at 35yds and landed at 95yds.
- The ball starts on the ground and ends on the ground.

(B) Why is the $y$-intercept not labelled on this graph?

It would be many yards below the ground.

(C) What horizontal distance does the ball travel before it hits the ground?

95 − 35 = 60yds.

**Graphing Quadratics Using The Vertex and a Table of Values**

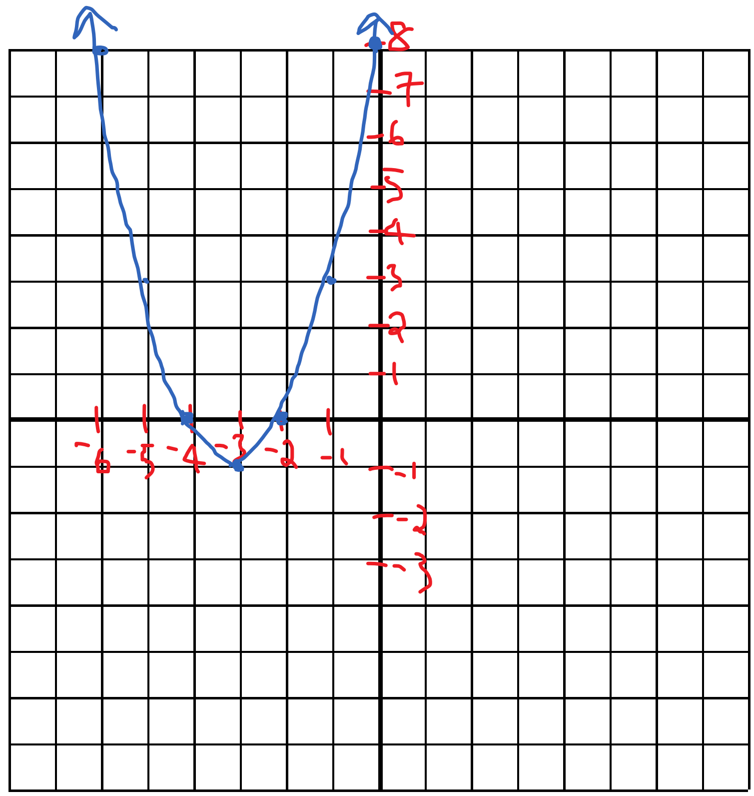
You can use the vertex formula discussed in the previous unit and a table of values to draw a graph.

**Example 4:**

Graph $y = x^2 + 6x + 8$ by finding the vertex and a table of values.

y-int: (0,8)

| $x$ | $y$ |
|---|---|
| -5 | 3 |
| -4 | 0 |
| -3 | -1 |
| -2 | 0 |
| -1 | 3 |



$h = \frac{-b}{2a} = \frac{-6}{2(1)} = -3$

$k = (-3)^2 + 6(-3) + 8 = -1$

Vertex: (-3,-1)

$y(-5) = (-5)^2 + 6(-5) + 8 = 3$ $\quad y(-2) = (-2)^2 + 6(-2) + 8 = 0$

$y(-4) = (-4)^2 + 6(-4) + 8 = 0$ $\quad y(-1) = (-1)^2 + 6(-1) + 8 = 3$

*You only need to find the two y-values to the left. (Symmetry.)

(A) Are the the $x$-intercepts exact values? Can they be clearly marked on the graph?

Yes. Yes.

(B) Can the $x$-intercepts be identified from the table of values?

Yes. The y-values are 0 at the x-intercepts.

(C) How many $x$-intercepts exist? Is this always true?

2. No. There can be 0, 1 or 2 x-intercepts, depending on where the vertex is and if $a>0$ or $a<0$.

(D) What is the $y$-intercept? Can this value be determined from the table of values?

(0, 8). It depends on if the y-intercept is one of the 5 points.

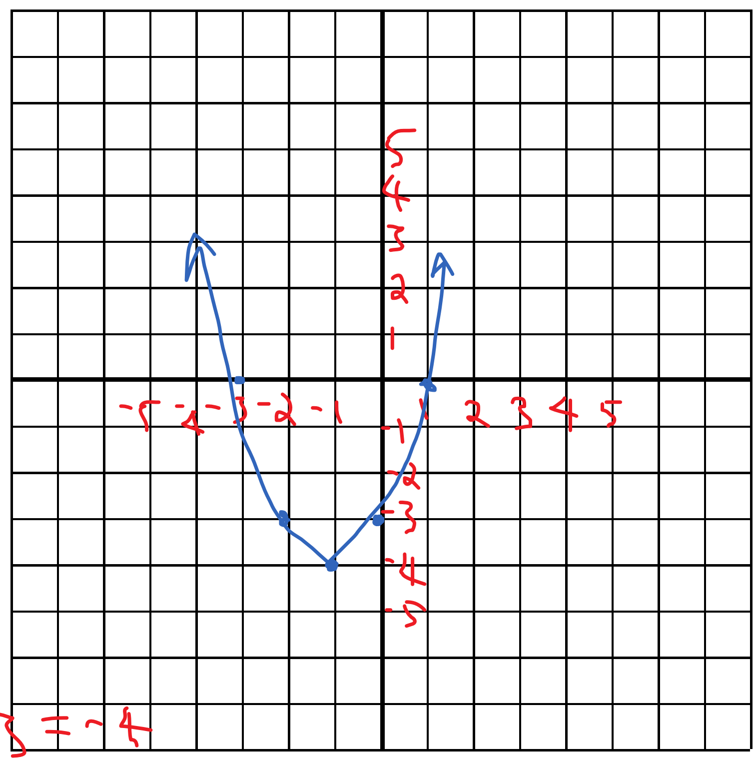
(E) How can the $x$-intercepts be determined if they are not exact?

Estimation.

**Example 5:**
Graph $y = x^2 + 2x - 3$ by finding the vertex and a table of values.

y-int: $(0, -3)$

| $x$ | $y$ |
|---|---|
| $-3$ | $0$ |
| $-2$ | $-3$ |
| $-1$ | $-4$ |
| $0$ | $-3$ |
| $1$ | $0$ |




$h = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

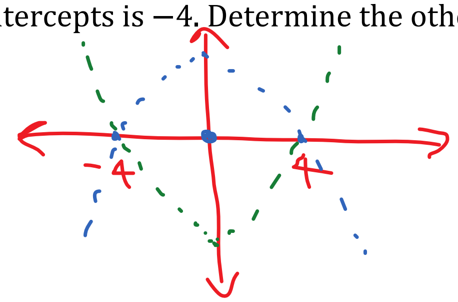
$k = (-1)^2 + 2(-1) - 3 = -4$

vertex $(-1, -4)$

$y(-3) = (-3)^2 + 2(-3) - 3 = 0$

$y(-2) = (-2)^2 + 2(-2) - 3 = -3$

**Example 6:**
The equation of the axis of symmetry of a quadratic function is $x = 0$ and one of the $x$-intercepts is $-4$. Determine the other $x$-intercept. Explain using a diagram.

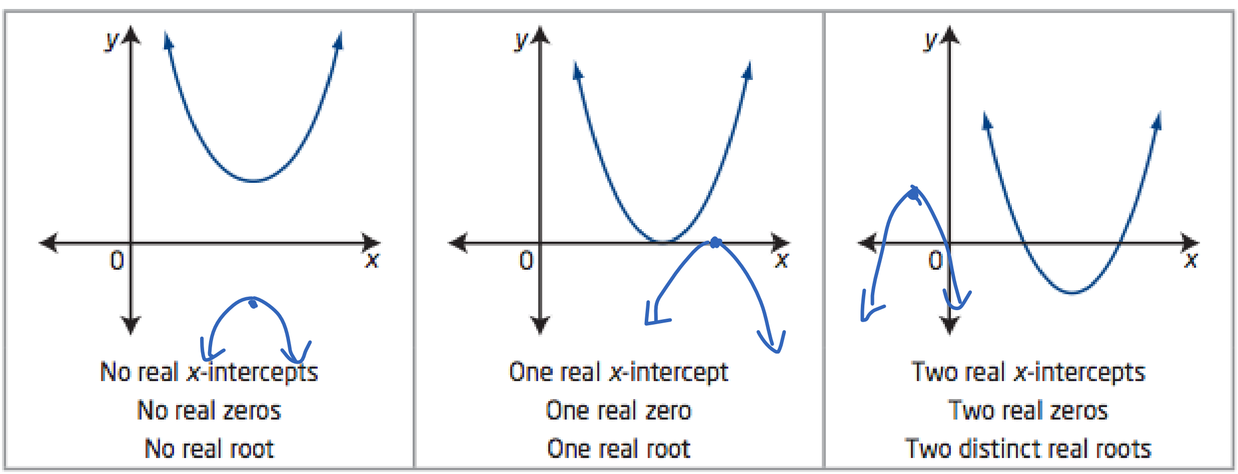



other x-intercept must be $x = 4$ because of symmetry.

**Determining the Number of $x$-intercepts**
In the previous unit, you studied the graphs of quadratic functions. A connection was made between the coefficient of the quadratic term and the direction of the opening of a quadratic. You also represented the maximum or minimum point of the graph as the vertex.

What will affect the number of $x$-intercepts?

- The direction of the opening of the parabola
- The location of the vertex




To determine how many zeros a quadratic function has, make a rough sketch of the graph using the vertex and direction of opening.

**Example 7:**
Complete the following table:

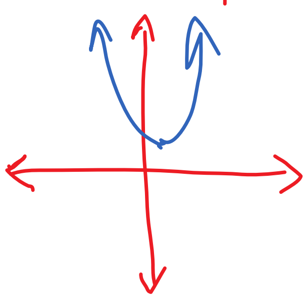
| Vertex | Direction of Opening | Graph | Number of $x$-intercepts |
|---|---|---|---|
| (2,-3) | upward | | 2 |
| (5,0) | downward | | 1 |
| (4,-2) | downward | | 0 |

**Example 8:**
Determine the number of roots/zeros/$x$-intercepts for the quadratic represented by the equation $y = 5x^2 - 3x + 4$.

$$h = \frac{-b}{2a} = \frac{-(-3)}{2(5)} = \frac{3}{10} = 0.3$$

$$k = 5(0.3)^2 - 3(0.3) + 4 = 3.55$$

Vertex $(0.3, 3.55)$   $a > 0$



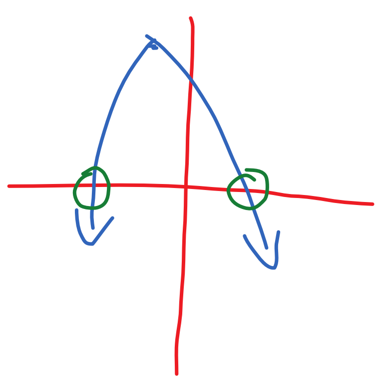
$\therefore$ No $x$-intercepts.

**Example 9:**
Determine the number of roots/zeros/$x$-intercepts for the quadratic represented by the equation $y = -2x^2 - 8x + 7$.

$$h = \frac{-b}{2a} = \frac{-(-8)}{2(-2)} = \frac{8}{-4} = -2$$

$$k = -2(-2)^2 - 8(-2) + 7 = 15$$

Vertex $(-2, 15)$   $a < 0$,

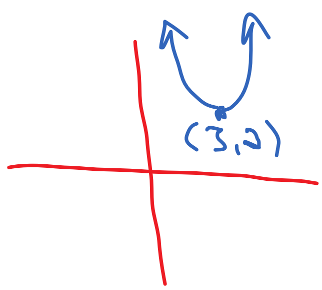


$\therefore$ 2 $x$-intercepts

**Example 10:**
Are the following statements are true or false. If a statement is false provide a counter-example to illustrate why the statement is false.

(A) Some quadratic equations have no real solutions.

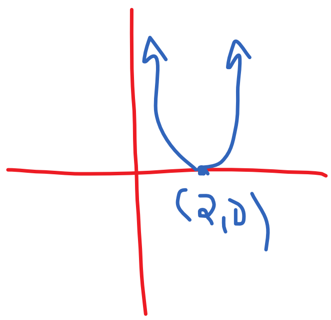
True.




No solutions.

(B) A quadratic equation will always cross the $x$-axis in two distinct places.

False.




Crosses in one distinct place.

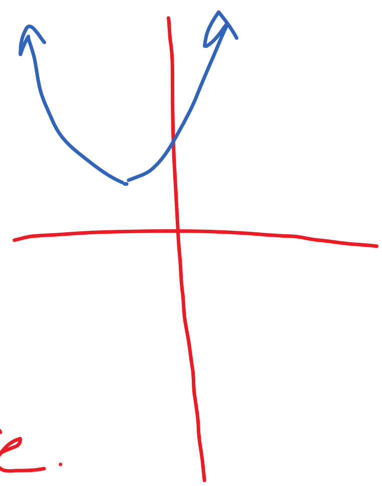
(C) The graph of $y = x^2 + 2x + 2$ intersects the $x$-axis twice.

$h = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

$k = (-1)^2 + 2(-1) + 2 = 1$

Vertex $(-1, 1)$

$a > 1$



False.

**Textbook Questions:** page 402 - 403 #1, 2, 3, 4, 5, 7, 9