Math 2201 **Date:**_______________

7.2 Solving Quadratic Equations by Factoring

Once a quadratic equation has been factored, students will use the **zero product property** to determine the roots. For example, $5x^2 + 14x - 3 = 0$ in factored form is:

$$(5x - 1)(x + 3) = 0$$

this means either $5x - 1 = 0$ or $x + 3 = 0$.

Solving each factor we get:

$5x - 1 = 0$

$\frac{5x}{5} = \frac{1}{5}$

$x = 1/5$

$(\frac{1}{5}, 0)$

$x + 3 = 0$

$x = -3$

$(-3, 0)$

Example 1:
The factors of the equation $x^2 + 5x + 6 = 0$ are $(x + 2)(x + 3)$. Use the zero product property to find the roots.

$x + 2 = 0$

$x = -2$

$(-2, 0)$

$x + 3 = 0$

$x = -3$

$(-3, 0)$

Review of Factoring

$y = ax^2 + bx + c$

$a = 1$

Type I Trinomial: $x^2 + bx + c$

We simply need two numbers that multiply to give us c and add to give us b. We usually refer to this as the **Product/Sum** method.

$x^2 + x - 6 = 0$

$(x-2)(x+3) = 0$

$x - 2 = 0, \; x + 3 = 0$

$x = 2, \; x = -3$

$\frac{6}{1,6 \quad 2,3}$

$(2,0), (-3,0)$

What relationship did you notice between the factored form of a quadratic equations and its roots? They have opposite signs.

Example 2:

Factor each quadratic equation:

(A) $x^2 + 7x + 12 = 0$

$\frac{12}{1,12 \quad 2,6 \quad 3,4}$

$(x+3)(x+4) = 0$

$x + 3 = 0, \; x + 4 = 0$

$x = -3, \; x = -4$

$(-3,0), (-4,0)$

(B) $x^2 + 3x - 10 = 0$

$\frac{10}{1,10 \quad 2,5}$

$(x-2)(x+5) = 0$

$x - 2 = 0, \; x + 5 = 0$

$x = 2, \; x = -5$

$(2,0), (-5,0)$

(C) $x^2 - 4x + 21 = 0$

$\frac{21}{1,21 \quad 3,7}$

$(x+3)(x-7) = 0$

$x + 3 = 0, \; x - 7 = 0$

$x = -3, \; x = 7$

$(-3,0), (7,0)$

(D) $x^2 - 8x + 12 = 0$

$\frac{12}{1,12 \quad 2,6 \quad 3,4}$

$(x-2)(x-6) = 0$

$x - 2 = 0, \; x - 6 = 0$

$x = 2, \; x = 6$

Type II Trinomial: $ax^2 + bx + c$

We typically use **decomposition** to factor this type of quadratic. The process involves multiplying $a \cdot c$ and choosing two factors of this product that can be added to make b.

$$\underset{a}{2x^2} - \underset{b}{9x} - \underset{c}{5} = 0$$

10

1, 10

2, 5

$(2x^2 - 10x)(+x - 5) = 0$

$2x(x - 5) + 1(x - 5) = 0$

$(x - 5)(2x + 1) = 0$

$x - 5 = 0, \quad 2x + 1 = 0$

$x = 5, \quad 2x + 1 = 0$

$\frac{2x}{2} = -\frac{1}{2}$

$(5, 0)$

$x = -1/2$

$(-\frac{1}{2}, 0)$

Example 3:

Factor each Type II Trinomial using decomposition:

(A) $5x^2 + 14x - 3 = 0$

15
1, 15
3, 5

$(5x^2 - x)(+15x - 3) = 0$

$x(5x - 1) + 3(5x - 1) = 0$

$(5x - 1)(x + 3) = 0$

$5x - 1 = 0$, $x + 3 = 0$

$5x = 1$, $x = -3$

$x = \frac{1}{5}$

$(\frac{1}{5}, 0)$ $(-3, 0)$

(B) $2x^2 - 5x - 3 = 0$

6
1, 6
2, 3

$(2x^2 - 6x)(+x - 3) = 0$

$2x(x - 3) + 1(x - 3) = 0$

$(x - 3)(2x + 1) = 0$

$x - 3 = 0$, $2x + 1 = 0$

$x = 3$, $2x = -1$

$(3, 0)$ $x = -\frac{1}{2}$

$(-\frac{1}{2}, 0)$

(C) $6x^2 - x - 15 = 0$

90
1, 90
2, 45
3, 30
5, 18
6, 15
9, 10

$(6x^2 - 10x)(+9x - 15) = 0$

$2x(3x - 5) + 3(3x - 5) = 0$

$(3x - 5)(2x + 3) = 0$

$3x - 5 = 0$, $2x + 3 = 0$

$3x = 5$, $2x = -3$

$x = \frac{5}{3}$, $x = -\frac{3}{2}$

$(\frac{5}{3}, 0)$, $(\frac{-3}{2}, 0)$

(C) $3x^2 - 4x - 7 = 0$

21
1, 21
3, 7

$3x^2 + 3x - 7x - 7 = 0$

$3x(x + 1) - 7(x + 1) = 0$

$(x + 1)(3x - 7) = 0$

$x + 1 = 0$, $3x - 7 = 0$

$x = -1$, $3x = 7$

$x = \frac{7}{3}$

$(-1, 0)$, $(\frac{7}{3}, 0)$

Example 4:

Answer the following:

(A) Find the zeros of $f(x) = 2x^2 + 5x - 7$

$2x^2 + 5x - 7 = 0$

14

$1, 14$

$2, 7$

$2x^2 - 2x + 7x - 7 = 0$

$2x(x-1) + 7(x-1) = 0$

$(x-1)(2x+7) = 0$

$x - 1 = 0$, $2x + 7 = 0$

$x = 1$, $2x = -7$

$(1, 0)$, $x = -\frac{7}{2}$

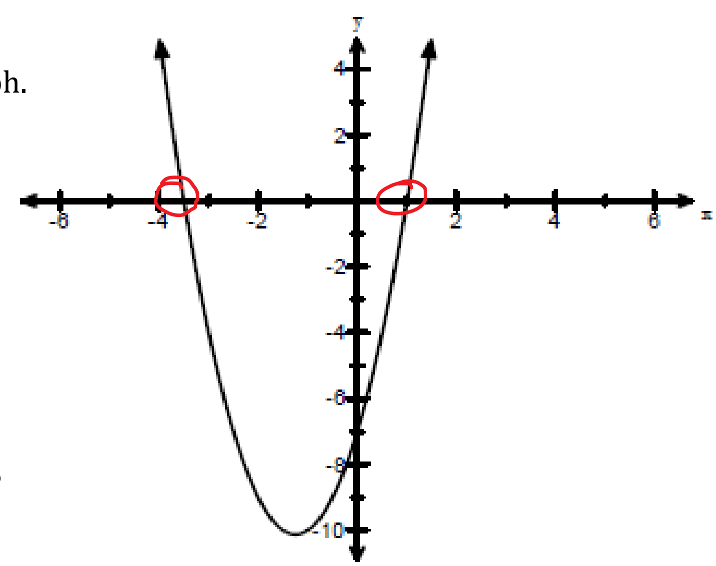
$(-\frac{7}{2}, 0)$

(B) Identify the x-intercepts of the graph.

$(1,0)$, $(-3.5, 0)$

or

$(-\frac{7}{2}, 0)$



(C) Find the roots of $2x^2 + 5x - 7 = 0$.

$x = 1$, $x = -\frac{7}{2}$

(D) What do you notice about the answers to the above questions?

The numbers are the same.

Difference of Squares:
These are quadratics of the form $a^2x^2 - b^2$. If you can recognize this pattern, factoring becomes quite easy.

$$x^2 - 4 = 0$$

$(x+2)(x-2)=0$

$x+2=0$, $x-2=0$

$x=-2$, $x=2$

$(-2,0)$, $(2,0)$

Example 5:
Factor each quadratic equation:

(A) $x^2 - 49 = 0$

$(x+7)(x-7)=0$

$x+7=0$, $x-7=0$

$x=-7$, $x=7$

$(-7,0)$, $(7,0)$

(B) $4x^2 - 25 = 0$

$(2x+5)(2x-5)=0$

$2x+5=0$, $2x-5=0$

$\frac{2x}{2}=-\frac{5}{2}$, $\frac{2x}{2}=\frac{5}{2}$

$x=-\frac{5}{2}$, $x=\frac{5}{2}$

$(-\frac{5}{2},0)$, $(\frac{5}{2},0)$

(C) $9x^2 - 16 = 0$

$(3x+4)(3x-4)=0$

$3x+4=0$, $3x-4=0$

$\frac{3x}{3}=-\frac{4}{3}$, $\frac{3x}{3}=\frac{4}{3}$

$x=-\frac{4}{3}$, $x=\frac{4}{3}$

$(-\frac{4}{3},0)$, $(\frac{4}{3},0)$

(D) $36x^2 - 81 = 0$

$(6x+9)(6x-9)=0$

$6x+9=0$, $6x-9=0$

$\frac{6x}{6}=-\frac{9}{6}$, $\frac{6x}{6}=\frac{9}{6}$

$x=-\frac{3}{2}$, $x=\frac{3}{2}$

$(-\frac{3}{2},0)$, $(\frac{3}{2},0)$

There is an alternate method to solving differences of squares. This is especially useful if a quadratic is a binomial but one term is not a perfect square. This method uses the **square root property.**

$$x^2 - 4 = 0$$

$(2)^2 = 4$

$\therefore \sqrt{4} = 2$

principle root

$x^2 = 4$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$(2, 0), (-2, 0)$

$* \ (-2)^2 = 4$

$\therefore \sqrt{4} = -2$

secondary root

Now let's try for a binomial that's not a difference of squares:

$$x^2 - 12 = 0$$

$x^2 - 12 = 0$

$x^2 = 12$

$\sqrt{x^2} = \sqrt{12}$

$x = \pm\sqrt{12}$

$x = \pm\sqrt{4}\sqrt{3}$

$x = \pm 2\sqrt{3}$

$(2\sqrt{3}, 0), (-2\sqrt{3}, 0)$

Example 6:
Factor each quadratic equation:

(A) $x^2 - 6 = 0$

$x^2 = 6$

$\sqrt{x^2} = \sqrt{6}$

$x = \pm\sqrt{6}$

$(\sqrt{6}, 0), (-\sqrt{6}, 0)$

(B) $x^2 - 8 = 0$

$x^2 = 8$

$\sqrt{x^2} = \sqrt{8}$

$x = \pm\sqrt{8}$

$x = \pm\sqrt{4}\sqrt{2}$

$x = \pm 2\sqrt{2}$

$(2\sqrt{2}, 0), (-2\sqrt{2}, 0)$

Greatest Common Factor

Sometimes it may be beneficial to factor the GCF out of a quadratic expression first. This is often the method to use when there is a missing b or c term from $ax^2 + bc + c = 0$. It's also important to look for other types of quadratic equations imbedded inside a quadratic that can have the GCF factored first.

$$x^2 + 5x = 0$$

$x(x+5)=0$

$x=0$, $x+5=0$

$(0,0)$ $\quad x=-5$

$(-5,0)$

Example 7:

Factor each quadratic equation:

(A) $x^2 - 10x = 0$

$x(x-10)=0$

$x=0$, $x-10=0$

$x=10$

$(0,0)$, $(10,0)$

(B) $4x^2 - 8x = 0$

$4x(x-2)=0$

$\frac{4x}{4}=\frac{0}{4}$, $x-2=0$

$x=0$, $x=2$

$(0,0)$, $(2,0)$

(C) $9x^2 - 18x = 0$

$9x(x-2)=0$

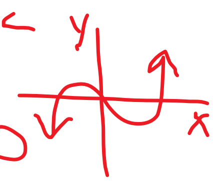
$\frac{9x}{9}=\frac{0}{9}$, $x-2=0$

$x=0$, $x=2$

$(0,0)$, $(2,0)$

(D) $x^3 - 49x = 0$

cubic



$x(x^2-49)=0$

$x(x+7)(x-7)=0$

$x=0$, $x+7=0$, $x-7=0$

$x=-7$, $x=7$

$(0,0)$, $(-7,0)$, $(7,0)$

Solving Quadratic Equations in Non-Standard Form

A quadratic equation may be given in a format which requires simplification to standard form before being able to determine the roots. For example:

$$2x^2 + 2x - 12 = 3(x - 2)$$

$$2x^2+2x-12=3x-6$$

$$2x^2+2x-12-3x+6=0$$

$$2x^2-x-6=0$$

12

1, 12

2, 6

3, 4

$$(2x^2-4x)(+3x-6)=0$$

$$2x(x-2)+3(x-2)=0$$

$$(x-2)(2x+3)=0$$

$$x-2=0, \quad 2x+3=0$$

$$x=2, \quad \frac{2x}{2}=\frac{-3}{2}$$

$$x=-3/2$$

$$(2,0), \left(-\frac{3}{2}, 0\right)$$

Example 8:
Simplify each expression, then factor the resulting quadratic: equation:

(A) $2x^2 + 10x + 2 = 5(1 + 3x)$

$2x^2+10x+2=5+15x$

$2x^2+10x-15x+2-5=0$

$2x^2-5x-3=0$

$\frac{6}{1,6}$
$2,3$

$2x^2-6x+x-3=0$

$2x(x-3)+(x-3)=0$

$(x-3)(2x+1)=0$

$x-3=0$, $2x+1=0$

$x=3$, $\frac{2x}{2}=\frac{-1}{2}$

$x=-\frac{1}{2}$

$(3,0),\left(-\frac{1}{2},0\right)$

(B) $2(5 + 3x^2) = 7x^2 + x - 2$

$10+6x^2=7x^2+x-2$

$0=7x^2-6x^2+x-2-10$

$x^2+x-12=0$

$(x-3)(x+4)=0$

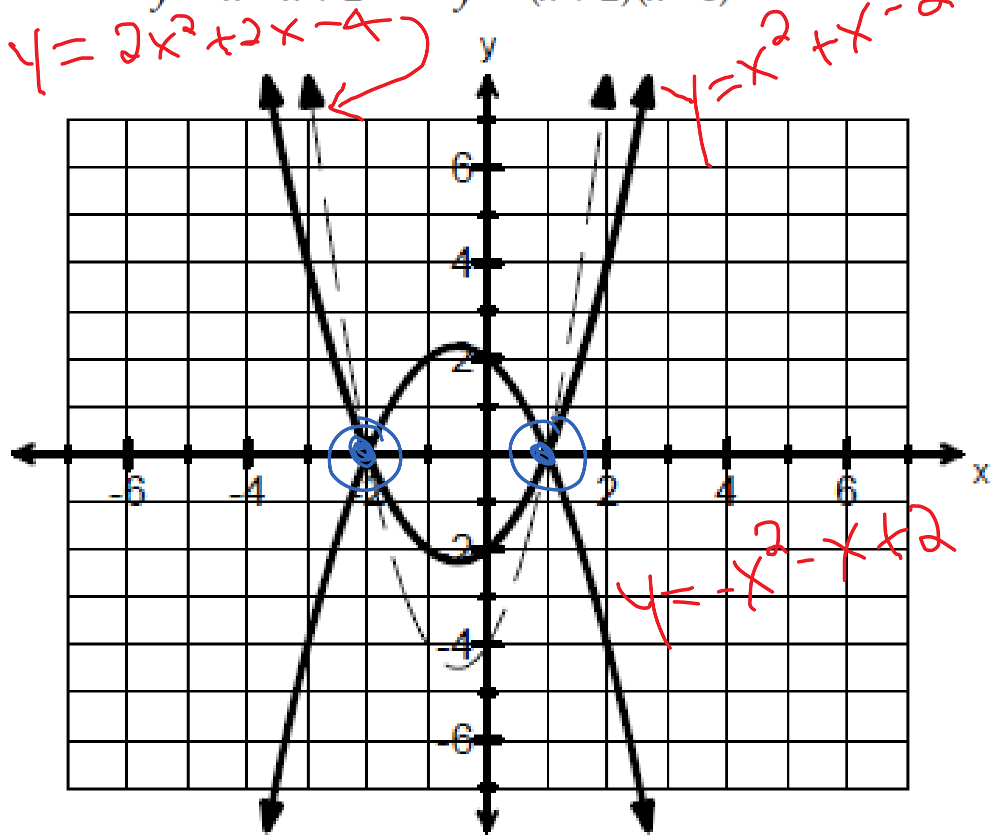
$x-3=0$, $x+4=0$

$x=3$, $x=-4$

$(3,0),(-4,0)$

In the previous unit, you had to determine a unique parabola given the x-intercepts and another point. Now you will write the quadratic equation based only on the x-intercepts. Manipulating the parameter a does not influence the value of the x-intercepts. Therefore, multiple quadratic equations exist. Consider the following equations resulting in x-intercepts -2 and 1.

$$y = x^2 + x - 2 \qquad y = (x + 2)(x - 1)$$
$$y = 2x^2 + 2x - 4 \qquad y = 2(x + 2)(x - 1)$$
$$y = -x^2 - x + 2 \qquad y = -(x + 2)(x - 1)$$



Example 9:
Write two different quadratic equations in standard form having roots -3 and 3.

$x = -3, \; x = 3$

$(x+3) = 0, \; (x-3) = 0$

$y = (x+3)(x-3)$

$y = x^2 \cancel{-3x} \cancel{+3x} - 9$

$y = x^2 - 9$

$y = 5(x+3)(x-3)$

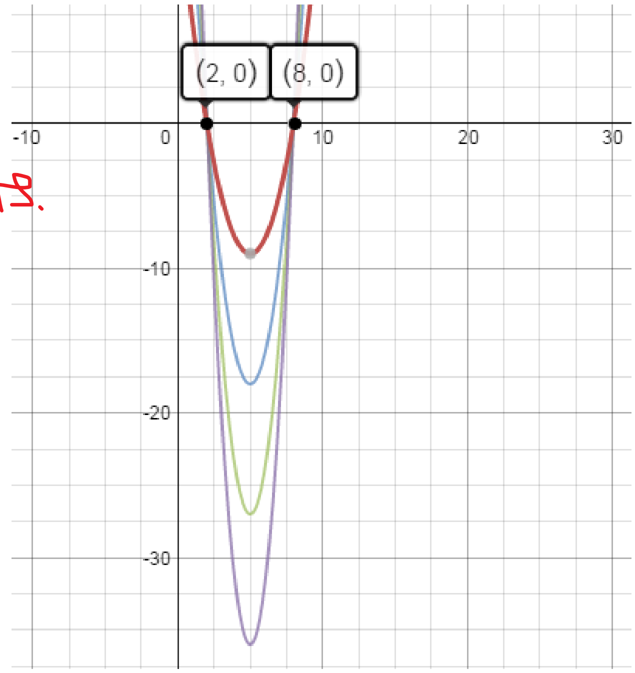
$y = 5(x^2 - 9)$

$y = 5x^2 - 45$

Example 10:
Using Desmos, graph $y = a(x-2)(x-8)$ for $a = 1, 2, 3$ and 4.

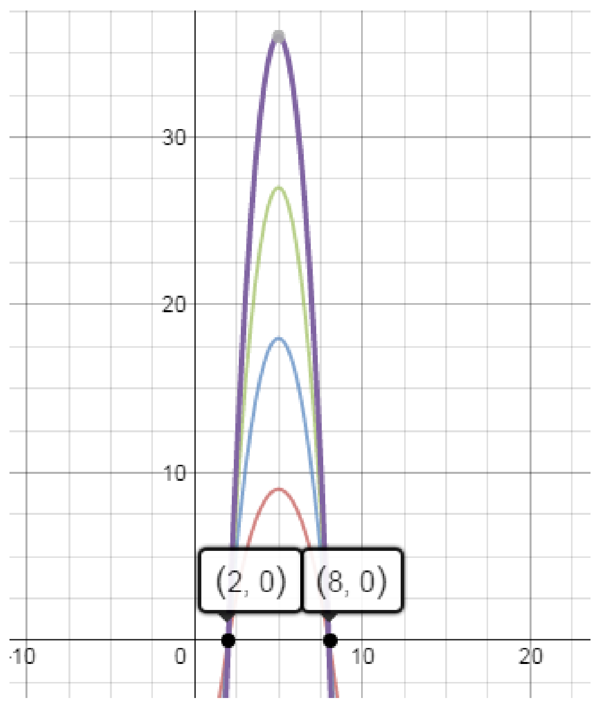
(A) What do these parabolas have in common?

They all have the same x-intercepts.



(B) Does the same property hold when a is negative? Explain.

Yes. Note the direction of opening changes.



pTextbook Questions: page 411 - 413 #1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14