Math 2201 Date:__________________

7.4 Solving Problems Using Quadratic Equations

Quadratic equations can be used to model a variety of situations such as projectile motion and geometry-based word problems.

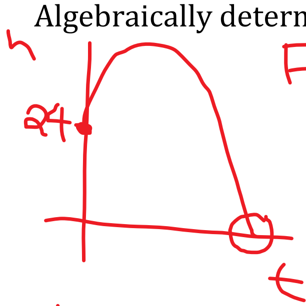
When solving a given quadratic equation using the quadratic formula, we give the answers in exact form, in other words, no decimals. In application problems however, it is acceptable to write the answers in decimal form.

It is important to recognize that the context of the problem dictates inadmissible roots. Different scenarios can produce inadmissible roots. For example, time, height and length, would not make sense if they have a negative numerical value. However, temperature would make sense to have both negative and positive solutions.

Just like with Quadratic Functions, there are two main types of applications associated with Quadratic Equations. Type I, the equation is provided and for Type II, the object is to come up with the equation yourself.

Type I – Equation Provided

Example 1:
A golf ball is hit from the top of a tower that is 24 m high. The ball follows a parabolic path defined by the function, $h(t) = -5t^2 + 14t + 24$, where t represents the time in seconds since the ball was hit, and $h(t)$ represents the height of the ball above the ground in metres. Algebraically determine how long the ball is in the air.



Find roots. * Positive a value.

$0 = -5t^2 + 14t + 24$

$5t^2 - 14t - 24 = 0$

$t = \dfrac{-(-14) \pm \sqrt{(-14)^2 - 4(5)(-24)}}{2(5)}$

$t = \dfrac{14 \pm \sqrt{196 + 480}}{10}$

$t = \dfrac{14 \pm \sqrt{676}}{10} \rightarrow t = \dfrac{14 \pm 26}{10}$

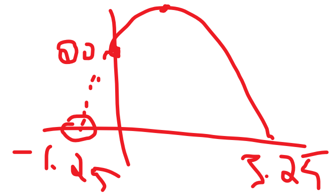
$t = \dfrac{14+26}{10}, \quad t = \dfrac{14-26}{10}$

$t = \dfrac{40}{10}, \quad t = -\dfrac{12}{10}$

$\boxed{t = 4}$ $\quad \cancel{t = -1.2}$

The ball hits the ground at 4s.

Example 2:
A diver's path when diving off a platform is given by $d = -5t^2 + 10t + 20$, where d is the distance above the water, in feet, and t is the time from the beginning of the dive, in seconds.



(A) How high is the diving platform?

20 ft

(B) After how many seconds is the diver 25 feet above the water?

$$25 = -5t^2 + 10t + 20$$
$$5t^2 - 10t - 20 + 25 = 0$$
$$\frac{5t^2 - 10t + 5}{5} = \frac{0}{5}$$
$$t^2 - 2t + 1 = 0$$
$$(t-1)(t-1) = 0$$
$$t = 1 \leftarrow \text{one solution, vertex}$$

Diver is 25 ft above the water at 1s.

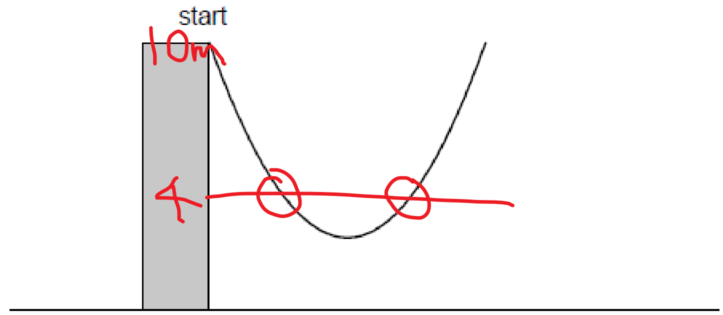
(C) When does the diver enter the water?

$$0 = -5t^2 + 10t + 20$$
$$\frac{5t^2 - 10t - 20}{5} = \frac{0}{5}$$
$$t^2 - 2t - 4 = 0$$
$$t = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$
$$t = \frac{2 \pm \sqrt{20}}{2}$$
$$t = \frac{2 \pm 4.5}{2}$$

$$\rightarrow t = \frac{2+4.5}{2}, \quad t = \frac{2-4.5}{2}$$
$$t = \frac{6.5}{2}, \quad t = \frac{-2.5}{2}$$
$$\boxed{t = 3.25}, \quad \cancel{t = -1.25}$$

Diver hits the water at 3.25s.

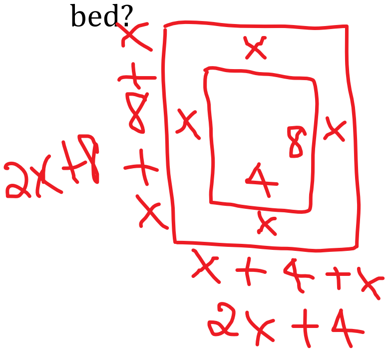
Example 3:
A blue jay swoops down from the top of a 10m tree to chase away some house sparrows. The blue jay's path follows a parabolic path given by the function $h(t) = 2t^2 - 8t + 10$ where t is time in seconds and $h(t)$ is height in metres. Algebraically, determine the times when the blue jay reaches a height of 4m.



$$4 = 2t^2 - 8t + 10$$
$$0 = 2t^2 - 8t + 10 - 4$$
$$\frac{0}{2} = \frac{2t^2 - 8t + 6}{2}$$
$$0 = t^2 - 4t + 3$$
$$0 = (t-1)(t-3)$$
$$t - 1 = 0, \quad t - 3 = 0$$
$$t = 1, \quad t = 3$$

Type II – No Equation Given

Example 4:
A rectangular lawn measuring 8 m by 4 m is surrounded by a flower bed of uniform width. The combined area of the lawn and flower bed is 165 m^2. What is the width of the flower bed?



$$l \cdot w = A$$

$$(2x+8)(2x+4)=165$$

$$4x^2+8x+16x+32-165=0$$

$$4x^2+24x-133=0$$

$$x=\frac{-24 \pm \sqrt{(24)^2-4(4)(-133)}}{2(4)}$$

$$x=\frac{-24 \pm \sqrt{2704}}{8}$$

$$x=\frac{-24 \pm 52}{8}$$

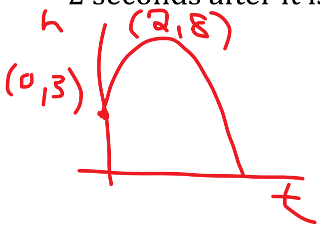
$$x=\frac{-24+52}{8} \quad , \quad x=\frac{-24-52}{8}$$

$$x=\frac{28}{8} \quad , \quad x=\frac{-76}{8}$$

$$\boxed{x=3.5} \quad , \quad \cancel{x=-9.5}$$

The width of the flower bed is 3.5m.

Example 5:
A baseball is thrown from an initial height of 3 m and reaches a maximum height of 8 m, 2 seconds after it is thrown. At what time does the ball hit the ground?



$y = a(x-h)^2 + k$

$3 = a(0-2)^2 + 8$

$3 - 8 = 4a$

$\frac{-5}{4} = \frac{4a}{4}$

$a = -1.25$

$y = -1.25(x-2)^2 + 8$

$y = -1.25(x-2)(x-2) + 8$

$y = -1.25(x^2 - 2x - 2x + 4) + 8$

$y = -1.25(x^2 - 4x + 4) + 8$

$y = -1.25x^2 + 5x - 5 + 8$

$y = -1.25x^2 + 5x + 3$

$0 = -1.25x^2 + 5x + 3$

$1.25x^2 - 5x - 3 = 0$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1.25)(-3)}}{2(1.25)}$

$x = \frac{5 \pm \sqrt{25 + 15}}{2.5}$

$x = \frac{5 \pm \sqrt{40}}{2.5}$

$x = \frac{5 \pm 6.3}{2.5}$

$x = \frac{5 + 6.3}{2.5}, \quad x = \frac{5 - 6.3}{2.5}$

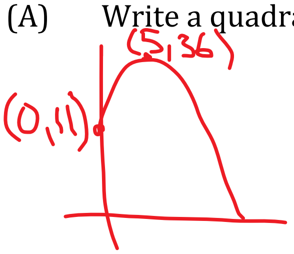
$x = \frac{11.3}{2.5}, \quad x = \frac{-1.3}{2.5}$

$\boxed{x = 4.5}, \quad \cancel{x = -0.52}$

The ball hits the ground at 4.5 seconds.

Example 6:
A ball is thrown from a building at an initial height of 11 metres and reaches a maximum height of 36 metres, 5 seconds after it is thrown.

(A) Write a quadratic equation which models this situation.



$y = a(x-h)^2 + k$

$11 = a(0-5)^2 + 36$

$11 - 36 = 25a$

$\frac{-25}{25} = \frac{25a}{25}$

$a = -1$

$y = -(x-5)^2 + 36$

$y = -(x-5)(x-5) + 36$

$y = -(x^2 - 10x + 25) + 36$

$y = -x^2 + 10x - 25 + 36$

$y = -x^2 + 10x + 11$

(B) Three targets are placed at different locations on the ground. One is at (10, 0), another at (11,0) and a final target is placed at (12, 0). Which target does the ball hit?

$0 = -x^2 + 10x + 11$

$x^2 - 10x - 11 = 0$

$(x+1)(x-11) = 0$

$\cancel{x = -1}, x = 11$

The ball hits the target at (11, 0).

Example 7: 1, 2

Find two consecutive whole numbers such that the sum of their squares is 265.

$x, x+1$

$x^2 + (x+1)^2 = 265$

$x^2 + (x+1)(x+1) = 265$

$x^2 + x^2 + x + x + 1 - 265 = 0$

$\frac{2x^2 + 2x - 264}{2} = \frac{0}{2}$

$x^2 + x - 132 = 0$

$(x-11)(x+12) = 0$

$x = 11, \cancel{x = -12}$

$\boxed{x = 11}, \cancel{x = -12}$

$x = 11$

$x + 1 = 11 + 1 = 12$

The two numbers are 11 and 12.

Example 8:

Find two consecutive odd, natural numbers such that their product is 63.

$x, x+2$

$x(x+2) = 63$

$x^2 + 2x - 63 = 0$

$(x-7)(x+9) = 0$

$x = 7, \cancel{x = -9}$

$x = 7$

$x + 2 = 7 + 2 = 9$

The two numbers are 7 and 9.

Textbook Questions: page 430 - 431 #3, 5, 8, 9