

8.3 Scale Diagrams

In Grade 9, you were introduced to scale factors and scale diagrams and you explored the concepts of enlargements and reductions. You also determined the scale factor given the scaled diagram of two dimensional images, and used a scale factor to create an image from its original figure.

In this unit, we will use scale diagrams involving 2-D shapes before moving on to 3-D objects.

Scale Factor

Scale factor, k , can be calculated as:

$$k = \frac{\text{scaled diagram measurement}}{\text{actual measurement}}$$

Where:

- If $0 < k < 1$, then the new shape is a reduction of the original shape
- If $k > 1$, then the new shape is an enlargement of the original shape

Example 1

A dinosaur model has a scale of 1:12.

- (A) If the head of the dinosaur model is 8 cm in length, how long is the head of the real dinosaur?

logically:

$$8 \times 12 = 96$$

$$k = \frac{\text{scaled}}{\text{actual}}$$

$$\frac{1}{12} = \frac{8}{x}$$

$$x = 8 \cdot 12$$

$$x = 96 \text{ cm}$$

- (B) What is the scale factor?

$$k = \frac{1}{12} = 0.0833$$

(C) What does this value mean?

The model is 0.0833 times the size of the actual.

(D) Will the value of the scale factor result in an enlargement or reduction? Explain.

Since $0 < k < 1$, the scale factor will result in reduction.

(E) How did you determine the length of the head of the dinosaur?

Multiplicated the model measurement by 12.

Example 2:

A circle has been transformed so that its image radius is 14 cm. If the scale factor is 0.4, what is the radius of the original circle?

$$k = \frac{\text{scale}}{\text{actual}}$$
$$\frac{0.4}{1} = \frac{14}{x}$$
$$\rightarrow \frac{0.4x}{0.4} = \frac{14}{0.4}$$
$$x = 35$$

The original circle has a radius of 35 cm.

Example 3:

Determine the scale factor from the diagram:

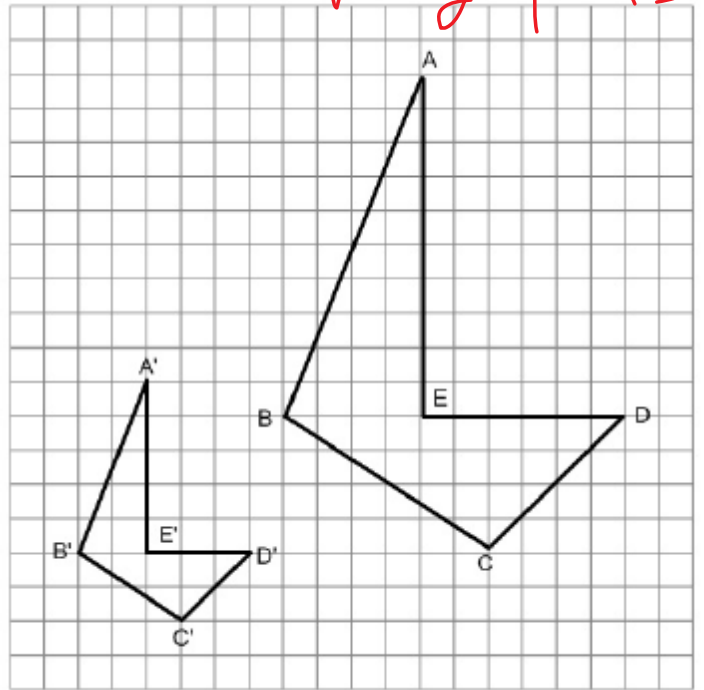
** use corresponding parts.*Scale

$$AE' : AE$$

$$5 : 10$$

$$1 : 2$$

$$\text{Scale factor} : \frac{1}{2} = 0.5$$

**Example 4:** ΔABC is similar to ΔSTU . If the sides of ΔABC measure 5.2, 3.8, and 6.4 respectively, and the scale factor is 2:3, what is the perimeter of ΔSTU ?

$$\Delta ABC : \Delta STU$$

$$2 : 3$$

$$k = \frac{\text{scale}}{\text{actual}}$$

$$\frac{2}{3} = \frac{5.2}{x}$$

$$2x = 3(5.2)$$

$$\frac{2x}{2} = \frac{15.6}{2}$$

$$x = 7.8$$

$$\frac{2}{3} = \frac{3.8}{x}$$

$$2x = 3(3.8)$$

$$\frac{2x}{2} = \frac{11.4}{2}$$

$$x = 5.7$$

$$\frac{2}{3} = \frac{6.4}{x}$$

$$2x = 3(6.4)$$

$$\frac{2x}{2} = \frac{19.2}{2}$$

$$x = 9.6$$

Perimeter:

$$7.8 + 5.7 + 9.6$$

$$= 23.1 \text{ units}$$

$$\text{or perimeter of } \Delta ABC = 5.2 + 3.8 + 6.4 = 15.4$$

$$\frac{2}{3} = \frac{15.4}{x} \rightarrow \frac{2x}{2} = \frac{46.2}{2}$$

$$2x = 3(15.4) \rightarrow x = 23.1 \text{ units}$$

Word Problems Involving Scale Diagrams

Examples of real-world applications are maps, sewing patterns, car models, and construction blueprints.

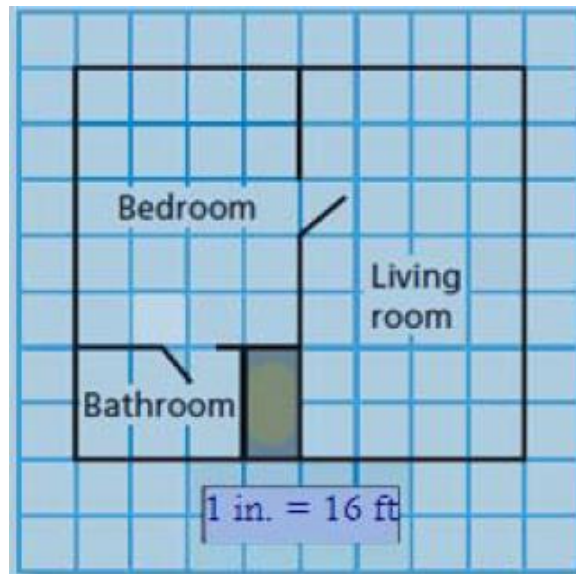
Floorplans for areas such as kitchens or bedrooms, are a great example of real world applications of scale diagrams. Architects and designers have to consider what to include in their drawings, the scale they will use and the measurements needed. It is important for the measurements to be realistic. If a blueprint is using a scale of 1 in. = 1 ft, for example, the doorway should be at least 3 in. wide on the drawing so that in real-life it would be 3 ft wide.

Things that must be considered are:

- dimensions need to be recorded on the diagram
- the doors, windows, closets and walls must be represented
- the scale must be indicated
- a key should be included to identify the symbols used in the drawing
- furniture will need to be included

Example 5:

In the following blueprint, each square has a side length of $\frac{1}{4}$ in.



Answer the following questions:

(A) Ceramic tile costs \$5 per square foot. How much would it cost to tile the bathroom?

Bathroom: 2 x 3

$$\text{width: } \frac{1}{4} \times 2 = 0.5 \text{ in} \times 16 \text{ ft/in} = 8 \text{ ft}$$

$$\text{length: } \frac{1}{4} \times 3 = 0.75 \text{ in} \times 16 \text{ ft/in} = 12 \text{ ft}$$

$$\text{Area} = 8 \text{ ft} \times 12 \text{ ft} = 96 \text{ ft}^2$$

Cost:

$$96 \text{ ft}^2 \times \frac{\$5}{\text{ft}^2}$$

$$= \$480$$

$$3 \begin{array}{|c|} \hline 1 \text{ yd}^2 \\ \hline \end{array} \begin{array}{l} (3\text{ft})^2 \\ = 9\text{ft}^2 \end{array}$$

- (B) Carpet costs \$18 per square yard. How much would it cost to carpet the bedroom and living room?

Bedroom: 5×4

$$\text{width} = \frac{1}{4} \cdot 5 = 1.25 \text{ in} \times 16 \text{ ft/in} = 20 \text{ ft}$$

$$\text{length} = \frac{1}{4} \cdot 4 = 1 \text{ in} \times 16 \text{ ft/in} = 16 \text{ ft}$$

$$\text{Area} = 20 \text{ ft} \times 16 \text{ ft} = 320 \text{ ft}^2$$

Living Room: 4×7

$$\text{width} = \frac{1}{4} \cdot 4 = 1 \text{ in} \times 16 \text{ ft/in} = 16 \text{ ft}$$

$$\text{length} = \frac{1}{4} \cdot 7 = 1.75 \text{ in} \times 16 \text{ ft/in} = 28 \text{ ft}$$

$$\text{Area} = 16 \text{ ft} \times 28 \text{ ft} = 448 \text{ ft}^2$$

Cost:

$$320 \text{ ft}^2 \times \frac{1 \text{ yd}^2}{9 \text{ ft}^2}$$

$$= 35.6 \text{ yd}^2$$

$$35.6 \text{ yd}^2 \times \$18/\text{yd}^2 = \$640.80$$

Cost:

$$448 \text{ ft}^2 \times \frac{1 \text{ yd}^2}{9 \text{ ft}^2}$$

$$= 49.8 \text{ yd}^2$$

$$49.8 \text{ yd}^2 \times \$18/\text{yd}^2 = \$896.00$$

$$\text{Total cost: } \$640.80 + \$896.00 = \$1536.80$$

- (C) Which has a higher unit cost, the tile or the carpet? Explain.

$$\text{Tile: } \$5/\text{ft}^2$$

$$\text{Carpet: } \$18/\text{yd}^2$$

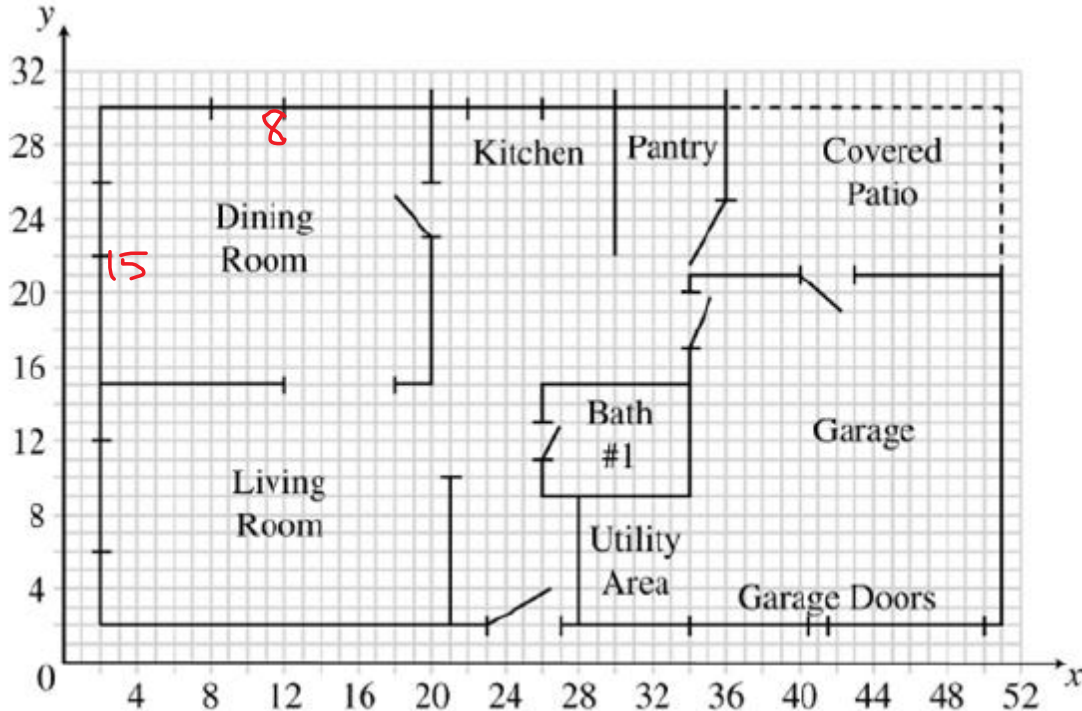
$$3 \begin{array}{|c|} \hline 1 \text{ yd}^2 \\ \hline \end{array} \begin{array}{l} 3 \times 3 \\ = 9 \text{ ft}^2 \end{array}$$

$$\frac{\$18}{\text{yd}^2} \times \frac{1 \text{ yd}^2}{9 \text{ ft}^2} = \$2/\text{ft}^2$$

Tiles cost more.

Example 6:

Mrs. Lewis intends to have the dining room redone with wood flooring that comes in 1 foot wide squares. The scale is 1 unit = 1ft. A 12 pack of squares sells for \$38.40 and single squares can be purchased for \$3.70 each. Determine the cost, in dollars, for Mrs. Lewis to buy just enough squares to cover her dining room floor?



$$15 \text{ ft} \times 18 \text{ ft} = 270 \text{ ft}^2$$

$$\frac{270 \text{ ft}^2}{12 \text{ ft}^2} = 22.5 \sim 22 \text{ boxes} \times 12 = 264$$

$$270 - 264 = 6$$

$$22 \times \$38.40 = \$844.80$$

$$6 \times \$3.70 = \$22.20$$

$$\underline{\$867.00}$$