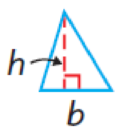
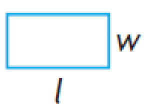
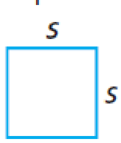
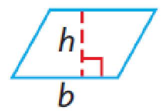
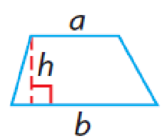
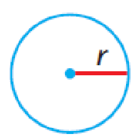
Math 2201 **Date:**___________________

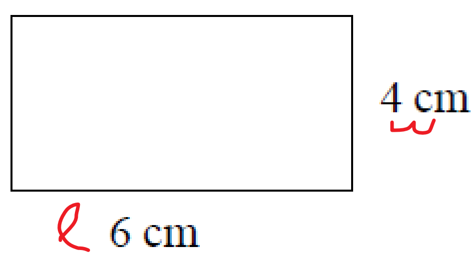
8.4 Scale Factors and Areas of 2-D Shapes

In this section we will focus on the relationship between scale factor and area of similar 2-D shapes. Later in this unit, we will solve problems that involve scale factor, surface area and volume of 3-D objects.

How is area affected when the lengths of shapes are enlarged or reduced by a particular scale factor? Before we let's review the area formulae for some common shapes:

triangle 	$A = \frac{1}{2}bh$
rectangle 	$A = lw$
square 	$A = s^2$
parallelogram 	$A = bh$
trapezoid 	$A = \frac{1}{2}h(a + b)$
circle 	$A = \pi r^2$

Example 1:
Consider the following rectangle:



(A) What is the area of the rectangle?

$A = l \cdot w$

$A = (6cm)(4cm)$

$A = 24\ cm^2$

(B) Increasing by a scale factor of 2, what are the new dimensions and area?

$l = 6cm \times 2 = 12cm$

$w = 4cm \times 2 = 8cm$

$A = lw$

$A = (12cm)(8cm)$

$A = 96cm^2$

(C) Increasing by a scale factor of 3, what are the new dimensions and area?

$l = 6cm \times 3 = 18cm$

$w = 4cm \times 3 = 12cm$

$A = (18cm)(12cm)$

$A = 216cm^2$

(D) Increasing by a scale factor of $\frac{1}{2}$, what are the new dimensions and area?

$l = 6cm \times \frac{1}{2} = 3cm$

$w = 4cm \times \frac{1}{2} = 2cm$

$A = (3cm)(2cm)$

$A = 6cm^2$

(E) What do you notice?

Original area: $24cm^2$

Scale factor 2: $96cm^2$

Scale factor 3: $216cm^2$

Scale factor $\frac{1}{2}$: $6cm^2$

$\frac{96cm^2}{24cm^2} = 4 = 2^2$

$\frac{216cm^2}{24cm^2} = 9 = 3^2$

$\frac{6cm^2}{24cm^2} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$

You should observe that the resulting areas are not directly proportional to the lengths. When you double the sides of a rectangle, for example, the area does not just double, it quadruples.

It is important to recognize that the scale factor is applied to each dimension of the 2-D shape. As a result, the area will either increase or decrease by a factor of k^2. Therefore:

$$k^2 = \frac{\text{area of 2-D Shape}}{\text{area of original shape}}$$

Example 2:
A 8 in by 12 in picture frame has dimensions that have been tripled. What is the area of the new frame?

Original area: 8in × 12in
= 96in²

$$k^2 = \frac{\text{scale area}}{\text{original area}}$$

$$(3)^2 = \frac{x}{96in^2}$$

$$\frac{9}{1} = \frac{x}{96}$$

$$x = 9 \times 96in^2$$

$$x = 864in^2$$

Example 3:
Chad and Charlene painted a mural on the wall, measuring 12 ft by 8 ft using an overhead projector. If the original sketch had an area of 216 in^2, what is the scale factor?

$$12\cancel{ft} \times \frac{12in}{1\cancel{ft}} = 144in$$

$$8\cancel{ft} \times \frac{12in}{1\cancel{ft}} = 96in$$

$$A_{mural} = (144in)(96in)$$
$$= 13824in^2$$

$$k^2 = \frac{\text{scale area}}{\text{original area}}$$

$$k^2 = \frac{13824\cancel{in^2}}{216\cancel{in^2}}$$

$$\sqrt{k^2} = \sqrt{64}$$

$$k = 8$$

Scale factor: 8

Textbook Questions: page 479 - 481 #1, 3, 4, 5, 6, 8, 9, 11, 12, 13