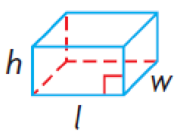
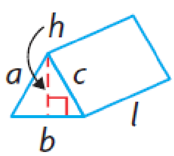
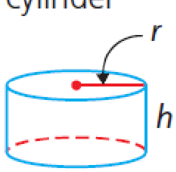
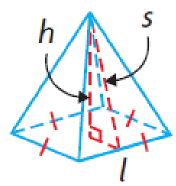
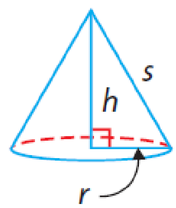
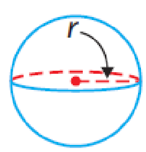
Math 2201 **Date:**__________________

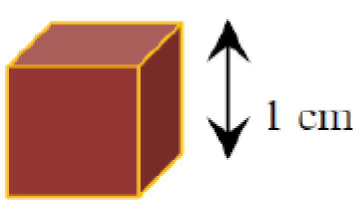
8.6 Similar Factors and 3-D Objects

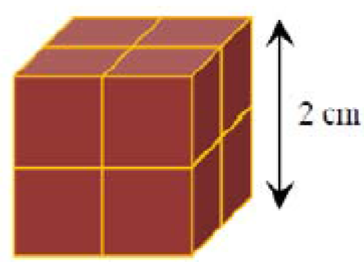
We will now investigate the relationship between the scale factor and the surface area of two similar 3-D objects, in addition to the relationship between the scale factor and the volume of two similar 3-D objects. First we'll review the formulae for 3-D objects from Math 1201.

rectangular prism 	$SA = 2(lw + lh + wh)$ $V = lwh$
right triangular prism 	$SA = bh + l(a + b + c)$ $V = \frac{1}{2}bhl$
right cylinder 	$SA = 2\pi r^2 + 2\pi rh$ $V = \pi r^2 h$

right pyramid 	$SA = l^2 + 2ls$ $V = \frac{1}{3}l^2h$
right cone 	$SA = \pi r^2 + \pi rs$ $V = \frac{1}{3}\pi r^2 h$
sphere 	$SA = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

What will happen to the surface area and the volume of a cube if every edge of the original cube doubles? Triples? To answer this question, lets consider the effects of enlarging a cube that measures 1 cm by 1 cm by 1cm.





The volume of this cube is:

$$V = s \cdot s \cdot s = s^3$$

$$V = 1cm \times 1cm \times 1cm = 1cm^3$$

The surface area is:

$$SA = 6 \times s \cdot s = 6s^2$$

$$SA = 6 \times (1cm)^2 = 6cm^2$$

What will happen if every edge of the cube is doubled?

Cube: $2cm \times 2cm \times 2cm$

$$V = 2cm \times 2cm \times 2cm = 8cm^3$$

$$SA = 6 \times (2cm)^2 = 24cm^2$$

What will happen if every edge of the cube is tripled?

Cube: $3cm \times 3cm \times 3cm$

$$V = (3cm)^3 = 27cm^3$$

$$SA = 6 \times (3cm)^2 = 54cm^2$$

The following chart summarizes the effects of scale factors 1 – 4 on a $1 \times 1 \times 1$ cube.

Scale factor	Length	Width	Height	Surface Area	Volume
1	1	1	1	6	1
2	2	2	2	24 $4 = 2^2$ (increases by a factor of 4)	8 $8 = 2^3$ (increases by a factor of 8)
3	3	3	3	54 $9 = 3^2$ (increases by a factor of 9)	27 $27 = 3^3$ (increases by a factor of 27)
4	4	4	4	96	64

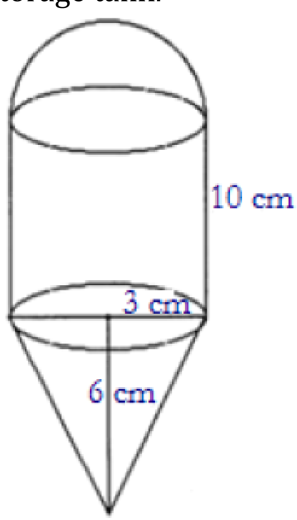
In summary, when the dimensions of similar 3-D objects are related by a scale factor k, their surface areas are related by k^2 and their volumes are related by k^3.

8.4 $\longrightarrow$ $$k^2 = \frac{\text{area of 2-D Shape}}{\text{area of original shape}}$$

New $\longrightarrow$ $$k^3 = \frac{\text{volume of 3-D Shape}}{\text{volume of original shape}}$$

In Mathematics 1201, you solved problems that involved the surface area and volume of 3-D objects.

You can use the dimensions of a scale diagram of a 3-D object, as well as the scale factor, to determine the surface area and volume of the enlarged/reduced object. Consider the scaled down diagram of the following storage tank:



1 cm = 1.5 m

Find the total surface area of the original tank.

Find the total volume of the original tank.

Was it necessary to determine the dimensions of the original drawing to answer the above questions?

How would you determine the surface area and volume of the original if the scale diagram was not given?

Example 1:
The surface area of a cone is 36 ft^2. What is the surface area of its image if a scale factor of 1:4 is applied?

$$k^2 = \frac{\text{scale area}}{\text{original area}}$$

$$\left(\frac{1}{4}\right)^2 = \frac{x}{36ft^2}$$

$$\frac{1}{16} = \frac{x}{36ft^2}$$

$$16x = 36ft^2$$

$$\frac{16x}{16} = \frac{36ft^2}{16}$$

$$x = 2.25ft^2$$

Example 2:
Find the volume of a cylinder if its image has a volume of 450 cm^3 and a scale factor of 2:3. $\frac{2}{3}$
Round your answer to the nearest cubic centimetre.

$$k^3 = \frac{\text{Scale volume}}{\text{original volume}}$$

$$\left(\frac{2}{3}\right)^3 = \frac{450\,cm^3}{x}$$

$$\frac{8}{27} = \frac{450\,cm^3}{x}$$

$$8x = 27 \cdot 450\,cm^3$$

$$8x = 12150\,cm^3$$

$$\frac{8x}{8} = \frac{12150\,cm^3}{8}$$

$$x = 1518.75\,cm^3 \sim 1519\,cm^3$$

Example 3:
What is the scale factor of the following pairs of similar spheres?

(A) Volume of the original is 450 mm^3 and its image is 1518.75 mm^3.

$$k^3 = \frac{\text{scale volume}}{\text{original volume}}$$

$$k^3 = \frac{1518.75}{450}$$

$$k^3 = 3.375$$

$$\sqrt[3]{k^3} = \sqrt[3]{3.375}$$

$$k = 1.5 \text{ or } \frac{3}{2} \text{ or } 3:2$$

(B) Surface area of the original is 248 in^2 and its image is 126.5306 in^2.

$$k^2 = \frac{\text{Scale area}}{\text{original area}}$$

$$k^2 = \frac{126.5306}{248}$$

$$k^2 = 0.5102$$

$$\sqrt{k^2} = \sqrt{0.5102}$$

$$k = 0.714$$

Example 4:
An oil tank has capacity of 32 m^3. A similar oil tank has dimensions that are larger by a scale factor of 3. What is the capacity of the larger tank?

$$k^3 = \frac{\text{Scale volume}}{\text{original volume}}$$

$$(3)^3 = \frac{x}{32m^3}$$

$$\frac{27}{1} = \frac{x}{32m^3}$$

$$x = 27(32m^3)$$

$$x = 864m^3$$

Example 5:
A cube has a base width of 10 cm and has a volume of 1800 m^3. It is enlarged until the base becomes 20 cm. Determine the volume of the enlarged cube.

10:20 -> 1:2

Scale Factor: 2

$$k^3 = \frac{\text{Scale volume}}{\text{original volume}}$$

$$2^3 = \frac{x}{1800m^3}$$

$$\frac{8}{1} = \frac{x}{1800m^3}$$

$$x = 8(1800m^3)$$

$$x = 14400m^3$$

Textbook Questions: page 500 - 503 #1, 2, 3, 4, 6, 8, 13, 15